

Modeling of Transport and Barrier Formation in Toroidal Plasmas

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- **Transport Modeling: CDBM Transport Model**
- **Modeling of Internal Transport Barrier**
- **Modeling of Edge Transport Barrier**
- **Modeling of Transport in Helical Plasmas**
- **Summary**

Transport Modeling

- **Recent progress of plasma operation and diagnostic technique in torus plasma experiments has revealed various improved confinement mode.**
- **Reliable transport model should reproduce experimental observations:**
 - **Global confinement time scaling**
 - **Nonlinear relation between heat flux and temperature gradient**
 - **Radial profile data base**
 - **Fluctuation type and level**
 - **Formation of transport barrier**

Modeling of Transport Barrier Formation (1)

- **Mechanism of Turbulence Suppression**

- **$E \times B$ rotation shear**

- E_r generation through radial force balance

$$E_r = -u_\theta B_\phi + u_\phi B_\theta + \frac{1}{e_s} \frac{d}{dr} P$$

- $E \times B$ shearing rate (**Hahm and Burrell**)

$$\omega_E = \frac{RB_\theta}{B} \frac{d}{dr} \left(\frac{E_r}{RB_\theta} \right) \frac{k_\theta}{k_r}$$

- **Criteria for suppression**

$$\omega_E > \gamma_{\text{Lin}}$$

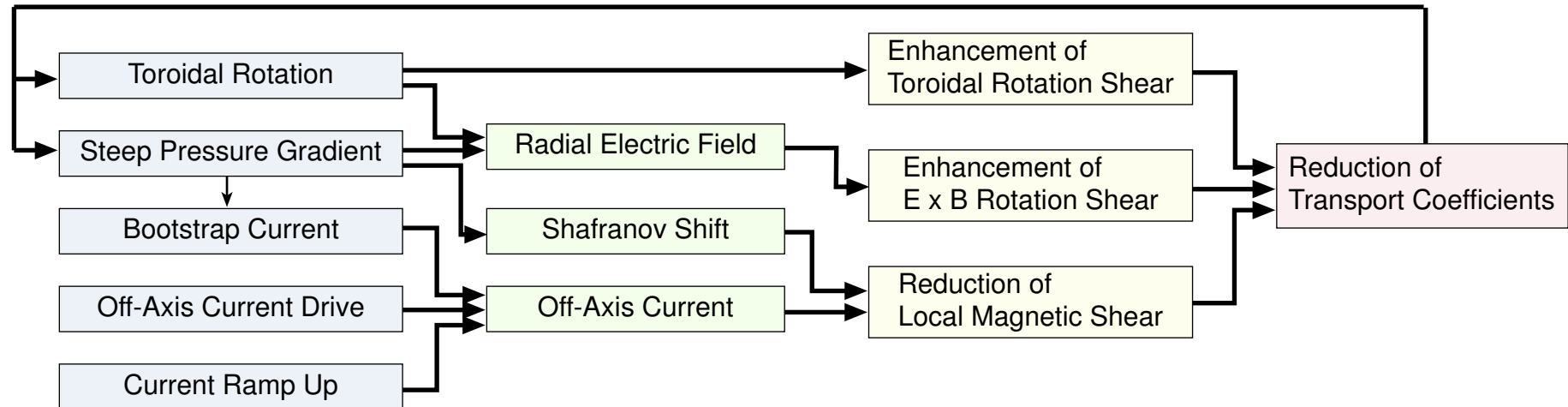
- **Magnetic shear s and normalized pressure gradient α**

- Marginal stability of self-sustained turbulence (CDBM model)

- Thermal diffusivity χ as a function of s and α

Modeling of Transport Barrier Formation (1)

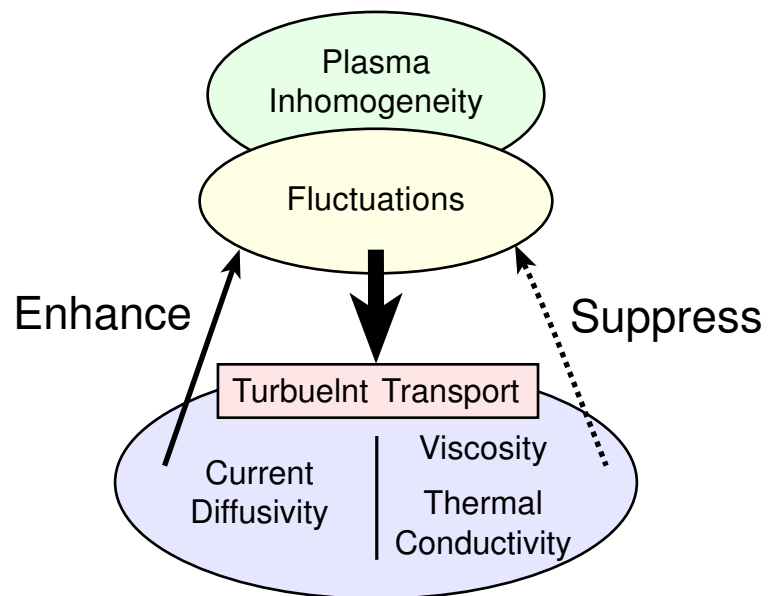
- **Positive feedback of pressure gradient increase**
 - In both models, χ depends on the pressure gradient. χ .



