Tomographic Reconstruction of Emissivity Profile from Tangentially Viewed Images Using Pixel Method

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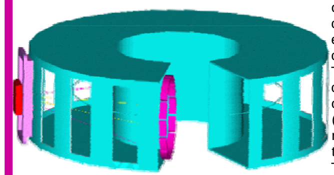
Abstract

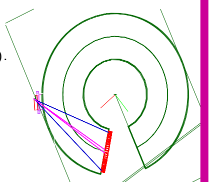
Fast camera systems for imaging tokamak plasmas are becoming increasingly popular. Edge fluctuations and plasma instabilities can be imaged in the visible and X-ray wavelengths using presently available cameras. While viewing the plasma tangentially, the lines of sight (LOS) pass through the plasma integrating the light through a number of flux surfaces. Here we report a reconstruction code for tomographic unfolding of the emissivity profile of the poloidal cross section from the tangential image, using pixel method. The poloidal cross section of the tokamak has been divided into pixels, each of which is a footprint of a subtorus. The emissivity of each of this subtorus (pixels) is assumed to be constant and uniform around the torus.

Introduction

- Plasma imaging getting increasingly popular with development of high speed cameras
- Tangential viewing through the plasma cross-section provides good spatial resolution
- This proves to be an useful tool to study edge fluctuations and plasma instabilities in the visible and X-ray wavelengths
- Reconstruction of the emissivity profile from the images is clearly a necessary task
  - Here an algorithm to infer the emissivity profile from the tangential images recorded on a camera chip is presented
  - Can easily be adapted to any machine parameter and camera location
  - Three non-collinear points, whose co-ordinates are known with respect to the machine center, serve as fiduciaries
  - line of sight (LOS) of each detector is defined through the lens center in terms of the detector unit vector v

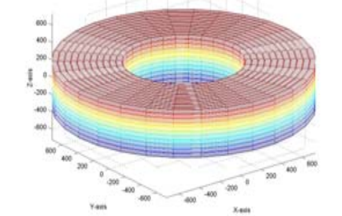


Tangential view of the camera covering almost entire poloidal cross section (L). Three non-collinear points on the limiter (dark magenta ring) serves as the fiduciaries; Top view (R)

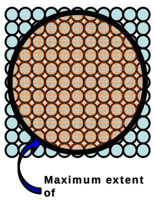


Pixel Approach

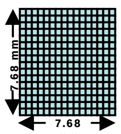
- A tangential camera with a view covering a considerable portion of the poloidal cross section of the plasma is considered
- Poloidal cross section of the tokamak torus has been divided into a matrix of 11 x 11 circles (hereinafter referred to as pixels; the term 'detector' being used to refer to the elements of the camera chip), each of 25 mm radius.
- Major radii of these sub torii vary from 500 mm to 1000 mm in a step of 50 mm, and the z-co-ordinates of their centers range between +250 mm to -250 mm
- The emissivity within each of these pixels is assumed to be uniform and remains constant toroidally
- The camera chip constitutes of a matrix of 16 x 16 macro elements (cluster of elements) of 0.48 x 0.48 mm dimension



Toroidal vessel of tokamak filled with 121 sub tori



Circular footprints of sub torii



256 detectors on the camera chip

Theoretical Background

- LOS of each detector passes through a number of pixels
- Calculation of brightness (f) i.e. power divided by étendue of the detector is given by:

$$I_{11}g_1 + I_{12}g_2 + \ldots + I_{1N}g_N = f_1$$

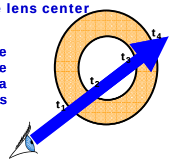
- I_{ij} is essentially the length of the LOS of detector I, passing through the pixel no. j
- The set of simultaneous linear equations can be written as

$$\mathbf{I}*\mathbf{g} = \mathbf{f}$$

- This is a *over determined* system since, number of equations (=256) are more than the number of unknowns (=121)
- LOS originating from each detector will pass through only a small number of pixels toroidally on its way, thus making the I matrix extremely sparse.

Now, each LOS can be represented by a ray, originating from the detector and passing through the lens center

Thus, populating the I matrix is to be done by solving a series of ray-torus intersection equations



- Inverting 'I' which is a sparse matrix cannot be done directly
- We could try in this case to minimize the chi-squared value

$$\chi^2 = (\tilde{I}*g - \tilde{f})^T * (\tilde{I}*g - \tilde{f})$$

- Solutions to the set of linear equations of this over-determined system are found by Singular Value Decomposition (SVD) method

Solving the Ray-Torus Equation

A torus is represented by the equation:

$$[R - \sqrt{(x^2 + y^2)}]^2 + (z - z_0)^2 = r^2$$

Where, R is the major radius, $z0$ is the shift along z axis and r is the minor radius

A ray in its parametric form is represented by the equation:

$$x = a_x + tv_x \quad y = a_y + tv_y \quad z = a_z + tv_z \quad \Rightarrow \quad \mathbf{a}_t = \mathbf{a} + t\mathbf{v}$$

