Binary Interaction Approximation to $N$-Body Problems

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The authors have developed an algebraic model for multibody problems, and have shown that the momentum transfer cross-section with our model is in excellent agreement with the exact one [1-2]. In a binary system with an impact parameter $b = b_0 \cot \chi/2$, a typical velocity change in the relative velocity $\Delta g$ is given by

$$\Delta g = 2g \sin \frac{\chi}{2} \sim \epsilon g, \quad \epsilon \equiv \frac{b_0}{\Delta \ell},$$

(1)

where $b_0$ corresponds to $\chi = \pi/2$ scattering, and $\Delta \ell$ is the average interparticle separation. In $N$-body systems with $\epsilon \ll 1$, such as the fusion plasma, Eq. (1) means that three-or-more body interaction is of order of $\epsilon^2$ and can be ignored. It should be noted that the Debye lengths $\lambda_D$ in fusion plasmas generally satisfy $\lambda_D \gg \Delta \ell$, thus typical interaction is characterized by the nondimensional parameter $\epsilon$. This parameter coincides approximately with $U/K$, where $U$ and $K$ stand for the potential and kinetic energies. The main computational difficulty in the full $N$-body calculation is that the number of floating point operation, FLOP, required for the force calculation is of order of $N^3$ to advance the system in time. If three-or-more interaction of order $\epsilon^2$ can be ignored, the FLOP reduces to order $N^2$. In this study, we compare numerically the binary approximation to an $N$-body system with the exact one, both using the Runge-Kutta-Fehlberg method. Fig.1 shows position and velocity of a field particle which rests at the origin initially. Here we have used the RKF method with an absolute tolerance of $10^{-16}$. Field particles are randomly distributed initially in the phase space. The 122-body system is integrated for $\Delta t \equiv \Delta \ell/g_{th}$, i.e. the time for a typical particle with a thermal speed to travel the average interparticle spacing. The CPU time for the binary approximation is only around 0.3 sec, while the full $N$-body system is around 90 sec.

Figure 1: $N = 122$-body system. The circles represent the initial position in the configuration space on the left, and the initial velocity on the right. Lines are exact trajectories of the full $N$-body system. The symbol '+' on the left and the symbol 'x' are the final points.