CDBM Transport Model

- **Self-Sustained Turbulence Theory**

- Turbulence sustained by the enhancement of transport coefficients due to the turbulence itself



- **Current-Diffusive Ballooning Mode**

- **Ballooning mode:** MHD mode localized in bad curvature region
 - **Ideal ballooning mode** (second stability)
 - **Resistive ballooning mode** (plasma near edge)
 - **Current-diffusive ballooning mode** (core plasma)

CDBM Turbulence Model

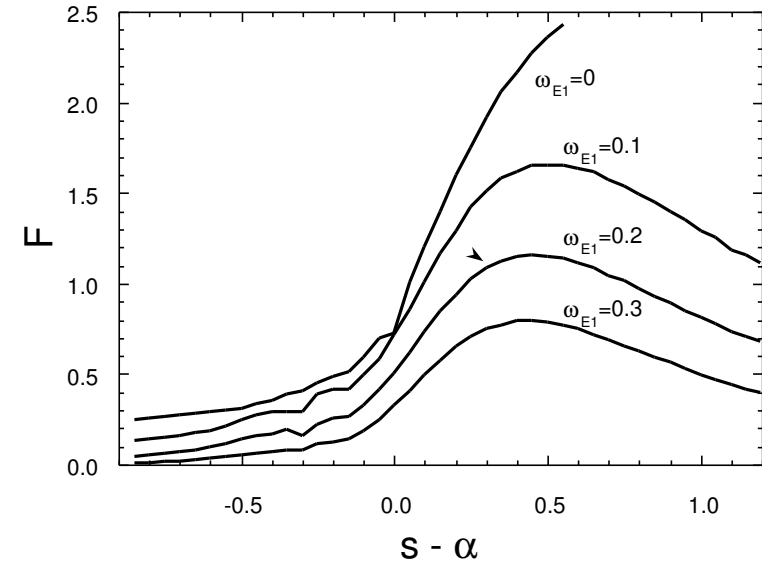
- **Marginal Stability Condition** ($\gamma = 0$)

$$\chi_{\text{TB}} = F(s, \alpha, \kappa, \omega_{E1}) \alpha^{3/2} \frac{c^2}{\omega_{pe}^2} \frac{v_A}{qR}$$

Magnetic shear	$s \equiv \frac{r}{q} \frac{dq}{dr}$
Pressure gradient	$\alpha \equiv -q^2 R \frac{d\beta}{dr}$
Magnetic curvature	$\kappa \equiv -\frac{r}{R} \left(1 - \frac{1}{q^2}\right)$
$E \times B$ rotation shear	$\omega_{E1} \equiv \frac{r^2}{sv_A} \frac{d}{dr} \frac{E}{rB}$

- **Weak and negative magnetic shear,**
Shafranov shift and
 $E \times B$ rotation shear
reduce thermal diffusivity.

$s - \alpha$ dependence of
 $F(s, \alpha, \kappa, \omega_{E1})$



Fitting Formula

$$F = \begin{cases} \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1}{\sqrt{2(1 - 2s')(1 - 2s' + 3s'^2)}} & \text{for } s' = s - \alpha < 0 \\ \frac{1}{1 + G_1 \omega_{E1}^2} \frac{1 + 9\sqrt{2}s'^{5/2}}{\sqrt{2(1 - 2s' + 3s'^2 + 2s'^3)}} & \text{for } s' = s - \alpha > 0 \end{cases}$$

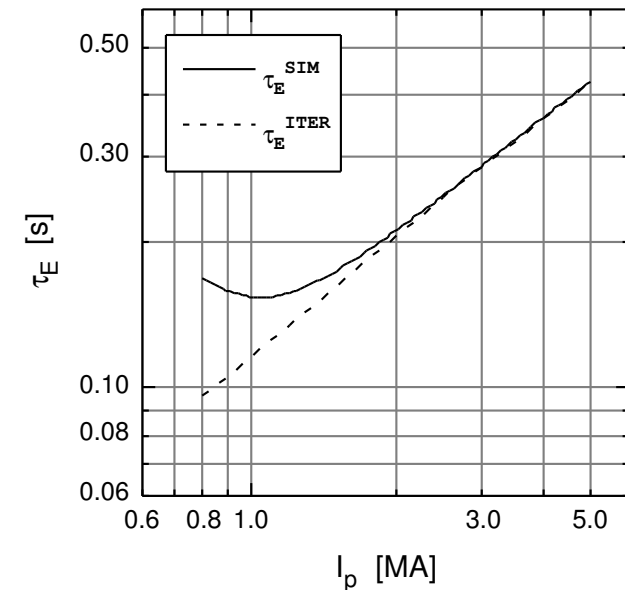
Simulation of L-mode and Improved Confinement

- **Zero-Dimensional Analysis** with fixed $F(s, \alpha, \kappa)$ (gyro-Bohm scaling)

$$\tau_E \propto F^{-0.4} A_i^{0.2} I_p^{0.8} n^{0.6} B^0 a^{1.0} R_0^{1.2} P^{-0.6}$$

- **Deviation from L-mode scaling at low I_p**

- Increase of P_{in}
- Increase of pressure gradient
→ Increase of α
- Increase of bootstrap current
→ Decrease of s
- Reduction of χ

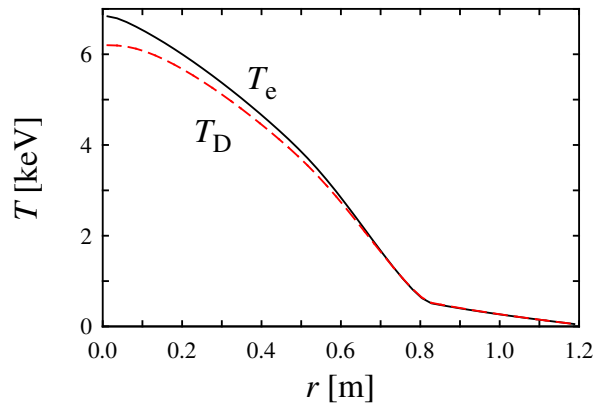


- **Various improved core confinement modes have been reproduced**
 - **High- β_p** mode
 - **PEP** (Pellet Enhanced Performance) mode
 - **LHEP** (Lower Hybrid Enhanced Performance) mode
 - **Negative Magnetic Shear** mode

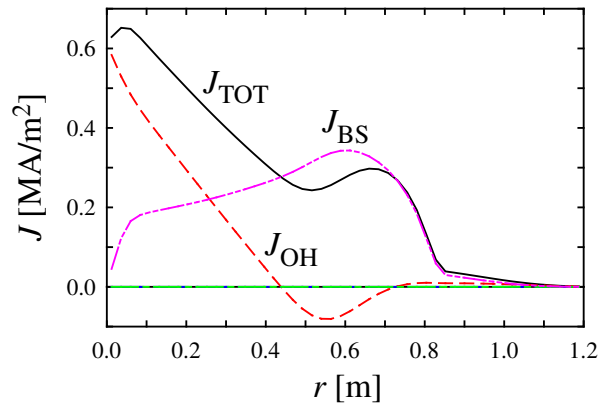
High β_p mode

- $R = 3 \text{ m}$, $a = 1.2 \text{ m}$, $\kappa = 1.5$, $B_0 = 3 \text{ T}$, $I_p = 1 \text{ MA}$
- one second after heating power of $P_H = 20 \text{ MW}$ was switched on

