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PS5 02

Effects of relativistic and absorption on ECE spectra in high temperature tokamak plasma

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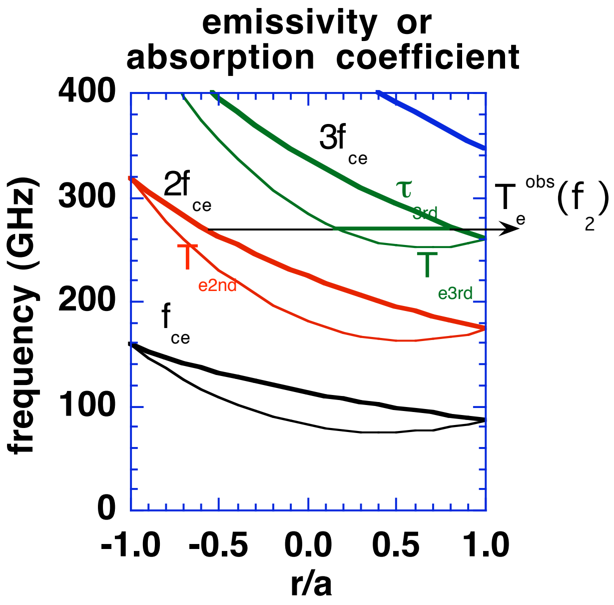
ECE spectra in tokamak

Electron cyclotron emission (ECE)

- The importance of a relativistic frequency downshift effect on ECE in high temperature plasma is well
- The characteristics of ECE spectra are evaluated in the high temperature plasma in the various sight line of observation.
  - Angle(θ_B) between sight line and B_t.
  - Angle(θ_{port}) between sight line and equatorial plane.



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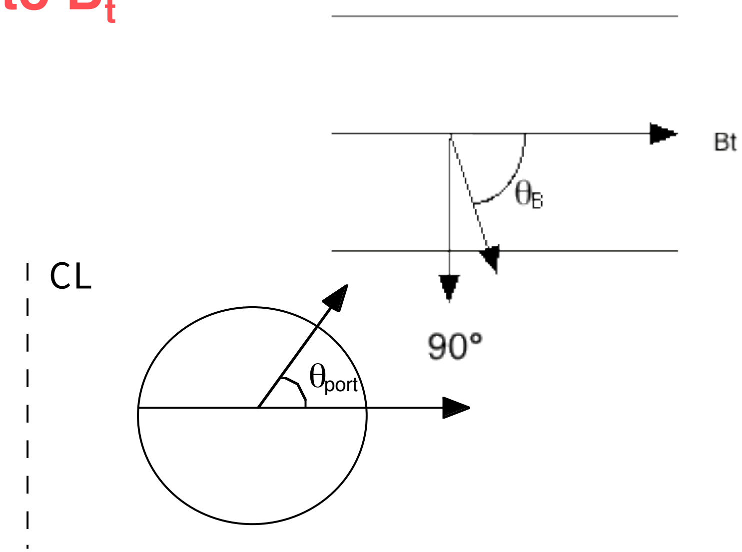
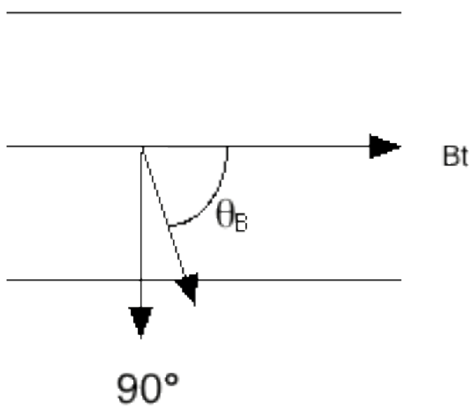
- Expression of emissivity for oblique propagation to B_t in the case of spherically symmetric relativistic Maxwellian. (Extension from the Trubnikov's eq.)
- Emissivity (θ_B Scan).
- ECE Spectra (θ_B & θ_{port} Scans).



Emissivity for oblique propagation to B_t (Extension from the Trubnikov's Eq.)

- Assumption:

  Velocity distribution f(p):Spherically symmetric relativistic Maxwellian
- Obtained Emissivity(X mode) $j_1(\omega)$:spectral power density



$$J_\omega(\omega,\theta_B) = \int \mathbf{dp}\ J_1(\theta) f(\mathbf{p}) = \int J_1(\theta) N(\varepsilon) d\varepsilon$$

$$j_\omega^{\perp}(\omega,\theta_B) = \frac{\omega_p^2}{\omega_H c} \frac{\omega^2 T_e}{8\pi^3 c^2} \frac{\pi\mu^2}{K_2(\mu)} \frac{1}{2}$$

$$\int_0^\pi d\theta_p \sum_{n \geq x(1-\beta\cos\theta_p\cos\theta_B)}^{\infty} \frac{n^2}{x^4} \sin\theta_p (\beta\sin\theta_p)^2 J_n'^2\left(\frac{n\beta\sin\theta_p\sin\theta_B}{1-\beta\cos\theta_p\cos\theta_B}\right)$$

$$\frac{1}{(1-\beta\cos\theta_p\cos\theta_B)^3} \sqrt{\left(\frac{n}{1-\beta\cos\theta_p\cos\theta_B}\right)^2 - x^2}\ \exp\left\{-\left[\frac{\mu}{X}\left(\frac{n}{1-\beta\cos\theta_p\cos\theta_B}\right)\right]\right\}$$

$$\beta_\pm = \frac{\cos\theta_B\cos\theta_p \pm \sqrt{(\cos\theta_B\cos\theta_p)^2 - \{(\cos\theta_B\cos\theta_p)^2 + (n\omega_H/\omega)^2\}\{1-(n\omega_H/\omega)^2\}}}{(\cos\theta_B\cos\theta_p)^2 + (n\omega_H/\omega)^2}$$

ω_H:non-relativistic EC angular frequency, ω_p:plasma angular frequency.
θ_p:azimuth coordinate in momentum space, J_n:Bessel function of nth order,
K_n :modified Bessel function of 2nd order, $\mu = mc^2/T_e$, $x=\omega/\omega_H$,