Where, a is the origin or eye point and v is the direction vector

Substituting the ray equation in the torus equation, we get a QUARTIC equation in t. 't' is essentially the distance traversed by the ray from its origin

Realization in MATLAB Code

Following algorithm is adopted to populate the geometry matrix I and reconstruct emissivity profile through a MATLAB based code

Module 1: Co-ordinate of each detector center *w.r.t.* the chip center (local co-ordinate system) is determined → Module 2: generates the unit vectors along the horizontal and vertical axis of the chip w.r.t. the machine center → Module 3: calculates the co-ordinates of each detector center point w.r.t. the global origin → Module 4: generates a emissivity profile across the poliodal cross-section

Main code: generates individual rays originating from each detector center and passing through the lens center, which is also defined w.r.t. the global origin. Finally, the code solves the quartic equation for each ray and all sub torii individually and calculates the difference of the real roots, if at all there is an intersection, thereby calculates the I matrix

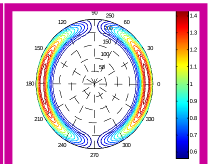
Simulated Emissivity

- An emissivity profile is created as an input to the code
- The emissivity profile assumed on the poloidal cross-section as:

$$g = 0.5 + A * e^{[-(r-r_0)^2/2\sigma^2]} * abs(\cos\theta)$$

Where, $0 \le r \le 250$ and $0 \le \theta \le 2\pi$; $A = 1$, $r_0 = 225$, $\sigma = 20$

- It is an edge peaked (@ $r_0/r \sim 0.9$) profile with a variation along θ. This has been assumed to replicate visible emission from tokamak plasma



Emissivity contour

Input Brightness and Noise Incorporation

- In the present scenario, two types of noises are addressed, viz. the read out like noise and the shot like noise
- Several amount of these noises are added separately to check the consistency of reconstruction. White noise with a normal distribution is added to the brightness profile
- For shot like noise we've introduced a predefined percentage of the brightness value itself and for read like noise, we've introduced a predefined percentage of the mean of the overall brightness as the standard deviation of the Gaussian random noise
- Percentages of both the noise amount to 0.1, 0.3, 0.5 and 1.

$$\mathbf{f}_{noisy\_shot} = \mathbf{f}_{clean} + \mathbf{f}_{clean} * s * randn(size(\mathbf{f}_{clean}))$$

$$\mathbf{f}_{noisy\_read} = \mathbf{f}_{clean} + \langle \mathbf{f}_{clean} \rangle * s * randn(size(\mathbf{f}_{clean}))$$

Results

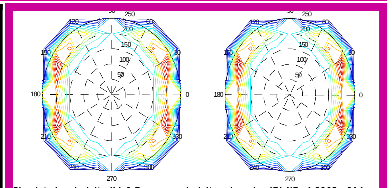
- Quality of reconstruction is reflected in the *normalized residual*
- Normalized Residual (NR) is given as:

$$NR = \sqrt{\sum [(\mathbf{I}*\mathbf{g}^R - \mathbf{f}_c)/\mathbf{f}_c]^2}$$

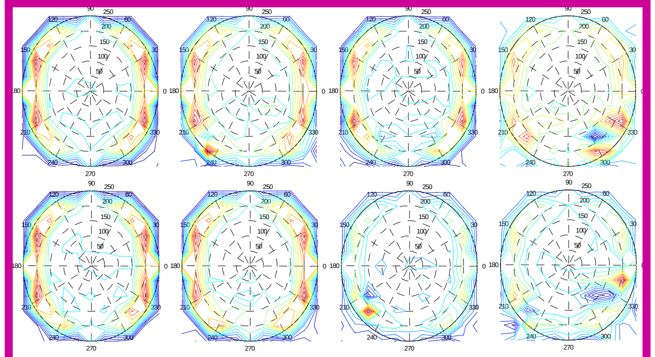
Where, g^R is the reconstructed emissivity, and f_c is the double precision input brightness

- Reconstruction starts deviating from the input emissivity profile at around a noise of 0.5 percent and degrades substantially at about 1 percent in both the cases

Normalized Residual			
Read out noise		Shot noise	
Percentage	NR	Percentage	NR
1	~ 0.1475	1	~ 0.1154
0.5	~ 0.0686	0.5	~ 0.0592
0.3	~ 0.0443	0.3	~ 0.0371
0.1	~ 0.0162	0.1	~ 0.0142



Simulated emissivity (L) & Recons. emissivity w/o noise (R) NR=4.2965e-014



Top row: read out like noise of (L to R) 0.1, 0.3, 0.5, 1%; Bottom row: shot like noise of (L to R) 0.1, 0.3, 0.5, 1%

Discussions

For analyzing tangential images in tokamak:

- Ray-torus intersection (quartic) equation is solved to populate the geometry matrix
- SVD used to solve sparse over-determined system
- At present the algorithm is robust to handle up to 0.5%, either of read out like and shot like noise
- Run time of the code on PIV 512 MB DDR2 RAM desktop computer is few seconds only
- Optimization of number of pixels and detectors to handle greater amount of noise is under progress

Ref. / Acknowledgement

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