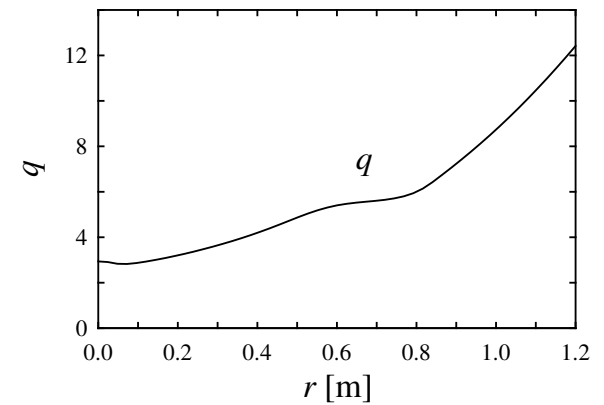
Temperater profile



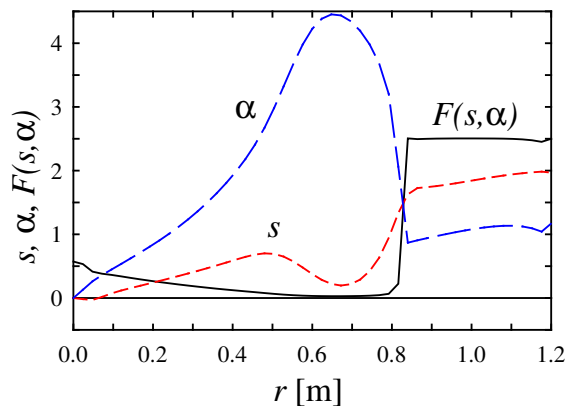
Current profile



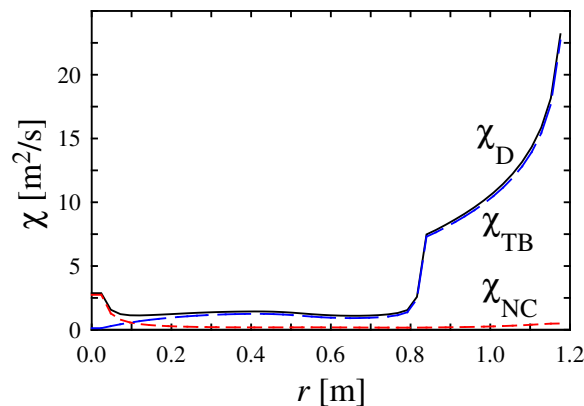
Safety factor



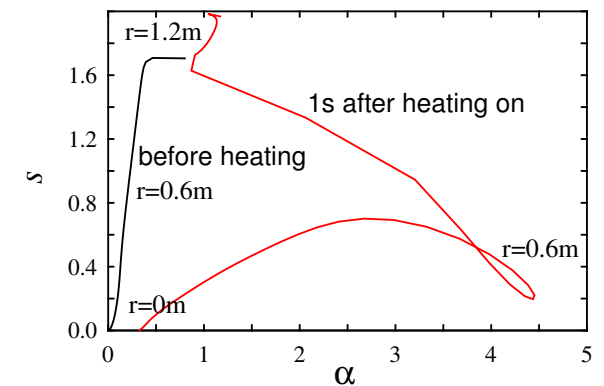
Shear and pressure



Thermal diffusivity



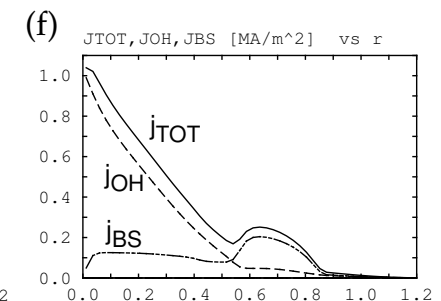
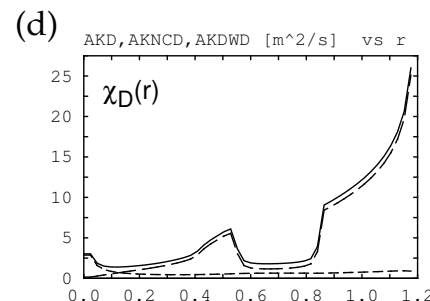
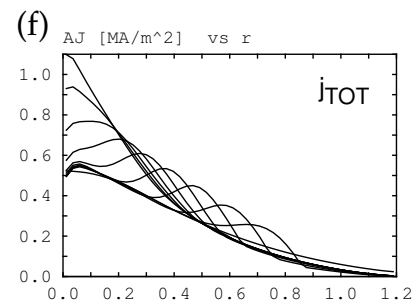
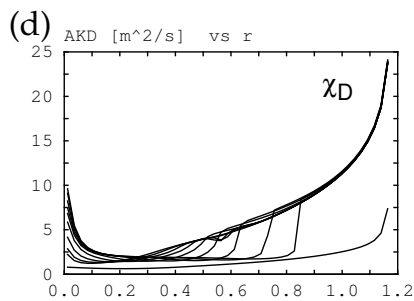
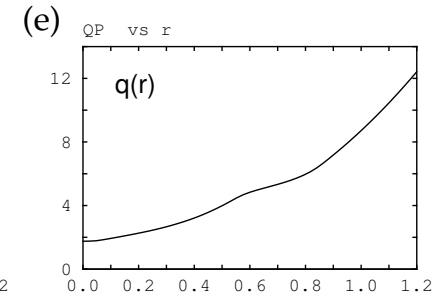
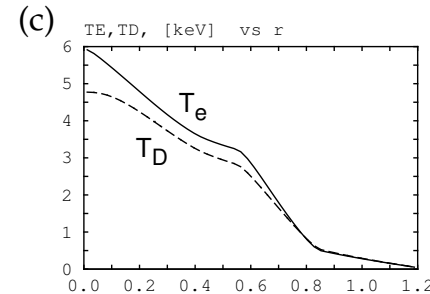
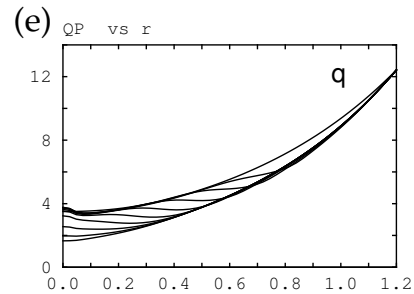
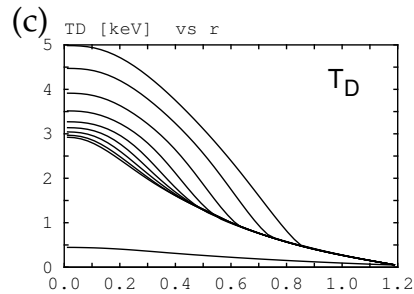
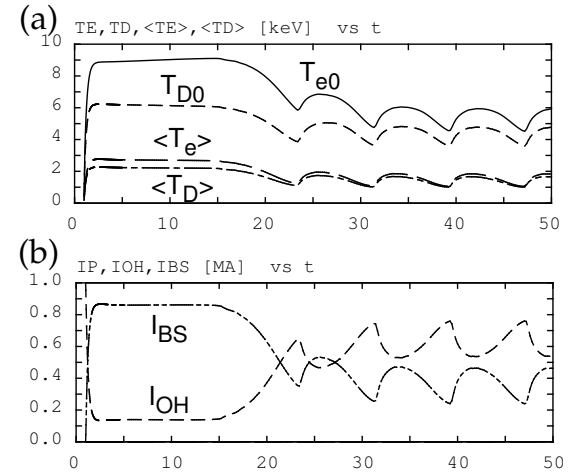
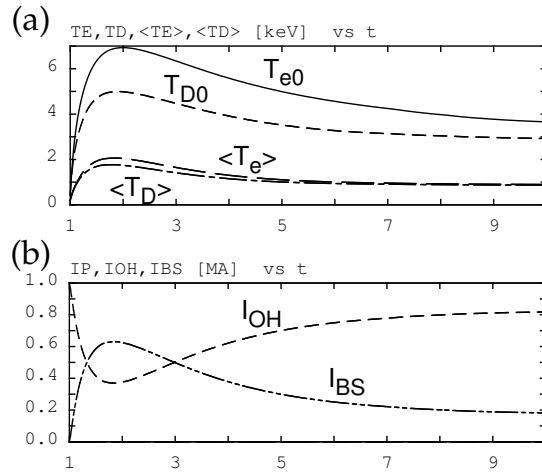
s - alpha diagram



Time Evolution of High β_p mode

• $P_H = 20 \text{ MW}$

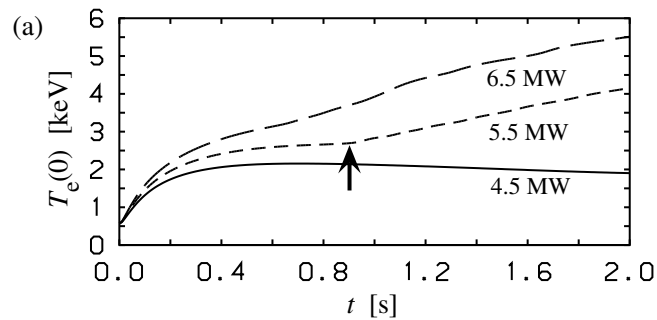
• $P_H = 24.2 \text{ MW}$



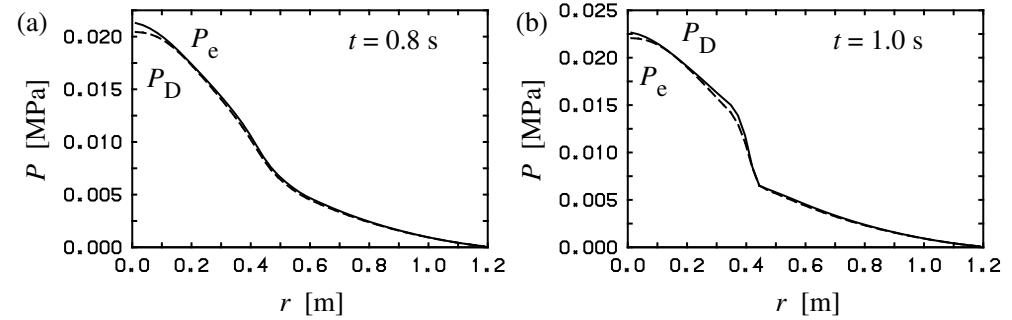
Effect of $E \times B$ Rotation Shear

- **Reduction of transport** due to small s and large α : $F(s, \alpha, \kappa)$
 - \Rightarrow **Rapid increase of rotation shear**: $1/[1 + G(s, \alpha)\omega_{E1}^2]$
 - \Rightarrow **Transition to enhanced ITB**