Obtained Emissivity(O mode)

$$j_\omega^{//}(\omega,\theta_B) = \frac{\omega_p^2}{\omega_H c}\frac{\omega^2 T_e}{8\pi^3 c^2}\frac{\pi\mu^2}{K_2(\mu)}\frac{1}{2}$$

$$\int_0^\pi d\theta_p \sum_{n \geq x(1-\beta\cos\theta_p\cos\theta_B)}^{\infty} \frac{n^2}{x^4} sin\theta_p \left(\frac{\cos\theta_B - \beta\cos\theta_p}{sin\theta_B}\right)^2 J_n^2\left(\frac{n\beta sin\theta_p sin\theta_B}{1-\beta\cos\theta_p\cos\theta_B}\right)$$

$$\frac{1}{(1-\beta\cos\theta_p\cos\theta_B)^3}\sqrt{\left(\frac{n}{1-\beta\cos\theta_p\cos\theta_B}\right)^2 - x^2}\exp\{-[\frac{\mu}{X}(\frac{n}{1-\beta\cos\theta_p\cos\theta_B})]\}$$

Extension of Trubnikov's eq,[1] to the oblique propagation to B_t.

Trubnikov's eq.	**->**	**Extension of Trubnikov's eq,**
n	**->**	$n/(1-\cos\theta_p\cos\theta_B)$
$d\theta_p$	**->**	$d\theta_p/(1-\cos\theta_p\cos\theta_B)$
β	**->**	$(\cos\theta_B - \beta\cos\theta_p)/\sin\theta_B$

[1] B. A. Trubnikov: *Magnetic Emission of High-Temperature Plasma, Thesis, Institute of Atomic Energy, Moscow, 1958.*

Emissivity (X mode, θ_B Scan)

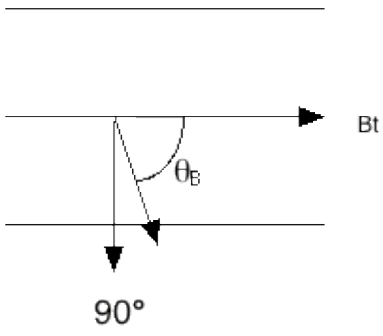
Emissivity $j^x(\omega, \theta_B)$

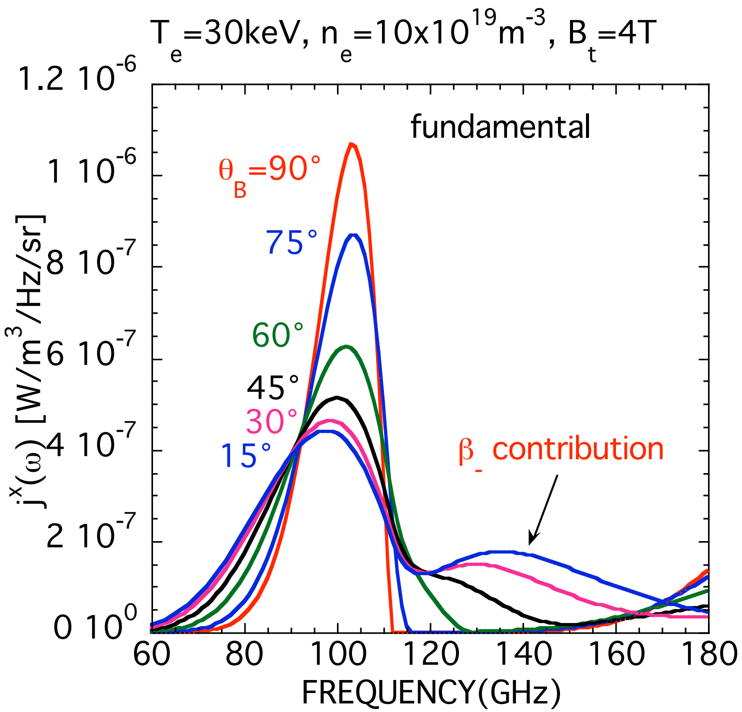
X mode Bt=4T, $T_e(0)$=30keV, $n_e(0)=10\times10^{19}m^{-3}$

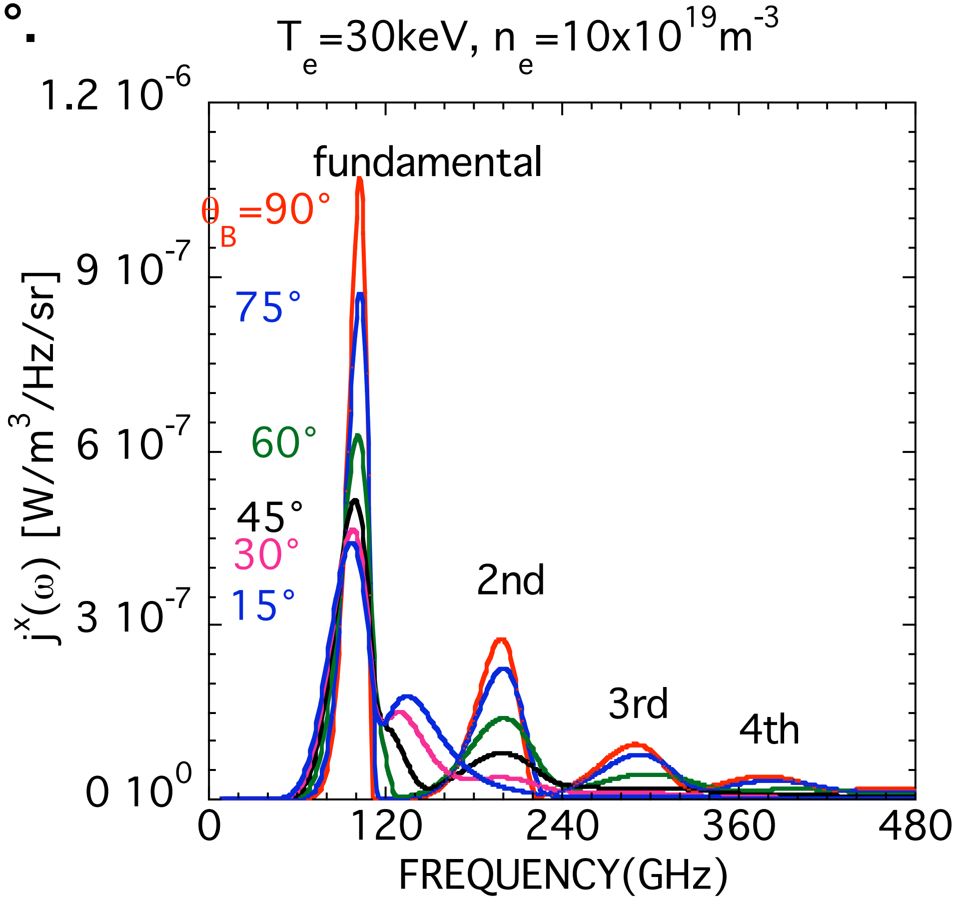
When θ_B decrease, peak of emissivity becomes small but the foot of emissivity expands.

