# Suppression of turbulence by mean flows in two-dimensional fluids

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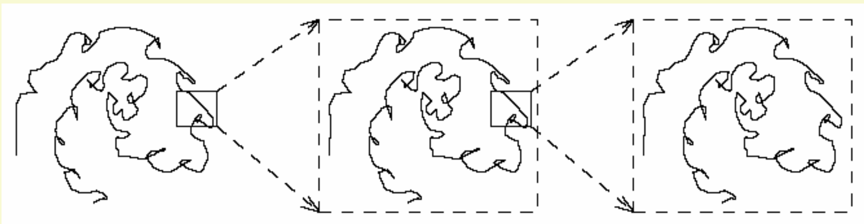
## Outline

- Introduction to 2D fluid turbulence
- Quasi-2D fluid experiment
- Suppression of turbulence by mean flow
  - Self-generated flow
  - Externally imposed flow
- Summary

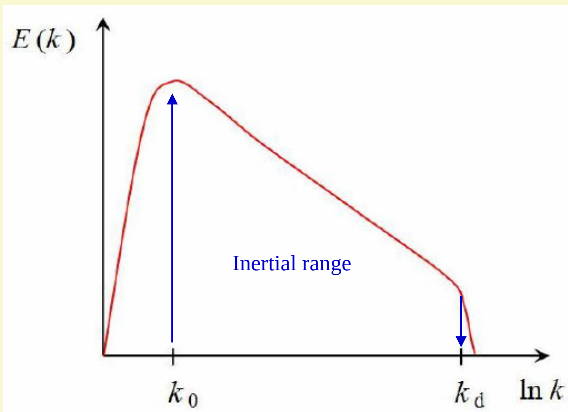
# Introduction

# Energy cascade in 3D turbulence

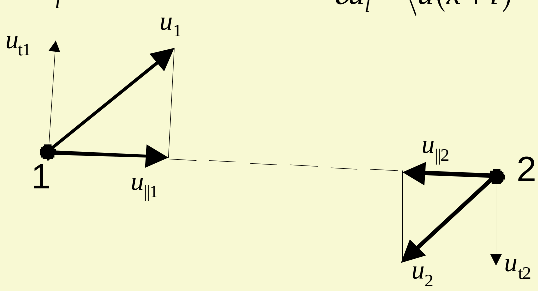


Self-similar cascade of eddies in a turbulent flow

Kolmogorov: energy cascade is the self-similar 'fractal' cascade process which transfers energy from the larger eddies, where it can not be dissipated, to the smaller ones where it can.




Label an 'eddy' by a velocity increment $\delta u_l$ across a distance $l$: $\delta u_l = \langle u(x+l) - u(x) \rangle$




Structure functions of the n[th] order $S_n$ :

$$S_n(l) = \left\langle (\delta u_l)^n \right\rangle = \left\langle \left(u(x+l) - u(x)\right)^n \right\rangle$$

E.g., root-mean-square velocity of eddies of scale l = $\sqrt{S_2(l)} = \sqrt{\left\langle (\delta u_l)^2 \right\rangle}$

# Main results of the Kolmogorov 1941 theory

**Four-fifths law.**

In the limit of infinite Reynolds number, the third order (longitudinal) structure function of homogeneous isotropic turbulence, evaluated for increments $l$ small compared to the integral scale, is given in terms of the mean energy dissipation per unit mass $\varepsilon$ (assumed to remain finite and nonvanishing) by

$$S_3(l) = \left\langle \delta v_{\parallel}^3 (r, l) \right\rangle = -\frac{4}{5}\varepsilon l$$

**Kolmogorov energy spectrum.**

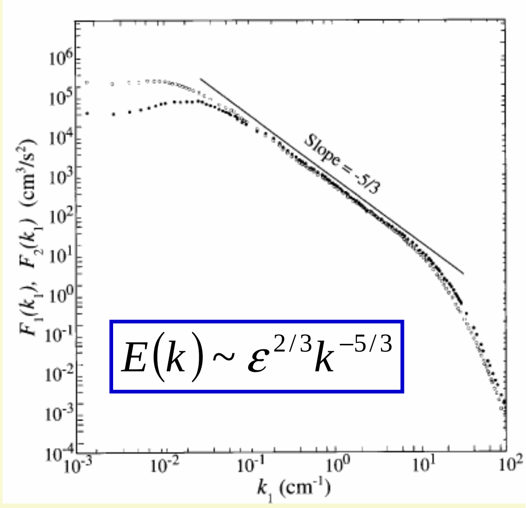
The second order structure function follows the $l^{2/3}$ law :

$$S_p(l) = C_p \varepsilon^{p/3} l^{p/3} \longrightarrow S_2(l) = C_2 \varepsilon^{2/3} l^{2/3}$$

If the energy spectrum is a power-law $E(k) = k^{-n}$ then the velocity field has the second-order spatial structure function which is also a power-law:

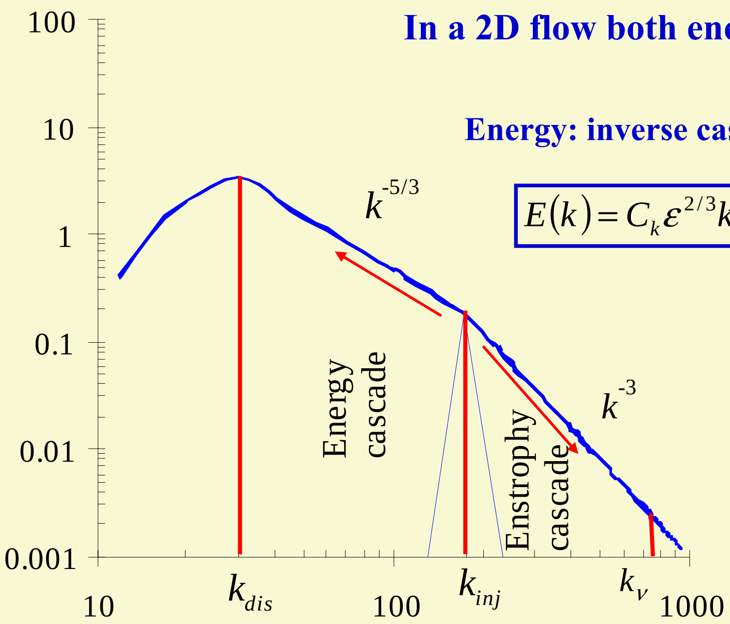
$$\left\langle \left( v(\boldsymbol{r} + l) - v(\boldsymbol{r}) \right)^2 \right\rangle \propto |l|^{n-1}$$

This implies a $k^{-5/3}$ law for the energy spectrum:




$$E(k) \sim \varepsilon^{2/3} k^{-5/3}$$

# 2D turbulence




**In a 2D flow both energy and enstrophy are conserved**

**Dual cascade**

**Energy: inverse cascade, Enstrophy: forward cascade**

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3}$$

$$E(k) = C_\omega \varepsilon_\omega^{2/3} k^{-3}$$

R. Kraichnan (1967):

$$\Omega = \frac{1}{2}\int_V |\boldsymbol{\omega}|^2 dV$$

enstrophy

**Kolmogorov law in 2D:**

$$S_3(l) = \left\langle \delta v_{\parallel}^3 (r, l) \right\rangle = \frac{3}{2}\varepsilon l$$

**Opposite to 3D, energy flows from smaller to larger structures.**

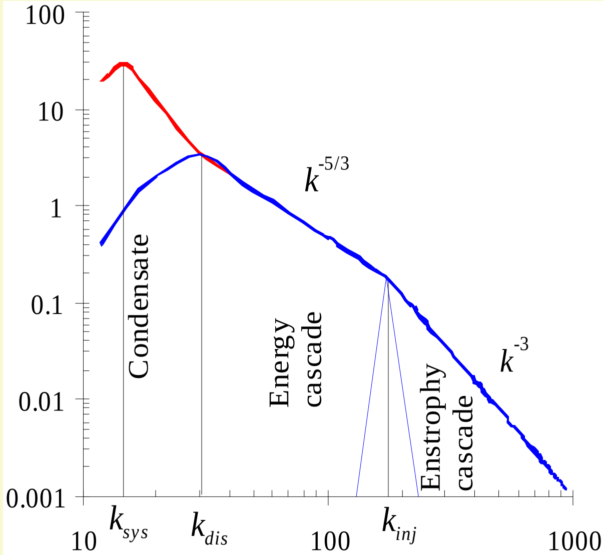
**The basis for self-organization**

# Spectral condensation of turbulence in 2D fluids

The maximum of the energy spectrum lies in the low-$k$ range, at $k_E$, and in the absence of the energy dissipation at large scales can not be constant in time since it accumulates spectral energy $k_E = f(\varepsilon, t)$

***System size < dissipation scale***

Damping for large scales (e.g.linear damping) $\mu$ stabilizes the maximum of the spectrum at the scale $k_{dis} \approx \left(\mu^3/\varepsilon\right)^{1/2}$




At low dissipation in a bound system, at $k_{dis} << k_{sys}$ spectral energy condenses into large (system size) coherent structure(s):
dipole (periodic boundary),
or monopole (no-slip boundary, experiments)

*Theory*: Kraichnan, 1967- qualitative => Bose condensate]

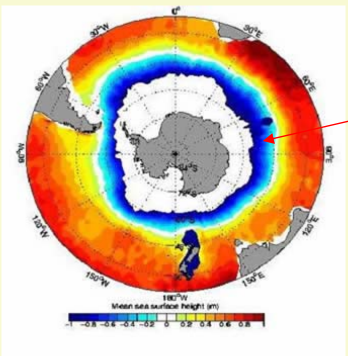
*Experiments*: Sommeria (1986), Paret&Tabeling (1998)
Shats et al (2005)

*Modelling:* Hossain (1983), Smith&Yakhot (1993)…
van Heijst, Clercx, Molenaar (2004-2006), Chertkov et al. (2007)

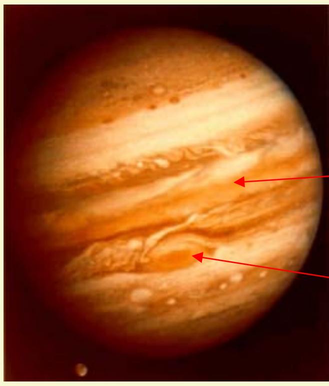
# Turbulence in two dimensions

**2D approximation relevant to systems in which a flow velocity in one of the dimensions can be neglected:**

*2D Navier-Stokes systems:* Thin layers of fluids. *Rotating fluids*. Planetary atmospheres. Oceanic flows *Stratified layers*. *Conducting fluids in magnetic field*. *Magnetically confined plasmas*.