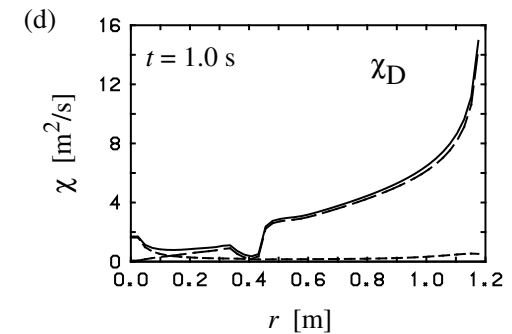
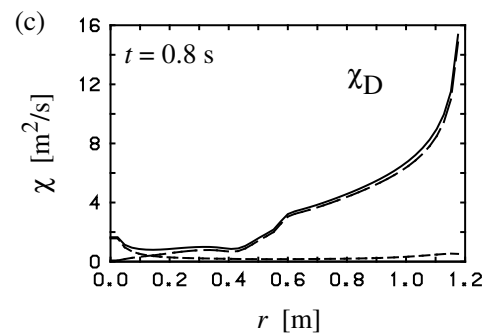
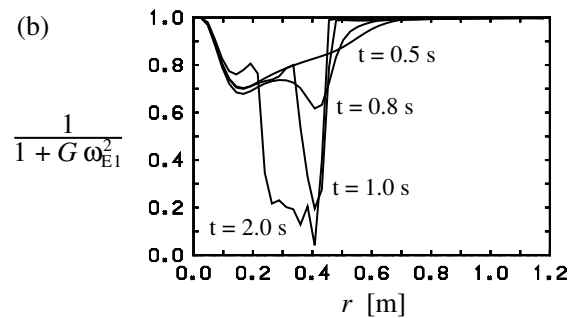
Time evolution of $T_e(0)$



