Proceedings of ITC18, 2008

Modeling of Collisional Transport in Ergodic Region

Ryutaro KANNO[1,2], Masanori NUNAMI[3], Shinsuke SATAKE[1], Hisanori TAKAMARU[4], Masao OKAMOTO[4] and Nobuyoshi OHYABU[1,2]

[1]*National Institute for Fusion Science, Toki 509-5292, Japan*
[2]*Department of Fusion Science, Graduate University for Advanced Studies, Toki 509-5292, Japan*
[3]*Research Institute for Sustainable Humanosphere, Kyoto University, Japan Science and Technology Agency, CREST, Uji 611-0011, Japan*
[4]*Department of Computer Science, Chubu University, Kasugai 487-8501, Japan*

In recent tokamak experiments it is found that so-called stochastic diffusion theory based on the "field line diffusion" overestimates the radial energy transport in collisionless edge plasma affected by resonant magnetic perturbations, though the perturbations induce chaotic behavior of the field lines. These results imply that the conventional modeling of the edge transport should be reconsidered for covering the range from lower to higher collisionalities. It is required to construct the modeling extracting information contributing to the transport in macro-scale from kinetic motions in micro-scale. A simulation study of collisional transport in the ergodic region is attempted for estimating the transport coefficients according to the modeling. By using a drift kinetic equation solver without the assumption of nested flux surfaces (the KEATS code), it is possible to execute the estimation. In this paper, we report the modeling constructed from the viewpoint of stochastic approach and the simulation study of the ion transport in the ergodic region under the assumption of neglecting effects of an electric field and neutrals.

Keywords: collisional transport, ergodic region, chaotic field line, stochastic analysis, diffusion process, δf simulation

1 Introduction

In recent tokamak experiments it is found that so-called stochastic diffusion theory based on the "field line diffusion" [1] overestimates the radial energy transport in the edge added resonant magnetic perturbations (RMPs) [2, 3]. This fact is discovered in the experiments of edge localized modes (ELMs) suppression by adding RMPs to the edge plasma. (The idea of suppressing the ELMs by using RMPs has been proposed in Ref. [4].) When the RMPs induce a chaotic behavior in the field lines, the theory predicts that a thermal diffusivity is given by "diffusion of the field lines." In collisionless edge ergodized plasma, the experimental thermal-diffusivity $\chi^{\rm exp} = -q_{\rm r}/(n\nabla T)$ is inconsistent with the prediction of the stochastic diffusion theory $\chi^{\rm ql}$; i.e. $\chi_{\rm e}^{\rm exp}/\chi_{\rm e}^{\rm ql} \ll 1/10$ for the electron thermal diffusivity [3], where $q_{\rm r}$ is the radial energy flux, n the density, and T the temperature measured in the experiments. The above experimental results imply that the conventional modeling of transport in the ergodic region should be reconsidered in torus plasmas, and kinetic modeling is required for understanding stochastic transport in the ergodic region [5].

For construction of kinetic modeling, statistical properties of the guiding center orbits in the ergodic region are previously studied in the monoenergetic test-particle simulations in detail [6]. The doubt on the validity of the stochastic diffusion theory for the collisionless limit has been reported; i.e. guiding center orbits in the ergodic region are not Brownian for lower-collisionality. On the other hand, for the collisional limit, the radial behavior of the guiding center orbits is numerically observed to be a standard diffusion process. These results mean that it is not a trivial problem whether the transport coefficients in the ergodic region can be always estimated by tracing monoenergetic test-particle orbits. We should note that the statistical properties of the neoclassical radial diffusion for the range from lower to higher collisionalities in a magnetic configuration having nested flux surfaces are confirmed through direct comparison with a Brownian process in configuration space given by tracing monoenergetic test-particle orbits [7]. It is important to construct the modeling extracting information contributing to the collisional transport in macro-scale from the kinetic motions in micro-scale, even if the guiding center orbits themselves are non-Brownian. The modeling of the transport should be reconsidered from the viewpoint of stochastic approaching the statistics of kinetic motions exposed to noise caused by the RMPs.

In order to estimate transport coefficients in the ergodic region, we develop a new transport simulation code without the assumption of nested flux surfaces; the code is named "KEATS" [8, 9, 10]. The code is programmed by expanding the well-known Monte-Carlo particle simulation scheme based on the δf method [11, 12, 13]. By

author's e-mail: kanno@nifs.ac.jp

using the KEATS code, it is possible to execute the estimation.