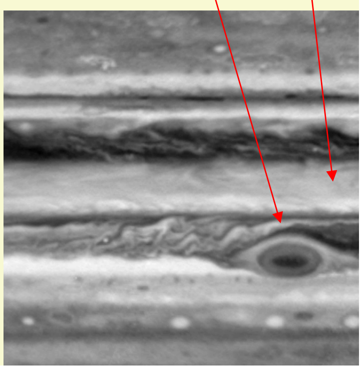


Antarctic Circumpolar Current

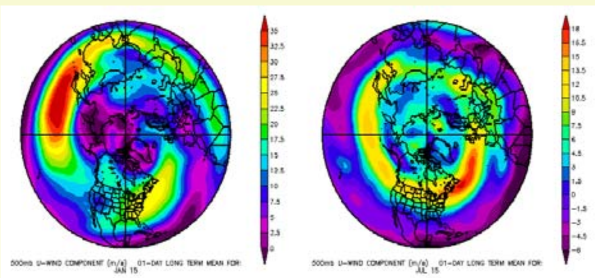


Jupiter's zonal flows

GRS



**Planetary atmospheres are dominated by turbulent structures (cyclones, zonal winds, *etc*)**



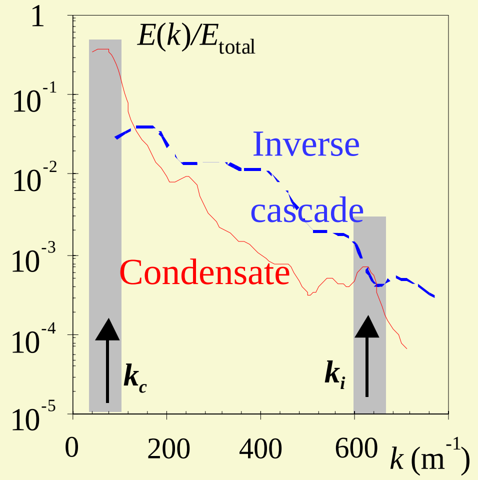
Earth: atmospheric zonal flows

# Plasma-fluid similarity

**The onset of a mean vortex flow (spectral condensation)**

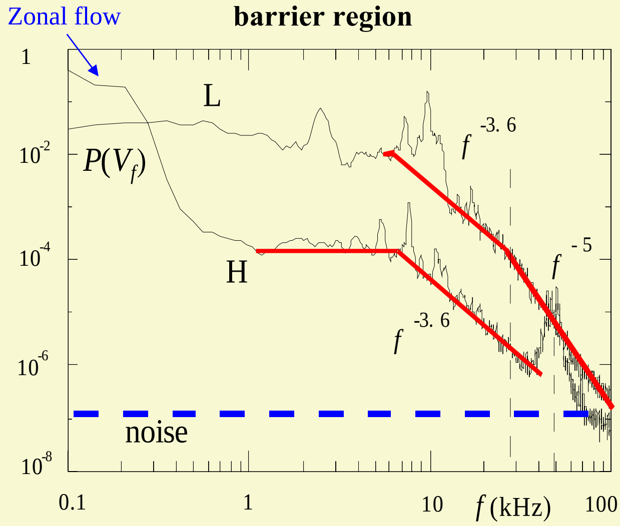
**seems to coincide with the reduction in the broad-band turbulence**

**During monopole formation in 2D bounded turbulence**




Shats, Xia, Punzman, *Phys. Rev. E*, 046409 (2005)

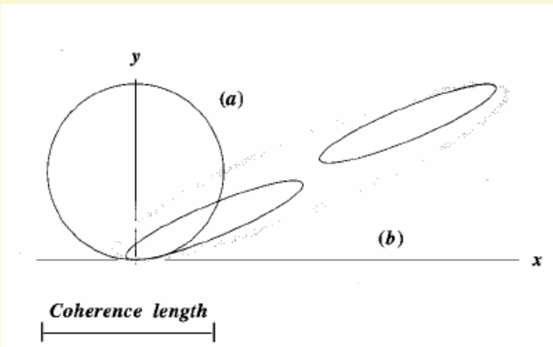
**During L-H transition in the transport barrier region**




Xia, Shats, Punzman, *Phys. Rev. Lett.* 255003 (2006)

Similarity with the L-H transition physics

# Shear flow suppression of turbulence




If the eddy is isolated it stretches into the shape indicated by the gray shaded curve. In turbulence, the eddy loses coherence in a coherence length, represented as a breakup into two eddies. The loss of coherence reduces the y scale relative to that of the reference eddy.