Pressure Profile Before and After Transition

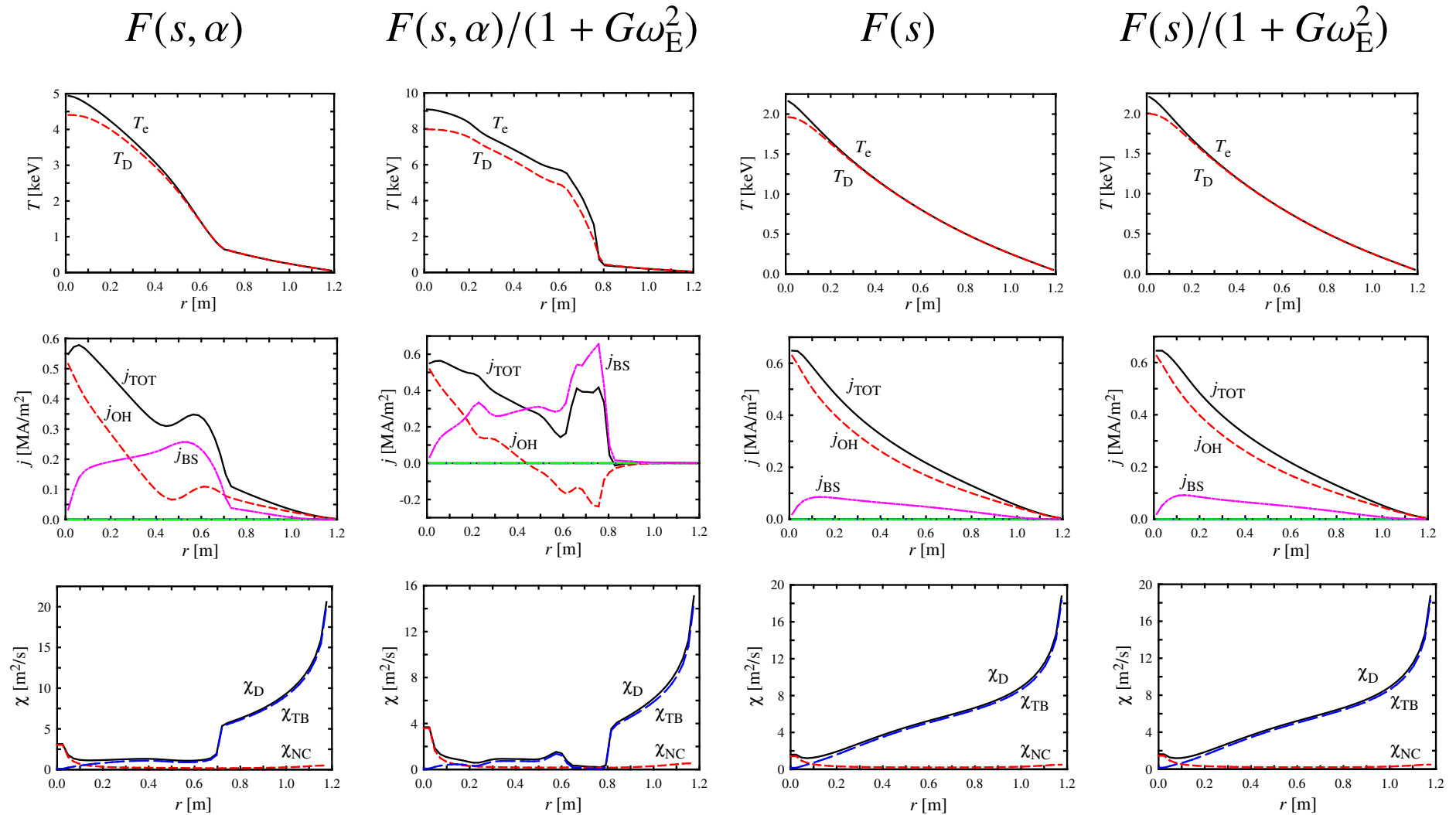


Rapid Change of Rotation Shear Thermal Diffusivity Before and After Transition



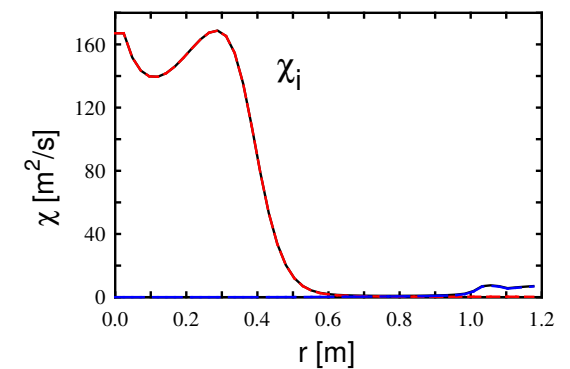
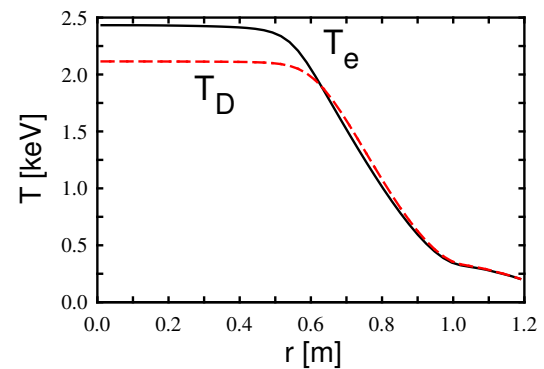
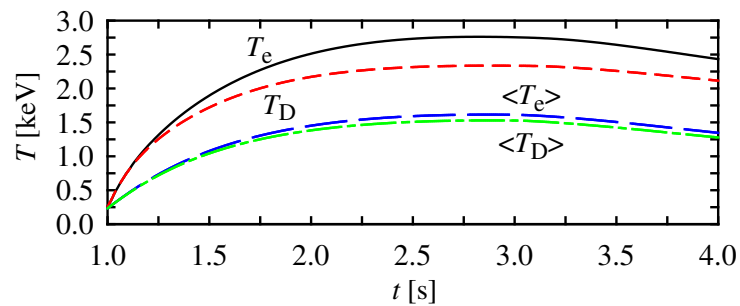
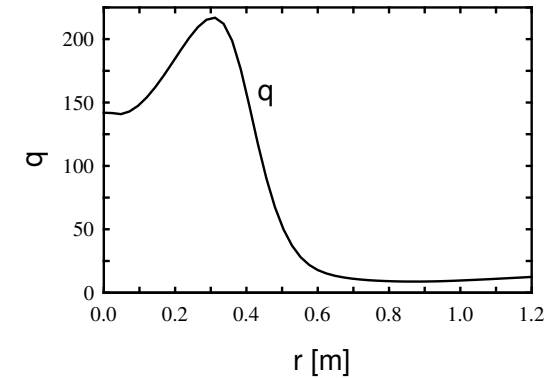
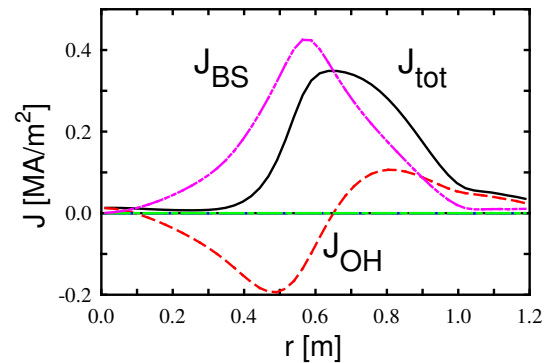
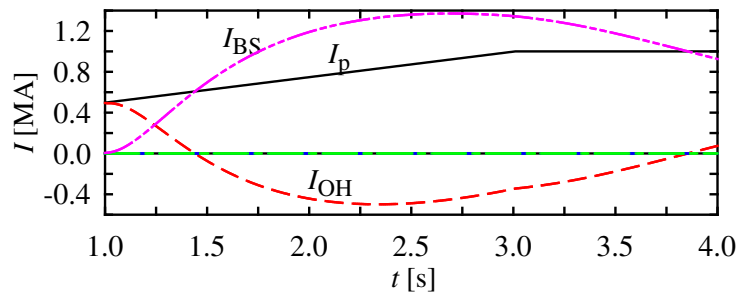
Shape of Transport Barrier

- Dependence on the model



Simulation of Current Hole

- **Current ramp up:** $I_p = 0.5 \rightarrow 1.0$ MA
- **Moderate heating:** $P_H = 5$ MW
- **Current hole** is formed.
- **The formation is sensitive to the edge temperature.**



Modeling of ETB Formation

- **Transport Simulation including Core and SOL Plasmas**
- **Role of Separatrix**
 - Closed magnetic surface \iff Open magnetic field line
 - Difference of dominant transport process
- **Radial Electric Field**
 - Poloidal rotation, Toroidal rotation
 - Polarization current
 - Poisson equation
- **Atomic Processes**
 - Ionization, Charge exchange, Recycling

Transport Model

- **1D Transport code (TASK/TX)** *Ref. Fukuyama et al.*
- **Two fluid equation for electrons and ions**
 - Flux surface average
 - Coupled with Maxwell equation
 - Neutral diffusion equation
- **Neoclassical transport**
 - Included as a poloidal viscosity term
 - Diffusion, resistivity, bootstrap current, Ware pinch
- **Anomalous transport**
 - Current diffusive ballooning mode
 - Ambipolar diffusion through poloidal momentum transfer