In this paper, we discuss the modeling of the transport exposed to noise caused by chaotic behavior of field lines, and apply the KEATS code to a torus plasma having the ergodic region for estimating the transport coefficients (in particular, thermal diffusivity). Here, because of a limited computational-time we treat ions (protons) in higher-collisionality for our numerical study of the transport in the ergodic region. The modeling and simulations are useful for understanding transport properties in the ergodic region generated in the edge of a helical plasma. The details of the modeling are discussed in Sec. 2. In Sec. 3, the simulation results are shown. Finally, summary is given in Sec. 4.

2 Modeling of Collisional Transport

First, we consider the collisional transport in macro-scale under the assumption of neglecting effects of chaotic field lines, an electric field, and neutrals. For a fluid quantity $u(t,\boldsymbol{x})$, i.e. the density $u = n(t,\boldsymbol{x})$ or energy $u = (3/2)nT$, the fluid equation is given as [14, 15]

$$\frac{\partial u(t,\boldsymbol{x})}{\partial t} + \nabla\cdot(\boldsymbol{V}u) - \frac{1}{2}\nabla\cdot(\mathsf{D}\cdot\nabla u) + \nu u = h(t,\boldsymbol{x}), \quad (1)$$

where $\nabla = \partial/\partial\boldsymbol{x}$, a position in Euclidean space $\boldsymbol{x}\in\mathbf{R}^3$, time $t\in[0,t_1)$, the initial condition $u(0,\boldsymbol{x}) = \phi(\boldsymbol{x})$ at $t = 0$, and the boundary condition $u(t,\boldsymbol{x}) = g(t,\boldsymbol{x})$ at the boundary. Here, the mean velocity $\boldsymbol{V}$ and the diffusion coefficient $\mathsf{D} = (D^{ij})$ are assumed to be given functions of t and $\boldsymbol{x}$.

The above fluid equation is the initial-boundary value problem (written for t replaced by $t_1 - t$):

$$(L+\nu_*)u + \frac{\partial u}{\partial t} = h_*(t,\boldsymbol{x}) \quad \text{in } Q = \mathcal{M}\times[0,t_1), \quad (2)$$

$$u(t_1,\boldsymbol{x}) = \phi(\boldsymbol{x}) \quad \text{on } \mathcal{M}, \quad (3)$$

$$u(t,\boldsymbol{x}) = g(t,\boldsymbol{x}) \quad \text{on } \mathcal{S}, \quad (4)$$

where $\mathcal{M}$ is a bounded domain with the boundary $\partial\mathcal{M}$, $\mathcal{S} = \partial\mathcal{M}\times[0,t_1)$, and

$$Lu := \left\{\frac{1}{2}D^{ij}\frac{\partial^2}{\partial x^i\partial x^j} + V_*^i\frac{\partial}{\partial x^i}\right\}u, \quad (5)$$

$$V_*^i = -V^i + \frac{1}{2}\frac{\partial D^{ij}}{\partial x^j}, \quad (6)$$

$$\nu_* = -\nu - \frac{\partial V_i}{\partial x^i}, \quad (7)$$

$$h_* = -h. \quad (8)$$

The solution of Eqs. (2)-(4) is given as [16]

$$\begin{aligned} u(t,\boldsymbol{x}) = {} & E_{t,x}g(\tau,\boldsymbol{\xi}(\tau))\exp\left[\int_t^\tau \nu_*(s,\boldsymbol{\xi}(s))\mathrm{d}s\right]\chi_{\tau<t_1} \\ & + E_{t,x}\phi(\boldsymbol{\xi}(T))\exp\left[\int_t^\tau \nu_*(s,\boldsymbol{\xi}(s))\mathrm{d}s\right]\chi_{\tau=t_1} \\ & - E_{t,x}\int_t^\tau h_*(s,\boldsymbol{\xi}(s))\exp\left[\int_t^s \nu_*(\lambda,\boldsymbol{\xi}(\lambda))\mathrm{d}\lambda\right]\mathrm{d}s, \quad (9) \end{aligned}$$

where $E_{t,x}$ is the expectation operator given by the diffusion process:

$$\mathrm{d}\xi^i(t) = \sigma^i_j(t,\boldsymbol{\xi}(t))\mathrm{d}w^j(t) + V^i_*(t,\boldsymbol{\xi}(t))\mathrm{d}t \quad (10)$$

having $D^{ij} = \sigma^i_k g^{k\ell}\sigma^j_\ell$, $g^{k\ell}$ is the metric, $\boldsymbol{w}(t)$ is a Brownian motion, χ_A is the indicator function of a set A, τ is the first time $\lambda\in[t,t_1)$ that $\boldsymbol{\xi}(\lambda)$ leaves $\mathcal{M}$ if such a time exists and $\tau = t_1$ otherwise. Therefore, the transport in macro-scale is expressed by using the diffusion process given as Eq. (10).