P. W. Terry, Rev. Mod. Phys. (2000)

The condition for the turbulence suppression

$$\tau_s < \tau_e$$

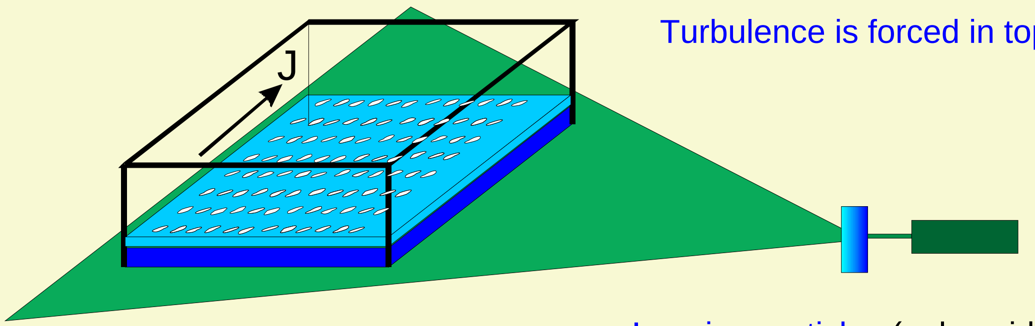
**Shear suppression model**

- has its origin in plasma physics;
- substantial (indirect) evidence in plasma experiments;
- vortex breaking has not been observed in plasma;
- not clear if it is universal or uniquely 2D effect;
- shear suppression in fluids is never observed;
- shear in fluids only generates new instabilities (turbulent boundary layers etc.)


•Biglari, Diamond, Terry, Phys. Fluids B (1990) 1
•Shaing, Crume, Houlberg, Phys. Fluids B (1990)
•P.W. Terry, Phys. Plasmas **7** (2000) 1653
•D.C. Montgomery, Phys. Plasmas 7 (2000) 4785.
•P.W. Terry, Phys. Plasmas **7** (2000) 4787;

# Quasi-2D fluid experiment

# Quasi 2D fluid: experimental setup




Stratified layer fluid layers

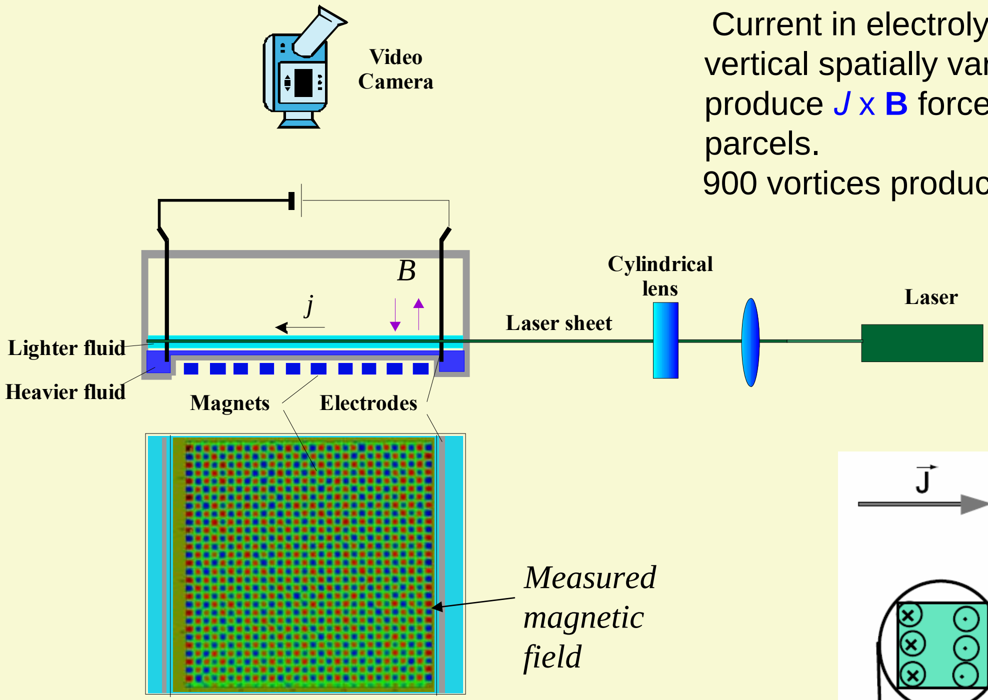
Turbulence is forced in top layer

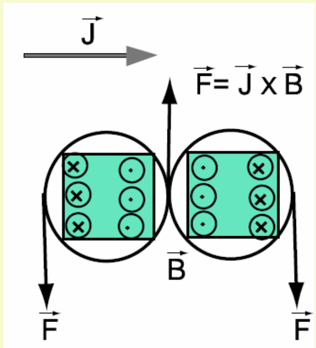
Imaging particles (polyamid $\rho$=1.04, 50 um) suspended in the top layer illuminated using laser sheet (1mm thick)

- Perspex cell 0.3 x 0.3 $m^2$
- Bottom layer: isolator Fluorinert FC-77 (resist. = $2 \times 10^{15}$ Ohm cm; SG = 1.78)
- Top layer: electrolyte NaCl solution (SG = 1.04)

# 2D turbulence forcing

Current in electrolyte interacts with vertical spatially varying *B* to produce *J* x **B** force acting on fluid parcels.
900 vortices produce turbulent flow