 - Perpendicular viscosity

Model Equation (1)

- Fluid equations (electrons and ions)**

$$\frac{\partial n_s}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r n_s u_{sr}) + S_s$$

$$\frac{\partial}{\partial t} (m_s n_s u_{sr}) = -\frac{1}{r} \frac{\partial}{\partial r} (r m_s n_s u_{sr}^2) + \frac{1}{r} m_s n_s u_{s\theta}^2 + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta) - \frac{\partial}{\partial r} n_s T_s$$

$$\frac{\partial}{\partial t} (m_s n_s u_{s\theta}) = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 m_s n_s u_{sr} u_{s\theta}) + e_s n_s (E_\theta - u_{sr} B_\phi) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^3 n_s m_s \mu_s \frac{\partial}{\partial r} \frac{u_{s\theta}}{r} \right)$$

$$+ F_{s\theta}^{\text{NC}} + F_{s\theta}^{\text{C}} + F_{s\theta}^{\text{W}} + F_{s\theta}^{\text{X}} + F_{s\theta}^{\text{L}}$$

$$\frac{\partial}{\partial t} (m_s n_s u_{s\phi}) = -\frac{1}{r} \frac{\partial}{\partial r} (r m_s n_s u_{sr} u_{s\phi}) + e_s n_s (E_\phi + u_{sr} B_\theta) + \frac{1}{r} \frac{\partial}{\partial r} \left(r n_s m_s \mu_s \frac{\partial}{\partial r} u_{s\phi} \right)$$

$$+ F_{s\phi}^{\text{C}} + F_{s\phi}^{\text{W}} + F_{s\phi}^{\text{X}} + F_{s\phi}^{\text{L}}$$

$$\frac{\partial}{\partial t} \frac{3}{2} n_s T_s = -\frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{5}{2} u_{sr} n_s T_s - n_s \chi_s \frac{\partial}{\partial r} T_e \right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})$$

$$+ P_s^{\text{C}} + P_s^{\text{L}} + P_s^{\text{H}}$$

Model Equation (2)

