Preliminary Study on Uncertainty-Driven Plasma Diffusion.

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A classical particle obeys the deterministic equation of motion which gives the particle trajectory in the phase space $(\boldsymbol{r}, \boldsymbol{v})$ at a time t. The actual trajectory of a particle, however, is stochastic in the phase space with uncertainties $\Delta\boldsymbol{r}$, $\Delta\boldsymbol{v}$, and ΔE in a time interval Δt because of the uncertainty relation:

$$\Delta r \Delta v > h/m, \;\; \Delta E > h/\Delta t, \tag{1}$$

where h, m and ΔE stand for Planck constant, the mass and uncertainty in energy, respectively. Equation (1) tells us that (i) lighter particle has larger uncertainty and (ii) uncertainty in energy ΔE is larger for shorter time intervals. If we choose $\Delta t = \Delta\ell/v\,(0)$, where $\Delta\ell$ stands for the average interparticle separation and $v\,(0)$ the initial particle speed, then the uncertainty in energy is given as $\Delta E \sim mv\,(0)\,\Delta v$. Thus, from Eq. (1), we have

$$\Delta r \sim \Delta\ell, \;\; \Delta v \sim h/m\Delta\ell. \tag{2}$$

Let us assume that a charged particle with a charge $q > 0$ is moving in the presence of a uniform magnetic field $\boldsymbol{B}$. The particle is assumed to be in a typical fusion plasma of $T = 10$ keV and $n = 10^{20}$ m^{-3}. First, we integrate the equation of motion for the particle for the time interval of Δt to get the classical position in the phase space $(\boldsymbol{r}\,(\Delta t)\,, \boldsymbol{v}\,(\Delta t))$. To $(\boldsymbol{r}\,(\Delta t)\,, \boldsymbol{v}\,(\Delta t))$ we add the uncertainty $(\Delta\boldsymbol{r}, \Delta\boldsymbol{v}))$, the magnitude of which is given by Eq. (2). These procedure are repeated until the time t reaches $\tau_c \equiv 2\pi m/qB$, i.e. the cyclotron period. Figure 1 shows the particle trajectory, in which a deviation $\delta\boldsymbol{r}$ form the

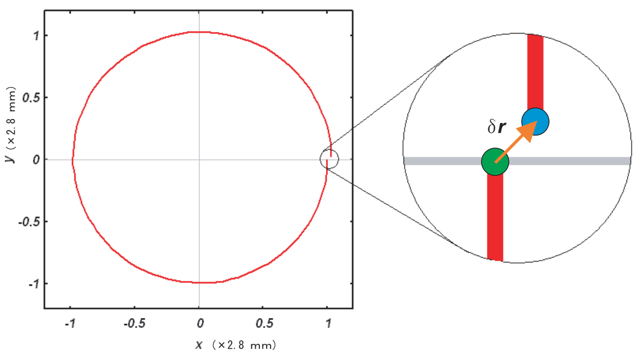

Figure 1: Deviation of cyclotron motion due to uncertainty in one gyration.

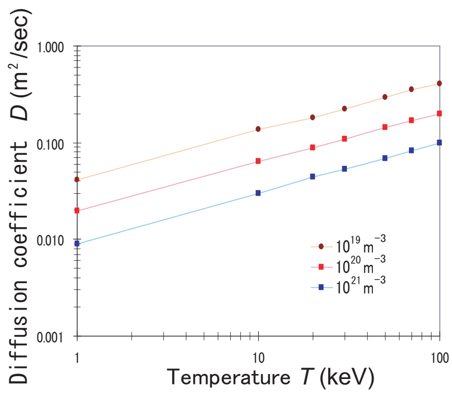

Figure 2: T and n dependence of diffusion coefficient.

classical motion is seen. The above calculation for a fixed T and n is repeated for $N = 10^4$ times, form which the diffusion coefficient $D \sim \langle \delta\boldsymbol{r}^2 \rangle / \tau_c$ can be obtained. Figure 2 shows the temperature and density dependence of the diffusion coefficient $D = D\,(T, n) \sim 0.1$ m^2/s.