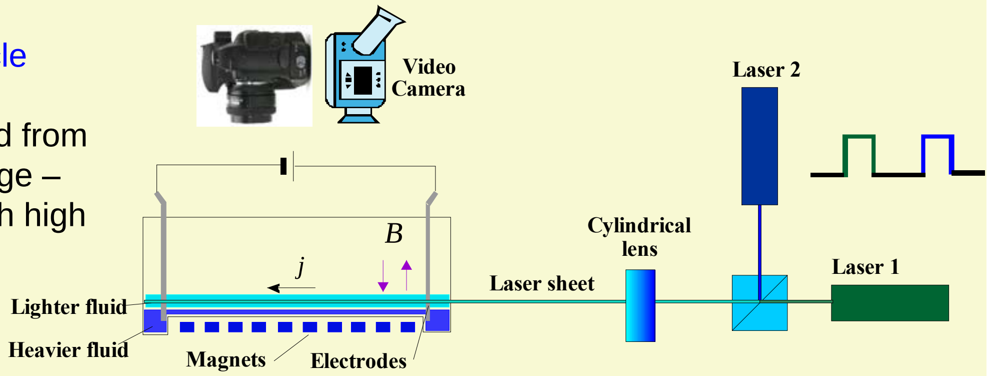



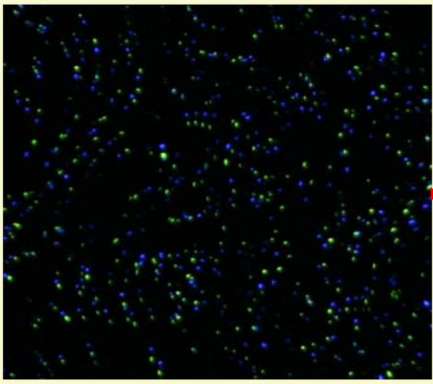

Matrix of 900 magnets (0.35 T) submerged in the bottom layer

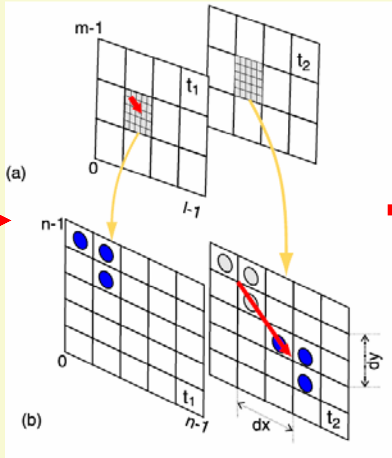
# Particle streaks in 2D turbulence

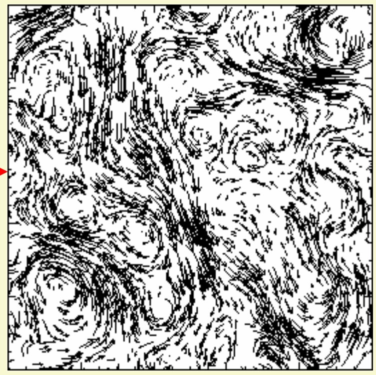
# Particle Image Velocimetry

Novel 2-colour particle image velocimetry : velocity fields derived from a single camera image – use still cameras with high resolution sensors





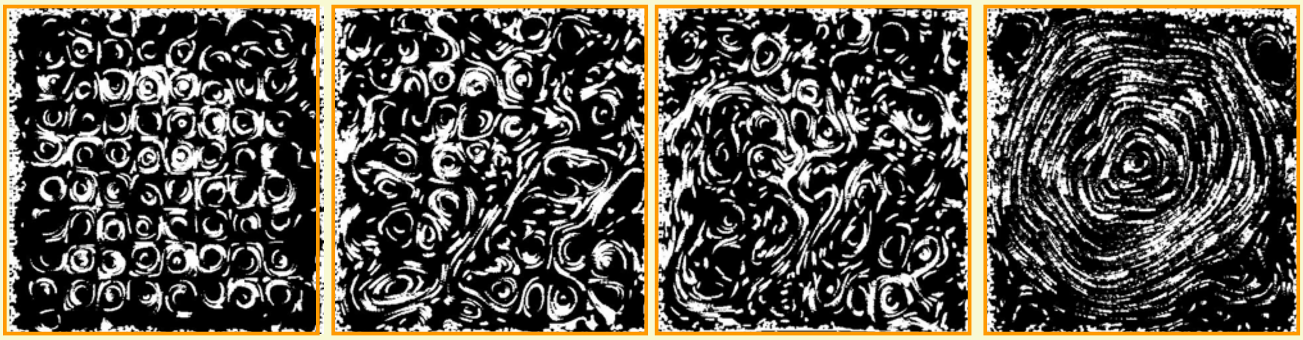
## Suppression of turbulence by mean flow

**(a) Self-generated flow (condensate)**

**(b) Externally imposed flow**

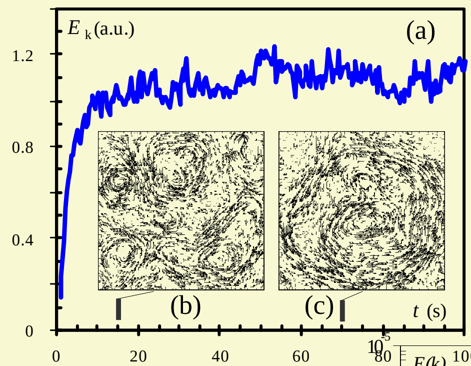
M.G. Shats, H. Xia, H. Punzmann and G. Falkovich
Suppression of turbulence by self-generated and imposed flows,
**Physical Review Letters**, **99**, *164502* (2007)