- **Neutral Transport**

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} \left(-r D_0 \frac{\partial n_0}{\partial r} \right) + S_0$$

- **Maxwell equations**

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) = \frac{1}{\epsilon_0} \sum_s e_s n_s$$

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \quad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi)$$

$$\frac{1}{c^2} \frac{\partial E_\theta}{\partial t} = -\frac{\partial}{\partial r} B_\phi - \mu_0 \sum_s n_s e_s u_{s\theta}, \quad \frac{1}{c^2} \frac{\partial E_\phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) - \mu_0 \sum_s n_s e_s u_{s\phi}$$

Transport Model (1)

- **Neoclassical transport**

- **Viscosity force arises when plasma rotates in the poloidal direction.**
- **Banana-Plateau regime**

$$F_{s\theta}^{\text{NC}} = - \sqrt{\pi} q^2 n_s m_s \frac{v_{Ts}}{qR} \frac{v_s^*}{1 + \nu_s^*} u_{s\theta}$$
$$\nu_s^* \equiv \frac{\nu_s q R}{\epsilon^{3/2} v_{Ts}}$$

- **This poloidal viscosity force induces**

- **Neoclassical radial diffusion**
- **Neoclassical resistivity**
- **Bootstrap current**
- **Ware pinch**

Transport Model (2)

- **Turbulent Diffusion**

- Poloidal momentum exchange between electron and ion through the turbulent electric field
- Ambipolar flux (electron flux = ion flux)

$$F_{i\theta}^W = - F_{e\theta}^W$$
$$= - ZeB_\phi n_i D_i \left[-\frac{1}{n_i} \frac{dn_i}{dr} + \frac{Ze}{T_i} E_r - \left\langle \frac{\omega}{m} \right\rangle \frac{ZeB_\phi}{T_i} - \left(\frac{\mu_i}{D_i} - \frac{1}{2} \right) \frac{1}{T_i} \frac{dT_i}{dr} \right]$$

- **Perpendicular viscosity**

- Non-ambipolar flux (electron flux \neq ion flux): $\mu_s = \text{constant} \times D$

- **Diffusion coefficient** (proportional to $|E|^2$)

- Current-diffusive ballooning mode turbulence model

Modeling of Scrape-Off Layer Plasma

- **Particle, momentum and heat losses along the field line**

- **Decay time**

$$v_L = \begin{cases} 0 & (0 < r < a) \\ \frac{C_s}{2\pi r R \{1 + \log[1 + 0.05/(r - a)]\}} & (a < r < b) \end{cases}$$

- **Electron source term**

$$S_e = n_0 \langle \sigma_{\text{ion}} v \rangle n_e - v_L (n_e - n_{e,\text{div}})$$

- **Recycling from divertor**

- **Recycling rate:** $\gamma_0 = 0.8$

- **Neutral source**

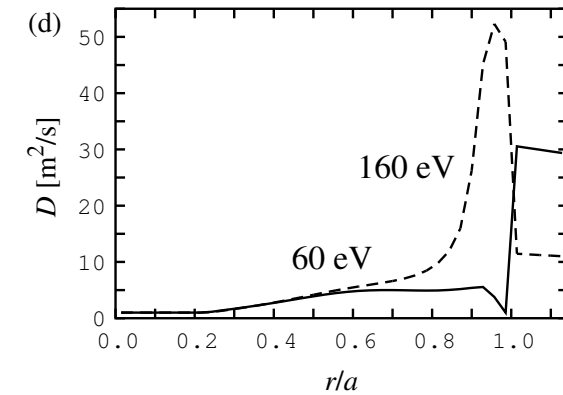
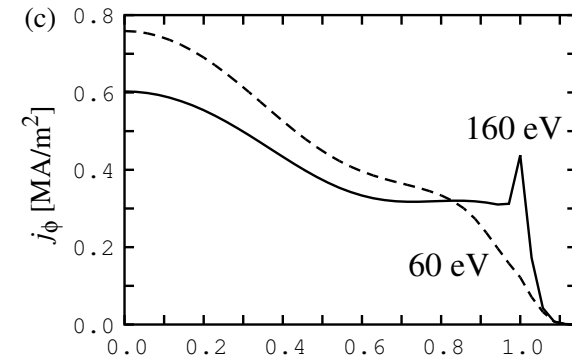
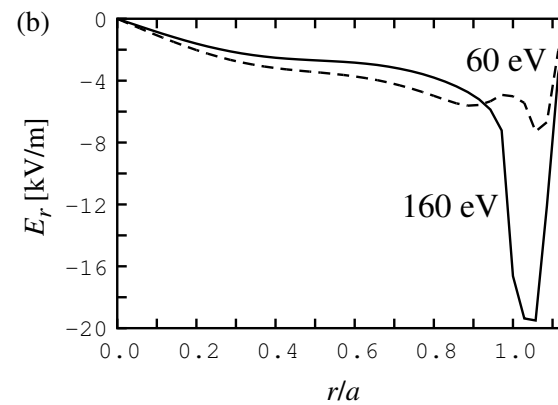
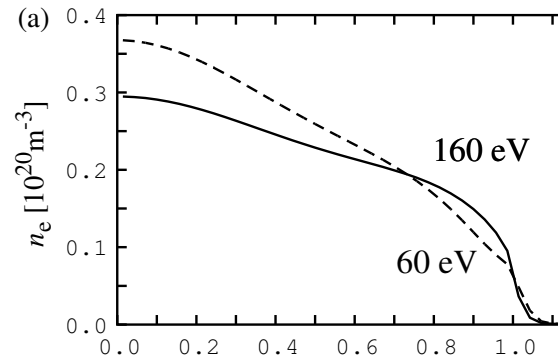
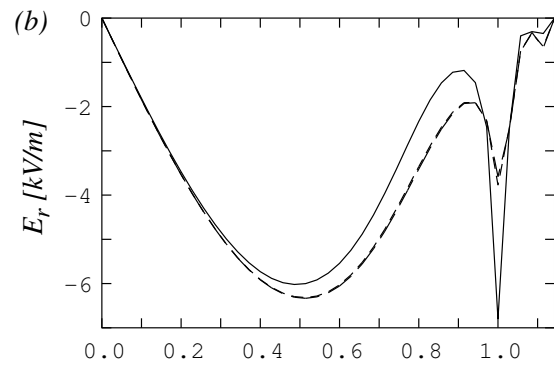
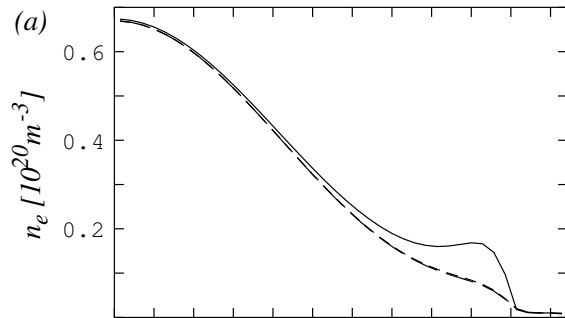
$$S_0 = \frac{\gamma_0}{Z_i} v_L (n_e - n_{e,\text{div}}) - \frac{1}{Z_i} n_0 \langle \sigma_{\text{ion}} v \rangle n_e + \frac{P_b}{E_b}$$