The diffusion process given as Eq. (10) is originally caused from the collision operator. Let us take the following collision operator $C(f)$:

$$C(f) = \nu_{\rm col}\frac{\partial}{\partial\boldsymbol{v}}\cdot\left\{\boldsymbol{v}f + v_{\rm th}^2\frac{\partial f}{\partial\boldsymbol{v}}\right\}, \quad (11)$$

where $f = f(t,\boldsymbol{x},\boldsymbol{v}) = f_{\rm M}(t,\boldsymbol{x},\boldsymbol{v};\boldsymbol{V}(t,\boldsymbol{x})) + \delta f(t,\boldsymbol{x},\boldsymbol{v})$ is a distribution function expressing statistics of kinetic motions in micro-scale, $f_{\rm M}$ is a shifted Maxwellian background, $\nu_{\rm col} = \nu_{\rm col}(\boldsymbol{x})$ is the collision frequency, $v_{\rm th}$ the thermal velocity, $(\boldsymbol{V}+\boldsymbol{v})$ the velocity of a guiding center, and $\boldsymbol{V} = \boldsymbol{V}(t,\boldsymbol{x})$ the mean velocity [17]. The operator (11) is simpler, but is used only to get a rough idea of collisional effects [18].

We consider the motion of a guiding center along a field line for estimation of radially spreading the guiding centers in a perturbed field. The guiding center motion exposed to the collisions (11) is given as an Ornstein-Uhlenbeck process:

$$\mathrm{d}\boldsymbol{x}(t) = (\boldsymbol{V}+\boldsymbol{v})\mathrm{d}t, \quad (12)$$

$$\mathrm{d}\boldsymbol{v}(t) = \sigma\mathrm{d}\boldsymbol{w}(t) - \nu_{\rm col}\boldsymbol{v}\mathrm{d}t, \quad (13)$$

where a perturbation field is neglected in the above equations, $\boldsymbol{V} = V_\parallel\boldsymbol{b}$, $\boldsymbol{b} = \boldsymbol{B}/B$ the unit vector along a field line, $\sigma = v_{\rm th}\sqrt{\nu_{\rm col}}$, and $\boldsymbol{B}$ the unperturbed magnetic field. Here, the effects of toroidal and helical ripples are neglected for simplicity. Here, $v_{\rm th}$ and $\nu_{\rm col}$ are assumed to be constant.

One may consider that effect of a perturbation field on the guiding center motion is interpreted as noise on the motion. If the effect is expressed as a linear operator $\tilde{\mathcal{N}}(\boldsymbol{V}+\boldsymbol{v})$, then instead of Eq. (12) the motion of a guiding center is described as

$$\mathrm{d}\boldsymbol{\zeta} = \left\{(\boldsymbol{V}+\boldsymbol{v}) + \tilde{\mathcal{N}}(\boldsymbol{V}+\boldsymbol{v})\right\}\mathrm{d}t. \quad (14)$$

After sufficient exposure to the collisions $t \gg 1/\nu_{\rm col}$ ($\nu_{\rm col}\to\infty$ and $v_{\rm th}/\sqrt{\nu_{\rm col}}$ = const.), the stochastic process $\boldsymbol{\zeta}(t)$ (written for t replaced by $t_1 - t$) becomes

$$\mathrm{d}\boldsymbol{\zeta}(t) \approx \frac{v_{\rm th}}{\sqrt{\nu_{\rm col}}}(1+\tilde{\mathsf{N}})\cdot\mathrm{d}\boldsymbol{w}(t) - (\boldsymbol{V}+\tilde{\boldsymbol{V}})\mathrm{d}t, \quad (15)$$

where $\tilde{\boldsymbol{V}} = \tilde{\mathcal{N}}\boldsymbol{V}$, and $\tilde{\mathsf{N}} = (N^i_j)$ is assumed to be a continuous function in t, together with its first t-derivative. Here, the noise expressed as $\tilde{\boldsymbol{V}}$ and $\tilde{\mathsf{N}}$ is bounded, i.e. there exist V_0 and N_0 satisfying $|\tilde{\boldsymbol{V}}|\le V_0$ and $|N^i_j|\le N_0$.