The contribution from $\beta-$ results in peaks between 120GHz and 150GHz in the cases of θ_B =15° and 30°.

When θ_B increases, emissivities increase.







Emissivity (O mode, θ_B Scan)

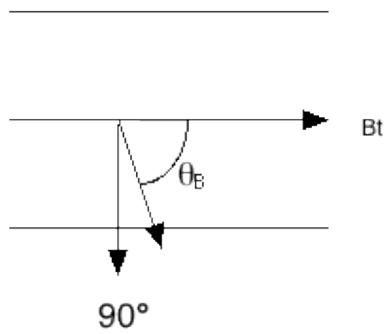
Emissivity $j^o(\omega, \theta_B)$

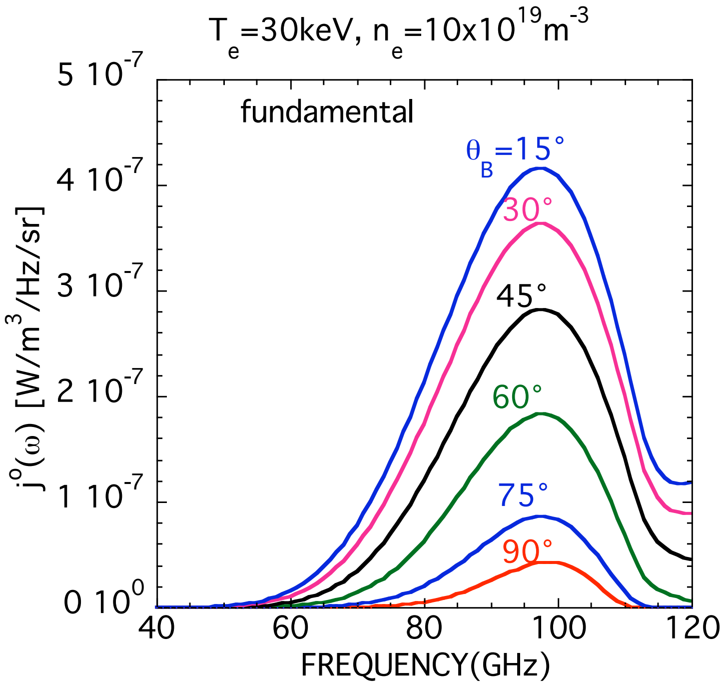
O mode Bt=4T, $T_e(0)$=30keV, $n_e(0)=10\times10^{19}m^{-3}$

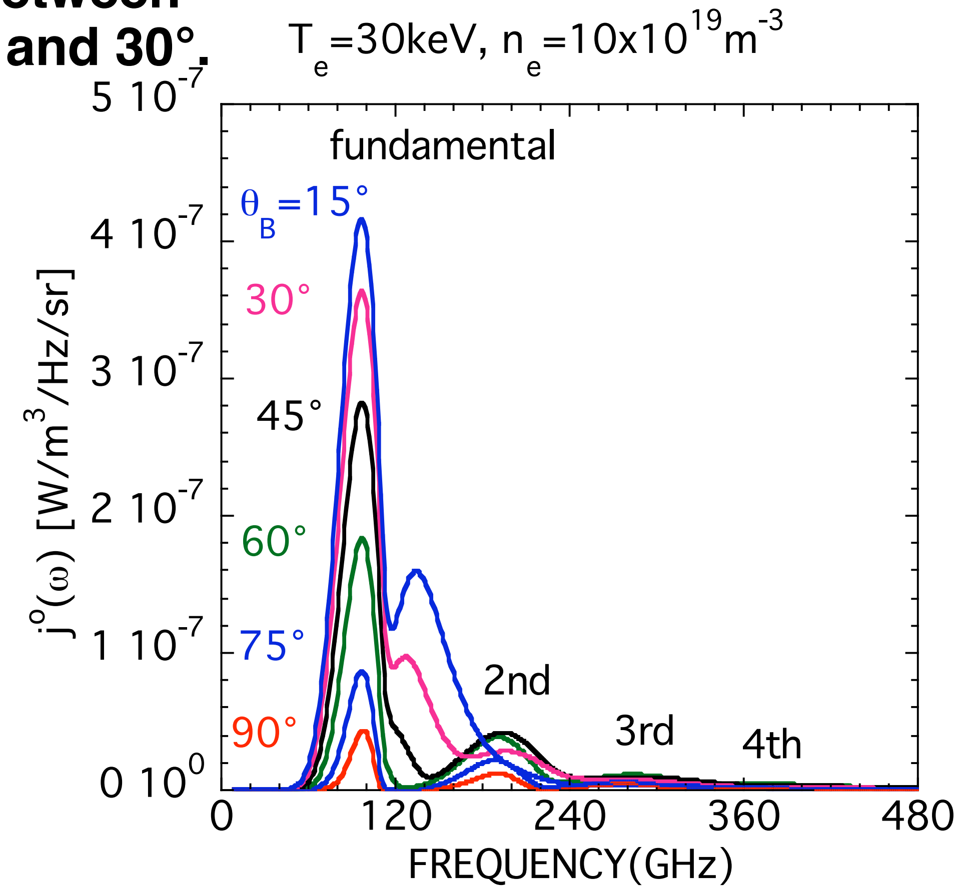
When θ_B increases, emissivities decrease.