# Evolution turbulence during spectral condensation



$t$ = 3 s $t$ = 9 s $t$ = 25 s $t$ = 60 s

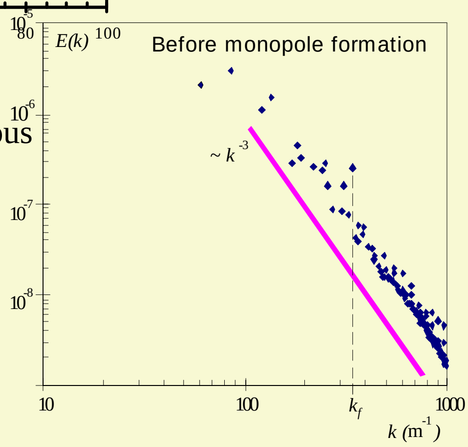
# Spectra of total velocity

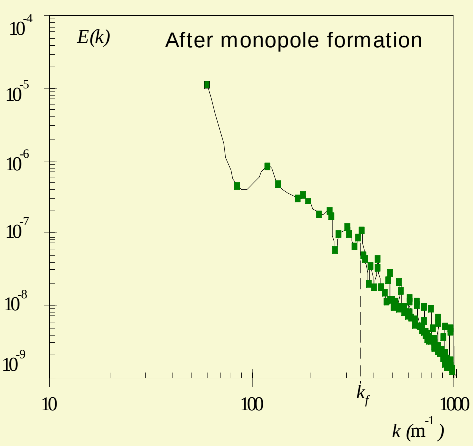



Total spectra include mean and fluctuating velocity components

$$E_{tot}(k) = 1/N \sum_{n=1}^{N} F(V)F^*(V)$$

Average of 100 instantaneous 2D Fourier Transform spectra, $\Delta T$ = 8 s



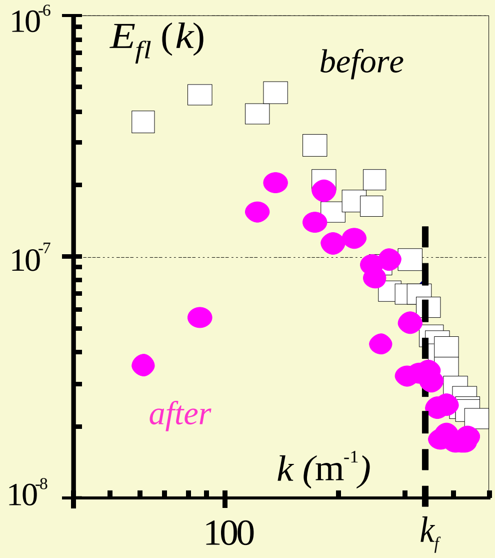

$k^{-3}$ power law is not due to the turbulent cascade, but rather due to coherent structures

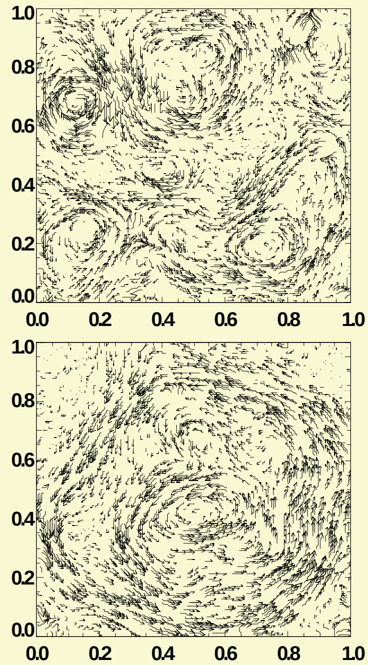
[M. Chertkov, C. Connaughton et al. (PRL 2007)]

# Spectra of turbulent velocity fields

Average of 100 instantaneous spectra of turbulent velocity fields, **mean flow subtracted**

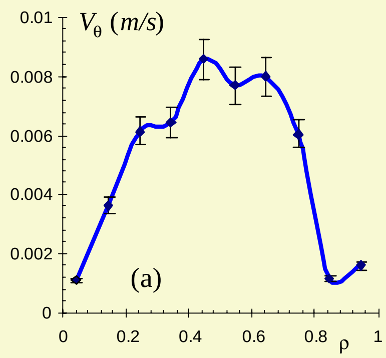
$$E_{fl}(k) = 1/N \sum_{n=1}^{N} F\left(V - \langle V \rangle_N\right) F^*\left(V - \langle V \rangle_N\right)$$

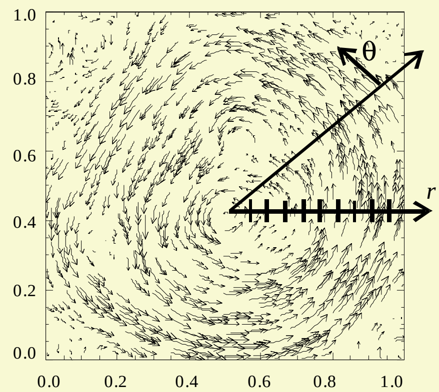


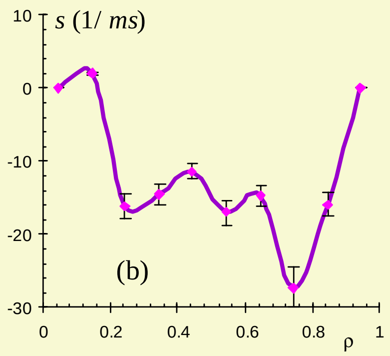




**Large scales with $k$ < 150 m$^{-1}$ suppressed**

# Mean flow and its shear



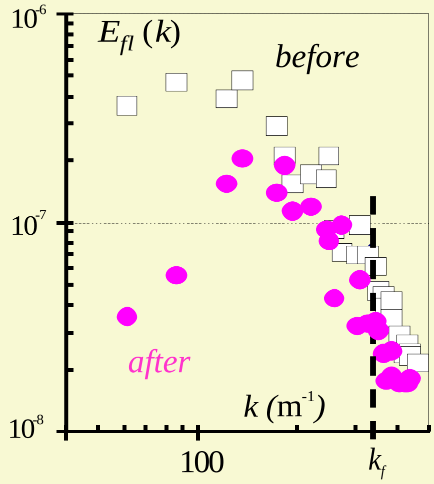

$$\Omega = V_\theta / r$$

$$\omega_s = l \frac{d\Omega}{dr} = l\left[\frac{1}{r}\frac{dV_\theta}{dr} - \frac{V_\theta}{r^2}\right] = sl$$

