Binary Interaction Approximation to N-Body Problems

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The authors have developed an algebraic model for multibody problems, and have shown that the momentum transfer cross-section with our model is in excellent agreement with the exact one [1-2]. In a binary system with an impact parameter $b = b_0 \cot \chi/2$, a typical velocity change in the relative velocity Δg is given by

$$\Delta g = 2g \sin \frac{\chi}{2} \sim \epsilon g, \; \epsilon \equiv \frac{b_0}{\Delta \ell}, \tag{1}$$

where b_0 corresponds to $\chi = \pi/2$ scattering, and $\Delta\ell$ is the average interparticle separation. In N-body systems with $\epsilon \ll 1$, such as the fusion plasma, Eq. (1) means that three-or-more body interaction is of order of ϵ^2 and can be ignored. It should be noted that the Debye lengths λ_D in fusion plasmas generally satisfy $\lambda_D \gg \Delta\ell$, thus typical interaction is characterized by the nondimensional parameter ϵ. This parameter coincides approximately with U/K, where U and K stand for the potential and kinetic energies. The main computational difficulty in the full N-body calculation is that the number of floating point operation, FLOP, required for the force calculation is of order of N^3 to advance the system in time. If three-or-more interaction of order ϵ^2 can be ignored, the FLOP reduces to order N^2. In this study, we compare numerically the binary approximation to an N-body system with the exact one, both using the Runge-Kutta-Fehlberg method. Fig.1 shows position and velocity of a *field*

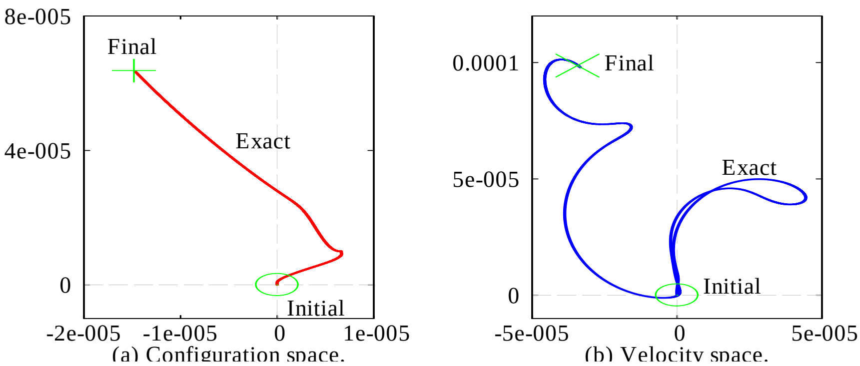

Figure 1: N =122-body system. The circles represent the initial position in the configuration space on the left, and the initial velocity on the right. Lines are *exact* trajectories of the full N-body system. The symbol '+' on the left and the symbol 'x' are the final points.

particle which rests at the origin initially. Here we have used the RKF method with an absolute tolerance of 10^{-16}. Field particles are randomly distributed initially in the phase space. The 122-body system is integrated for $\Delta t \equiv \Delta\ell/g_{\rm th}$, i.e. the time for a typical particle with a thermal speed to travel the average interparticle spacing. The CPU time for the binary approximation is only around 0.3 sec, while the full N-body system is around 90 sec.

[1] S. Oikawa, H. Funasaka, J. Plasma Fusion Res. **36** (2008) S1073

[2] K. Higashi, S. Oikawa, *this conference.*