When θ_B decrease, peak of emissivity become big but the foot of emissivity expands.

The contribution from β– results in peaks between 120GHz and 150GHz in the cases of θ_B =15° and 30°.







Calculation Method and Parameters

- **Solve the radiation transfer equation.**

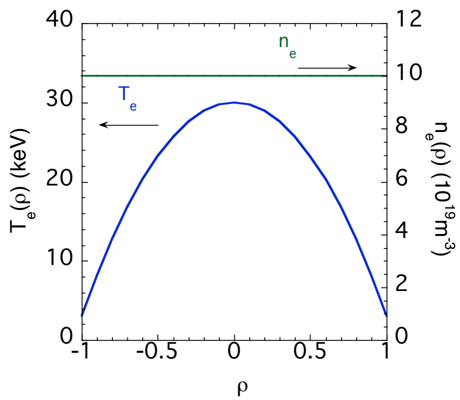
$$\frac{d}{d\tau}\left(\frac{I(\omega,s)}{N(\omega,s)^2}\right) = -\frac{I(\omega,s)}{N(\omega,s)^2} + S(\omega,s) \qquad S(\omega,s) \equiv \frac{1}{N(\omega,s)^2}\frac{j(\omega,s)}{\alpha(\omega,s)}$$

Assumptions:

- **Plasma parameters have the cylindrical symmetry.**
- **But,the radial dependence of the magnetic field field is taking into account.** $B_t(R)=B_t(0)*R_0/R$.
- **Emissivity is calculated using the extended Trubnikov's expression.**
- **Absorption coefficient ($\alpha(\omega,s)$) is obtained from the emissivity ($j(\omega,s)$) applying Kirchhoff's law.** $S(\omega,s)=I_{BB}(\omega,s)$ I_{BB} : **Black body**
- $N(\omega,s)=1$.
- **Refractive properties is taking into account after the calculation.**
  - **Upper hybrid resonance for 1st X mode etc,**
- **The T_e^{obt} is derived from the calculated radiance.**

$$T_e^{obt} = I(\omega,a) / \left(\frac{\omega^2 k}{8\pi^3 c^2}\right)$$

Calculation parameters:



$T_e(0)$=25~50keV, $n_e(0)$=0.2～20x10^{19}m^{-3} ,

Tokamaks : JT-60U A=3.4, R_0 =3.4m, a=1.0m, $B_t(0)$=4.0T, $2f_{ce}$=223.939GHz,

SlimCS A=2.6, R_0 =5.5m, a=2.1m, $B_t(0)$=6.0T, $2f_{ce}$=335.909GHz

ECE Spectra (X mode, θ_B Scan, LFS)

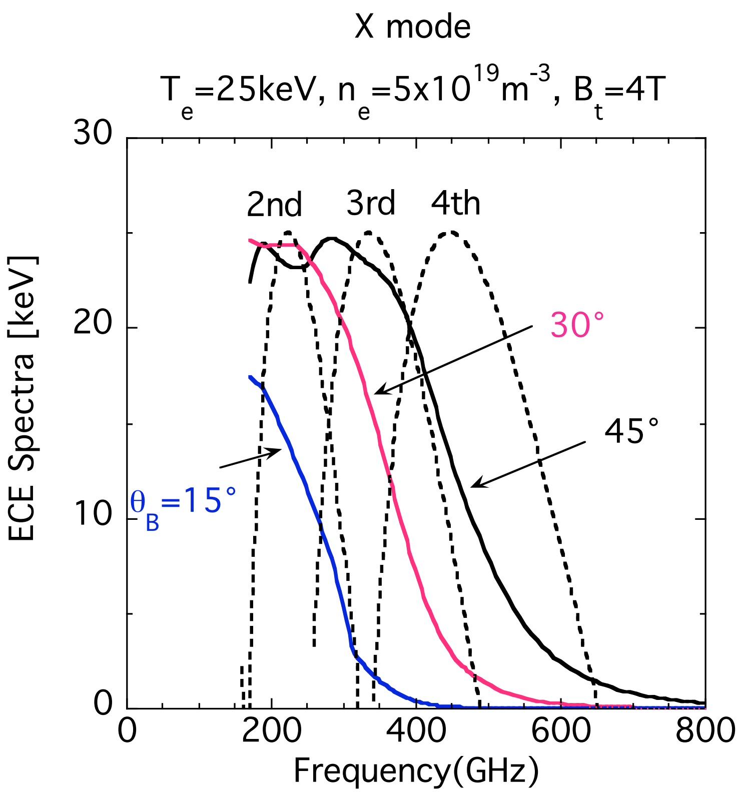
The observation angles are scanned in angle of θ_B.

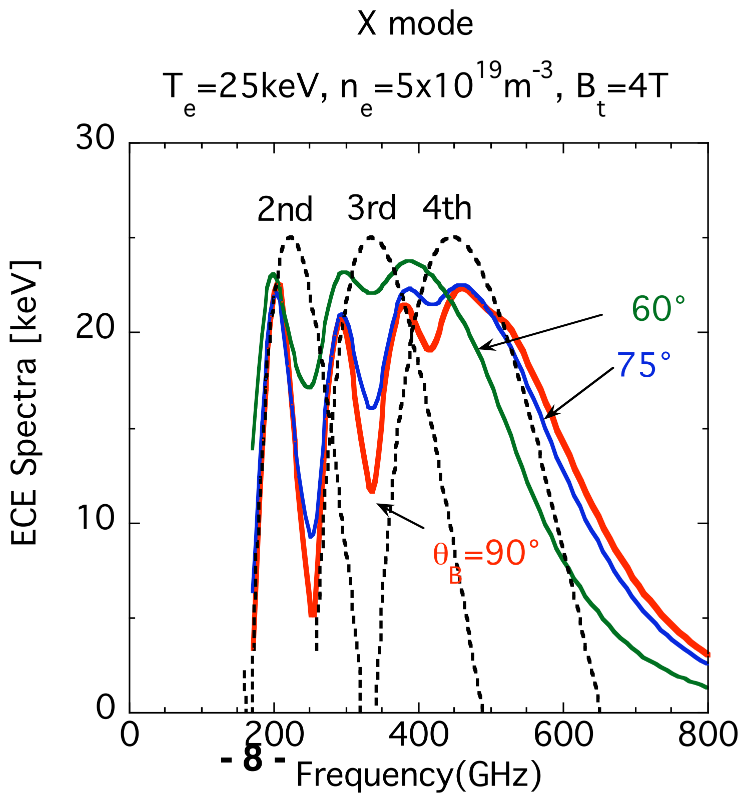
X mode JT-60U $T_e(0)$=25keV, $n_e(0)=5\times10^{19}m^{-3}$

$\theta_B \geq 75°$:Radiations are almost close the T_e in lower frequency side of assumed T_e.