Average shear of s = 15 m$^{-1}$s$^{-1}$ suppresses eddies with $k = \pi / l < 160$ m$^{-1}$

**Suppression means reduction in the eddy lifetime**

# Scale reduction by shear in turbulence




Shear suppression condition is satisfied in the middle of the energy inertial range

- Development of spectral condensate in the form of large vortex produces shear flow
- Shear is sufficient to affect eddies with $l$ >20 mm and lifetimes ~ (1-2) s
- Broadband spectrum of the turbulent velocity fluctuations is reduced at $k < 160 m^{-1}$

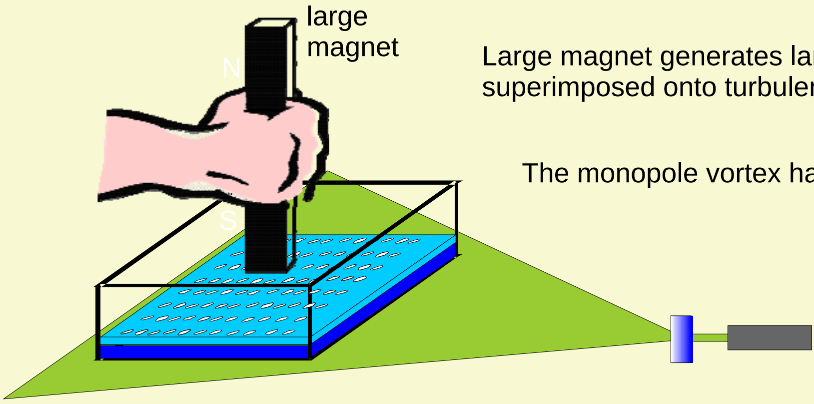
**The first experimental evidence of the shear turbulence suppression in fluids**

# Suppression of turbulence by mean flow

## (a) Self-generated flow (condensate)

## (b) Externally imposed flow

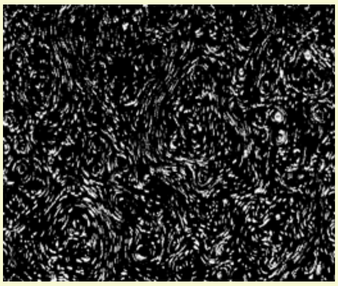
# Flow externally imposed on turbulence



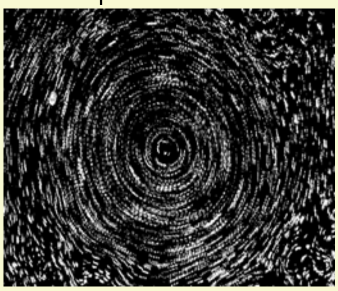

Large magnet generates large vortex superimposed onto turbulence.

The monopole vortex has slow decay time t>40sec.

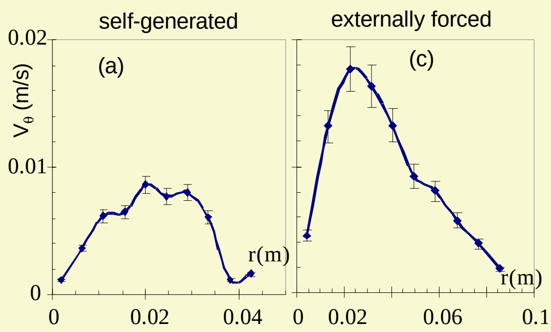
Particle streak photo of turbulent flow

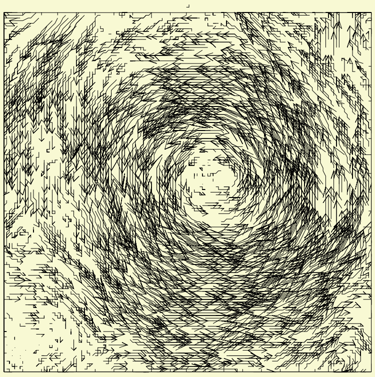


Turbulence with externally imposed mean flow



# Imposed mean flow



| | Self-organized | Externally imposed |
| --- | --- | --- |
| *Shearing rate at the forcing scale* $s = \omega_s \tau_e = \frac{l^2}{S_1}\frac{d\Omega}{dr}$ | 0.2 | 1.1 |
| *Sweeping rate at the forcing scale* $sw = \omega_{sw}\tau_e = \frac{V_\theta}{l}\frac{l}{S_1} = \frac{V_\theta}{S_1}$ | 0.75 | 7 |