- **Gas puff from wall**

Typical Profiles

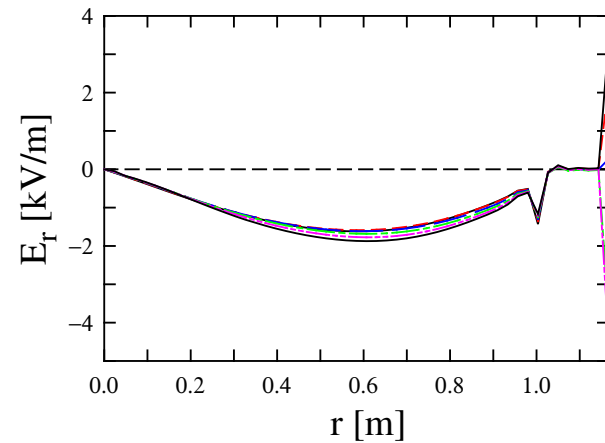
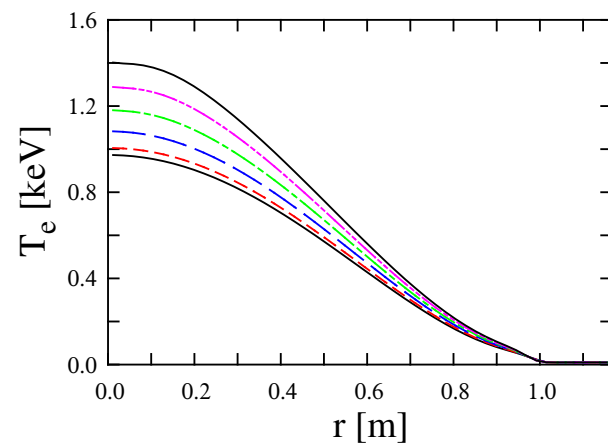
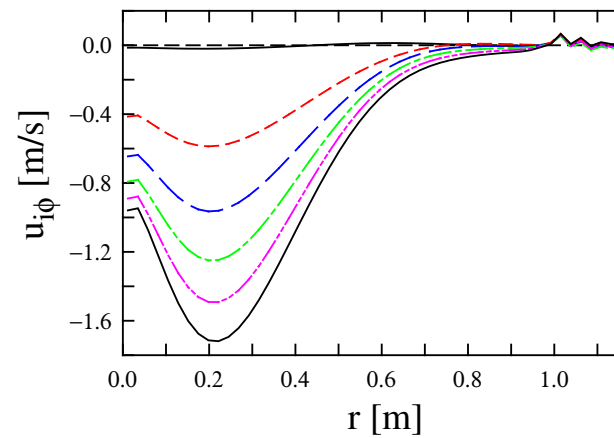
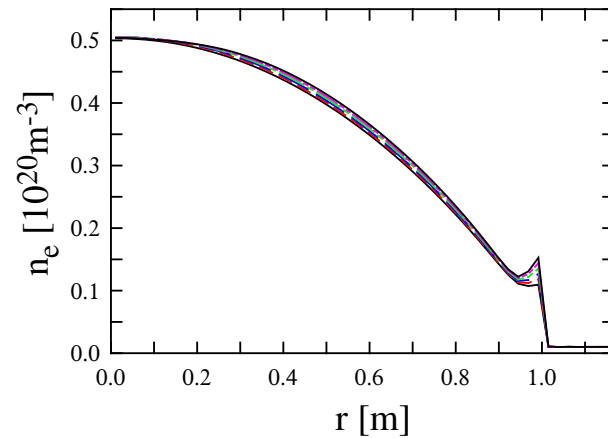
$$D_{TB} = 0$$

Edge Temperature Dependence



Transport Modeling in Helical Plasma

- Neoclassical toroidal viscosity
- Negative magnetic shear
- Preliminary Result
 - NBI heating ($P = 5$ MW) : Order of magnitude slower rotation



Summary

- Viable transport models reproduce **nonlinear relation between flux and gradient, L-mode confinement time scaling, radial profiles and its time evolution, fluctuation behavior and transport barrier formation.**
- Turbulence transport model driven by current diffusive ballooning mode satisfies most of these conditions and reproduces various kinds of improved confinement mode associated with **ITB formation.**
- **ETB formation** in H-mode was studied by using surface-averaged fluid equations in both core and SOL plasmas. The change of transport mechanism across the separatrix generates the radial electric field at the edge and suppress the transport.
- In **toroidal helical plasmas**, neoclassical toroidal viscosity impedes toroidal rotation and affects the transport. Preliminary result was presented.
- Further improvement in modeling of barrier formation is required:

Time evolution, Parameter dependence and Collapse