$\theta_B \leq 30°$:There is small amount of radiation in high frequency region.

I(@high ω):I(θ_B =90°)> I(θ_B =75°)> I(θ_B =60°)> I(θ_B =45°)> I(θ_B =30°)> I(θ_B =15°)





ECE Spectra (O mode, θ_B Scan, LFS)

There are continuous emissions in lower side of fundamental mode due to the Doppler and relativistic effect.

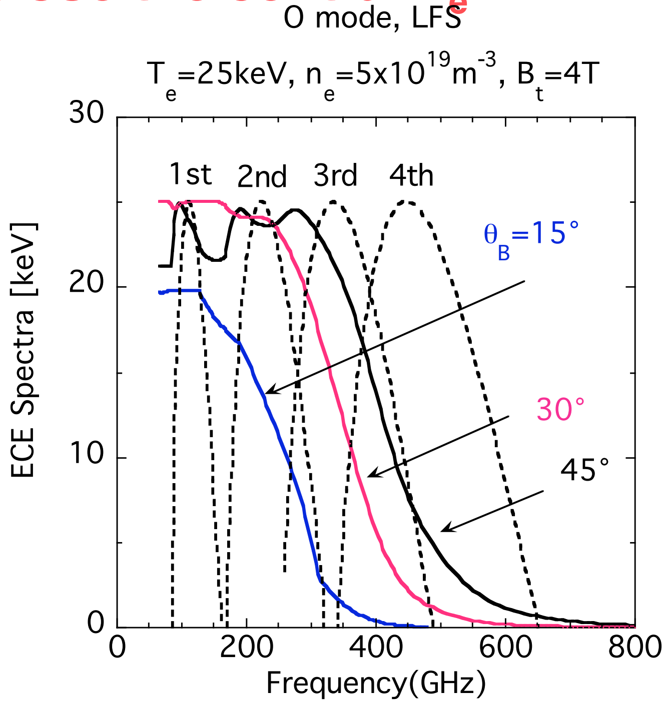
O mode JT-60U $T_e(0)$=25keV, $n_e(0)=5\times10^{19}m^{-3}$

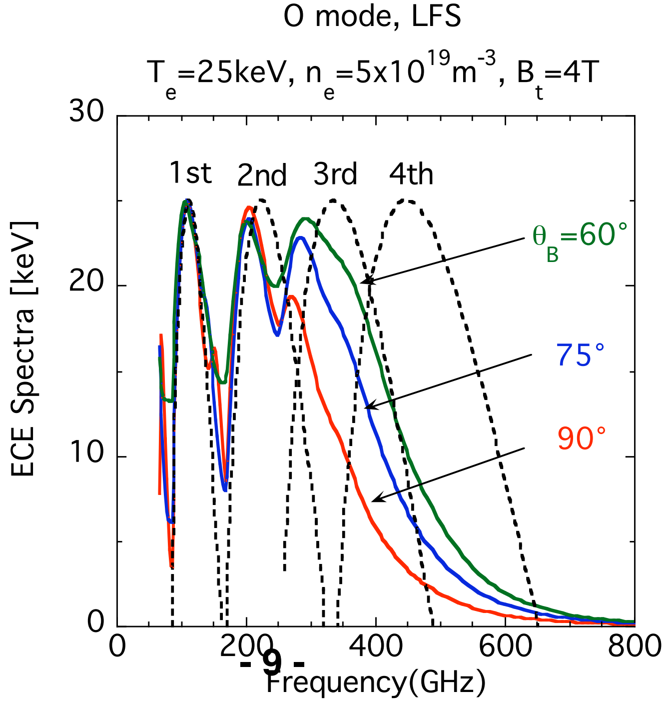
$\theta_B \geq 60°$:There are side peaks at both side of fundamental mode.

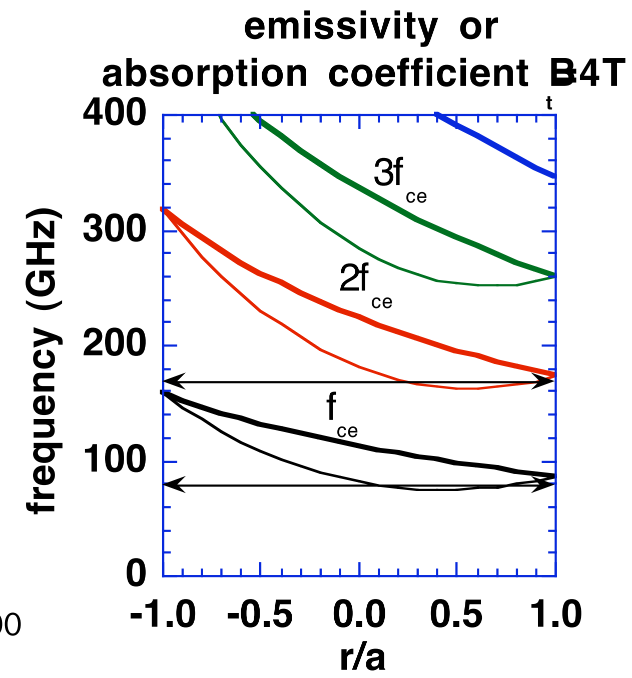
←Relativistic effect of fundamental & 2nd harmonics does not absorbed by another harmonics in plasma.