Imposed flow can affect turbulence in two ways:
Shearing
Sweeping

If the flow velocity is large, sweeping seems to play a dominant role

**Imposed flow affects scales down to the forcing scale**

# Sweeping due to mean flow

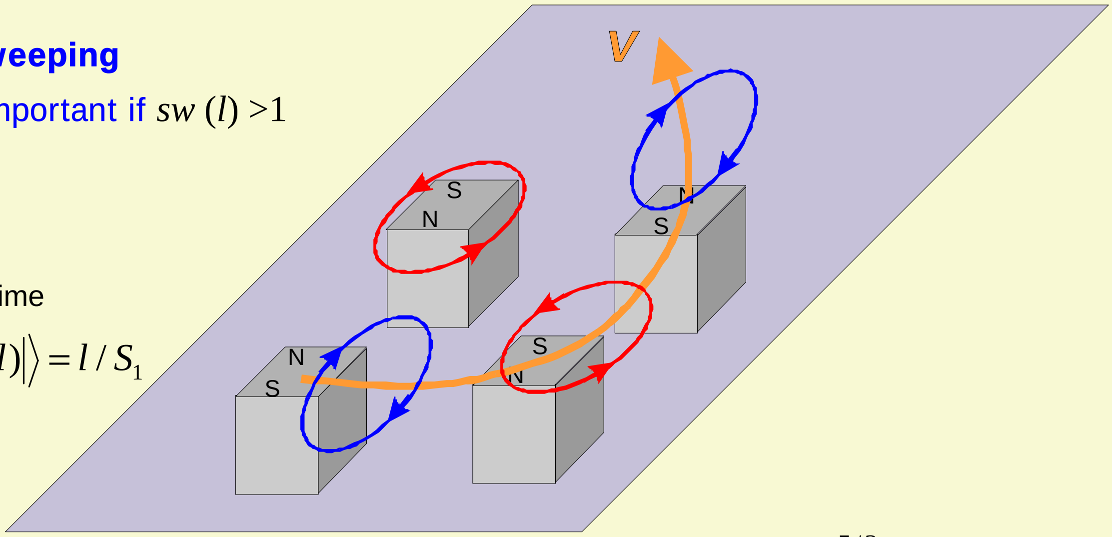
**Mean flow sweeps forcing scale vortices relative to magnets**

**Sweeping**

becomes important if $sw\,(l) > 1$

Eddy life time

$$\tau_e = l / \langle |\delta u(l)| \rangle = l / S_1$$




**Shearing acts more efficiently on large scales:** $s = \omega_s \tau_e \sim l^{5/3}$

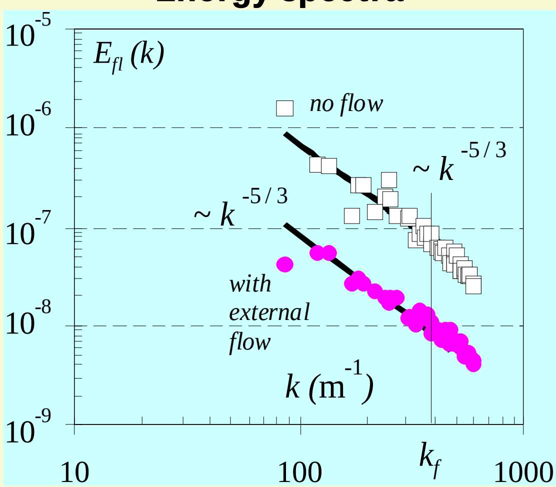
**Sweeping is more efficient on small scales:** $sw = \omega_{sw} \tau_e = \frac{V_\theta}{S_1} \propto l^{-1/3}$

Eddy life time: $\tau_e = l / \langle |\delta u(l)| \rangle = l / S_1$

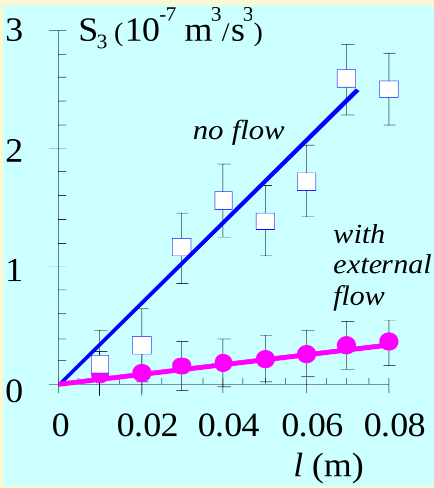
# Imposed flow reduces turbulence

**Externally imposed flow leads to turbulence reduction but retains $k^{-5/3}$ scaling**

**Energy spectra**




**3rd order structure function**




The energy flux through inertial range $\varepsilon = -2/3\, S_3(l)/l$

$\varepsilon$ is constant for all scales $l$

$\varepsilon$ is reduced by a factor of ~10 in the presence of the strong flow

**Mean flow reduces energy injected into turbulent cascade $\varepsilon$, leads to $E(k)$ drop:**

$$E(k) = C_k \varepsilon^{2/3} k^{-5/3}$$

# Summary

- Mean flows (both self-generated and imposed) suppress turbulence in 2D flows
- Weaker flow: shearing of larger turbulent eddies in the inertial range distorts the $E$(k) spectrum
- Stronger flow affects energy injection at the forcing scale via shearing and sweeping
- The inverse energy cascade persists in the presence of mean flow
- Energy spectra of turbulence do not change in the presence of strong flows
- No evidence in support of “eddy-breaking” in the presence of shear