I(@high ω):I(θ_B=60°)> I(θ_B=75°)> I(θ_B=90°), I(θ_B=45°)> I(θ_B=30°)> I(θ_B=15°)

θ_B =30°:Radiation temperature in lower frequency of fundamental mode is close the central T_e.







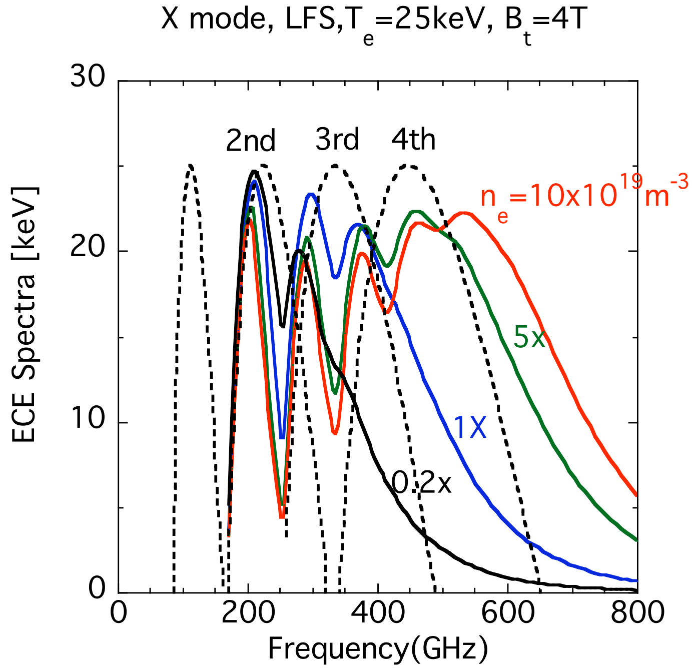
ECE Spectra (X&O mode, n_e Scan, LFS)

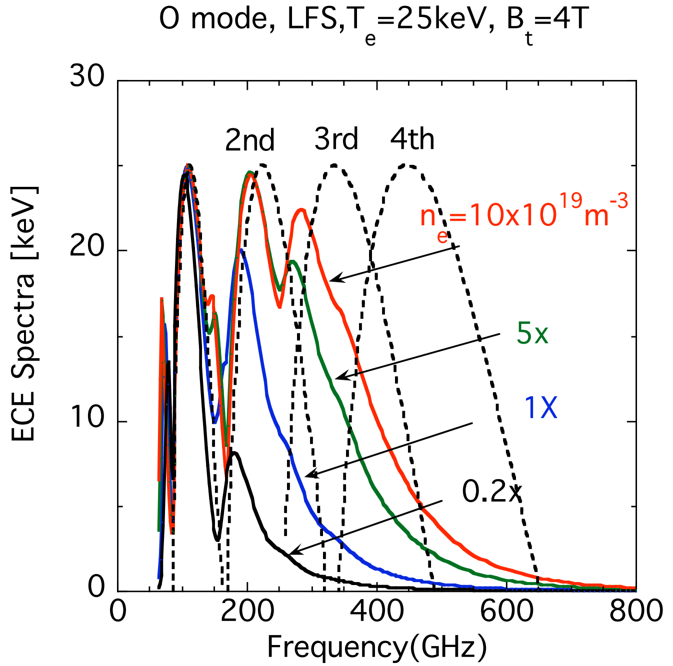
JT-60U $T_e(0)$=25keV, θ_B =90 °, θ_{port} =0 °.

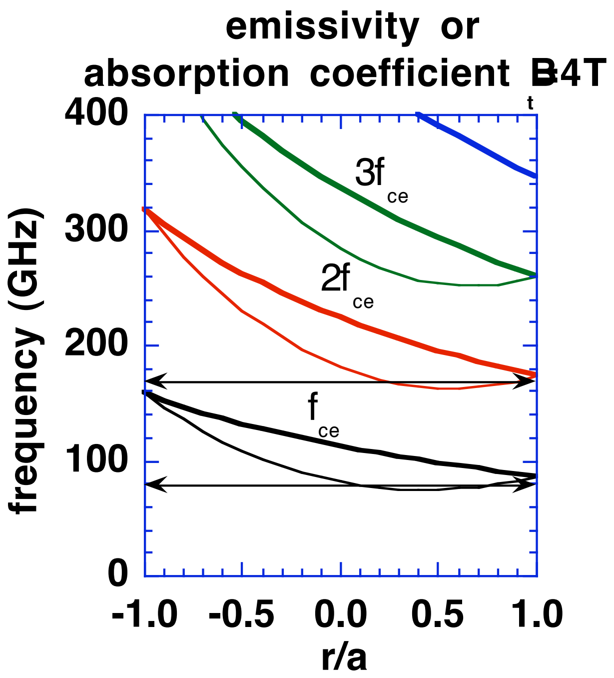
When n_e increases, radiation becomes bigger.

X mode:Radiations are almost close the T_e in lower frequency side of assumed T_e in the 2nd harmonic.
O mode:Radiations are almost close the T_e in both sides of assumed T_e in the fundamental mode.

→ T_e measurements.



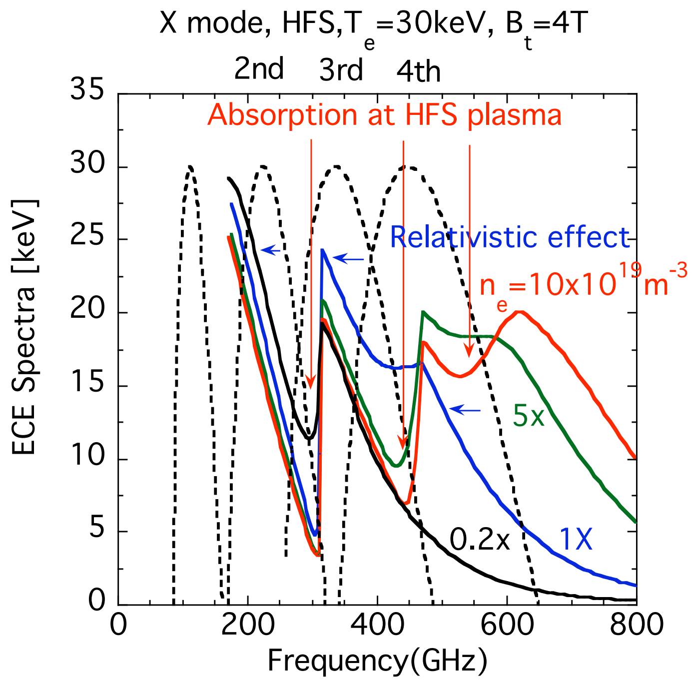


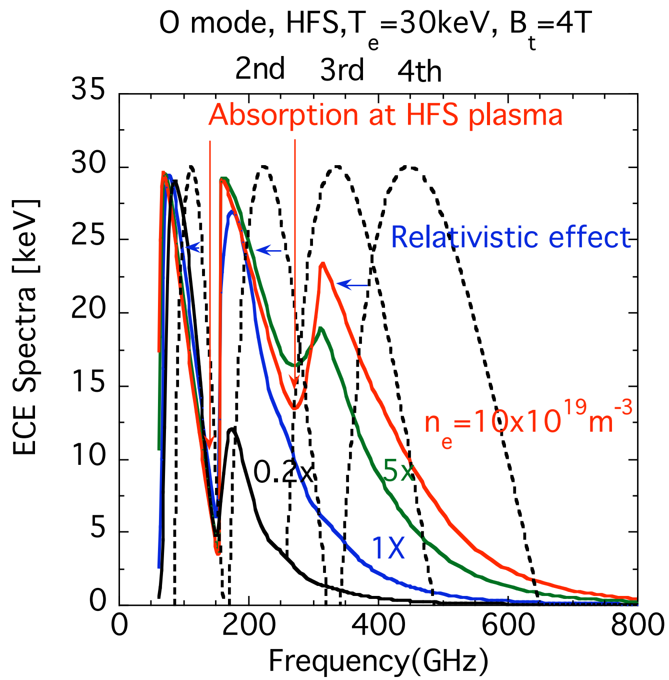


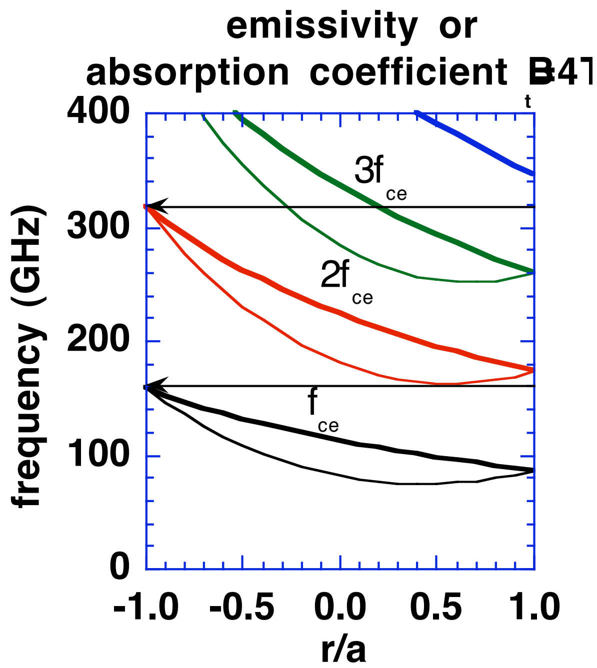
ECE Spectra (X&O mode, n_e Scan, HFS)

JT-60U $T_e(0)$=30keV, θ_B =90 °, θ_{port} =0 °.

- **Downshift frequency due to the relativistic effect in the HFS observation is bigger than that in the LFS observation.**
- **Absorption at the HFS plasma results in the deep dip.**





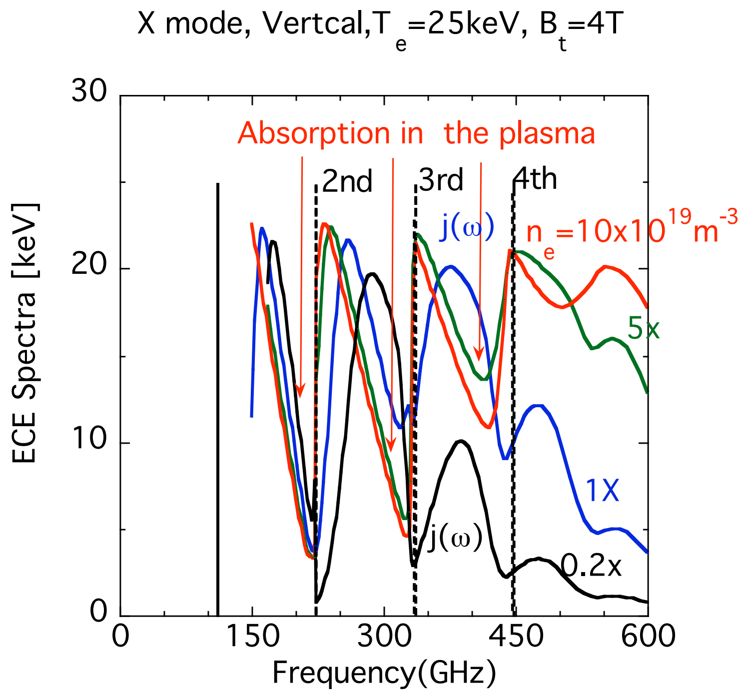


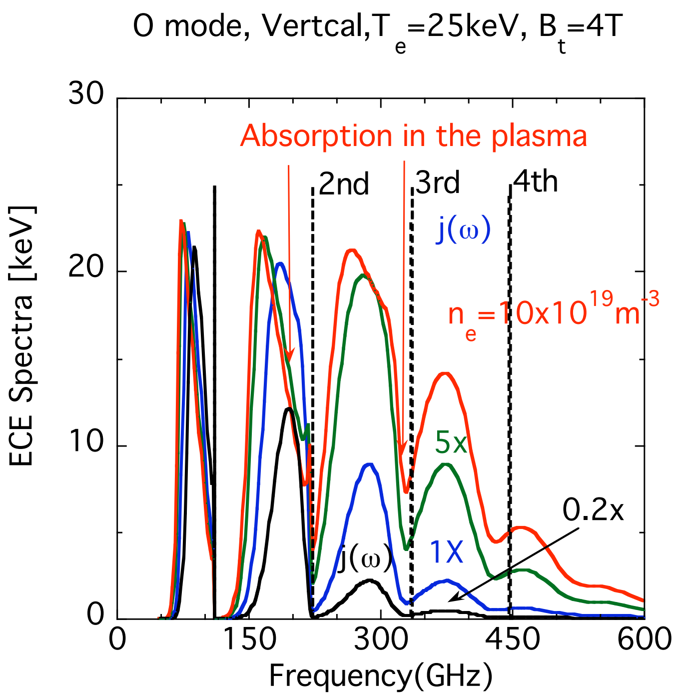
ECE Spectra (X&O mode, n_e Scan, Vertical SL)

JT-60U $T_e(0)$=25keV, $n_e(0)=5\times10^{19}m^{-3}$

In the case of optically thin case(EX. f between 3rd & 4th of X mode, $n_e(0) \leq 1\times10^{19}m^{-3}$), ECE spectra are similar to the emissivity profile.

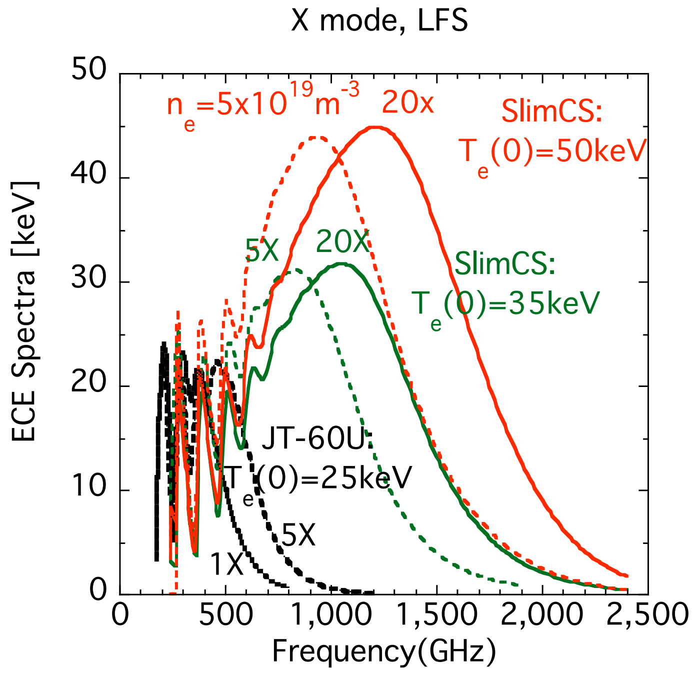
On the other hand, in the case of optically thick case (EX. f between 3rd & 4th of X mode, $n_e(0) \geq 5\times10^{19}m^{-3}$), ECE spectra are apart from the emissivity profile due to the absorption in plasma.

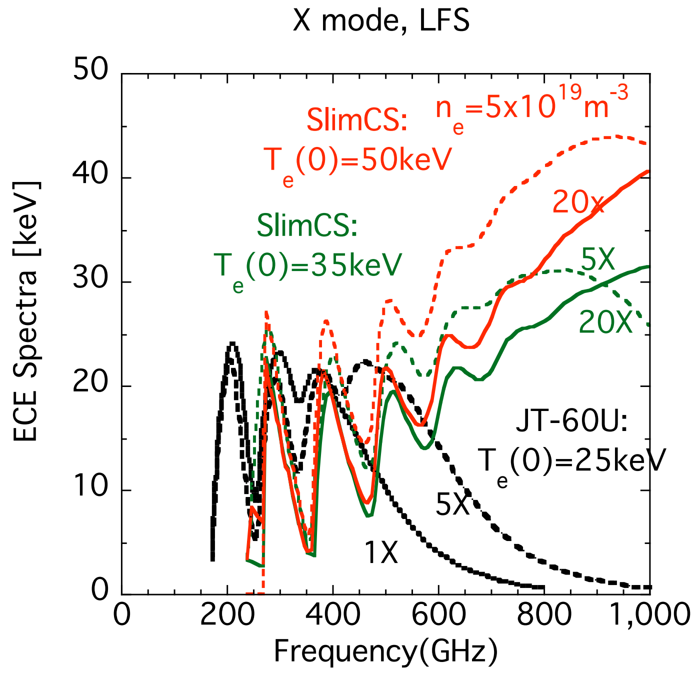




ECE Spectra (X mode,LFS JT-60U& SlimCS)

- **JT-60U $T_e(0)$=25keV, $n_e(0)$=1 & $5x10^{19}m^{-3}$ →100~800GHz**
- **SlimCS $T_e(0)$=35&50keV, $n_e(0)$=5 & $20x10^{19}m^{-3}$ →100~2500GHz**
- **ECE Diagnostics (Ex. detector of FTS) is modified from present ECE system because of the detection of high frequency emission.**





Summary

Extension of Trubnikov's eq. was obtained for oblique propagation to B_t in the case of spherically symmetric relativistic Maxwellian.

We evaluated the ECE spectra in the high temperature plasma using the extension of Trubnikov's eq.

Results

- **We evaluated the ECE spectra for various directions in the high temperature plasma.**
- **Feature of ECE spectra can be interpreted from the viewpoints of relativistic effect and absorption effect.**

  X mode
  - **LFS:Radiations are almost close the T_e in lower frequency side of assumed T_e in the 2nd harmonic. → T_e measurements.**

  O mode
  - **LFS:Radiations are almost close the T_e in both sides of assumed T_e in the fundamental mode. → T_e measurements.**

HFS

- Downshift frequency due to the relativistic effect in the HFS observation is bigger than that in the LFS observation. Absorption at the HFS plasma results in the deep dip.

Vertical Line

- In the case of optically thin case, ECE spectra are similar to the emissivity profile.When electron density is higher, ECE spectra are modified due to the absorption in plasma.

- In the case of SlimCS DEMO reactor, there is high frequency emission (~2000GHz). → ECE Diagnostics is modified from present ECE system because of the detection of high frequency emission.