From Eqs. (9) and (10), the collisional transport in macro-scale is described as diffusion phenomenon, thus the process $\boldsymbol{\zeta}(t)$ is projected onto a diffusion process:

$$\lim_{\epsilon\to 0+}\frac{1}{\epsilon}E\left[\zeta^{i}_{t-\epsilon,\boldsymbol{x}}(t)-x^{i}\middle|\mathcal{P}^{t-\epsilon}_{t}\right]=U^{i}_{*}(t,\boldsymbol{x}), \tag{16}$$

$$\lim_{\epsilon\to 0+}\frac{1}{\epsilon}E\Big[\left\{\zeta^{i}_{t-\epsilon,\boldsymbol{x}}(t)-x^{i}\right\} \times\left\{\zeta^{j}_{t-\epsilon,\boldsymbol{x}}(t)-x^{j}\right\}\Big|\mathcal{P}^{t-\epsilon}_{t}\Big]=\tilde{D}^{ij}(t,\boldsymbol{x}), \tag{17}$$

where $\boldsymbol{\zeta}_{t-\epsilon,\boldsymbol{x}}(t)$ is a path $\boldsymbol{\zeta}(t)$ satisfying $\boldsymbol{\zeta}(t-\epsilon)=\boldsymbol{x}$, and $\boldsymbol{U}_*$ is the mean velocity affected by the noise: $\boldsymbol{U}_*=-\{\boldsymbol{V}+E[\tilde{\boldsymbol{V}}|\mathcal{P}^{\zeta}_{t}]\}$. Here, $E[\,\cdot\,|\mathcal{P}^{t-\epsilon}_{t}]$ denotes the conditional expectation with respect to $\mathcal{P}^{t-\epsilon}_{t}$, and the σ-algebra $\mathcal{P}^{t-\epsilon}_{t}$ is generated by the set of sample paths $\{\boldsymbol{\zeta}_{t-\epsilon,\boldsymbol{x}}(s); t-\epsilon\le s\le t\}$ [19]. Note that $E[\,\cdot\,|\mathcal{P}^{t-\epsilon}_{t}]$ is $\mathcal{P}^{t-\epsilon}_{t}$ measurable, i.e. $\lim_{\epsilon\to 0+}E[\,\cdot\,|\mathcal{P}^{t-\epsilon}_{t}]$ is a function of t and $\boldsymbol{x}$. The diffusion process extracted from the process $\boldsymbol{\zeta}(t)$ is given as

$$\mathrm{d}\xi^{i}(t)=\tilde{\sigma}^{i}_{j}(t,\boldsymbol{\xi}(t))\mathrm{d}w^{j}(t)+U^{i}_{*}(t,\boldsymbol{\xi}(t))\mathrm{d}t, \tag{18}$$

i.e., the diffusion in velocity space for micro-scale becomes the diffusion in configuration space for macro-scale, see the first term in the right hand side of Eq. (18), where $\tilde{D}^{ij}=\tilde{\sigma}^{i}_{k}g^{k\ell}\tilde{\sigma}^{j}_{\ell}$. In this case, the partial differential operator L given by the diffusion process $\boldsymbol{\xi}(t)$ is derived as

$$\begin{aligned}-\frac{\partial u(t,\boldsymbol{x})}{\partial t} &= \lim_{\epsilon\to 0+}\frac{u(t-\epsilon,\boldsymbol{x})-u(t,\boldsymbol{x})}{\epsilon}\\ &= \lim_{\epsilon\to 0+}\frac{1}{\epsilon}E\left[u(t,\boldsymbol{\xi}_{t-\epsilon,\boldsymbol{x}}(t))-u(t,\boldsymbol{x})\right]\\ &= \frac{1}{2}\tilde{D}^{ij}(t,\boldsymbol{x})\frac{\partial^{2}u(t,\boldsymbol{x})}{\partial x^{i}\partial x^{j}}+U^{i}_{*}(t,\boldsymbol{x})\frac{\partial u(t,\boldsymbol{x})}{\partial x^{i}},\end{aligned} \tag{19}$$

where $u(t-\epsilon,\boldsymbol{x})=E[u(t,\boldsymbol{\xi}_{t-\epsilon,\boldsymbol{x}}(t))]$ and $E[u(t,\boldsymbol{x})]=u(t,\boldsymbol{x})$.

3 Results of KEATS Code

For estimation of the radial fluxes in the ergodic region, we use a magnetic configuration which is formed by adding RMPs into a simple tokamak field having concentric circular flux surfaces, where the major radius of the magnetic axis $R_{\rm ax}=3.6$ m, the minor radius of the plasma $a=1$ m, and the magnetic field strength on the axis $B_{\rm ax}=4$ T. The unperturbed magnetic field is approximately given as $B_R=-(B_{\rm ax}R_{\rm ax}/q)Z/R^2$, $B_\varphi=-B_{\rm ax}R_{\rm ax}/R$, and $B_Z=(B_{\rm ax}R_{\rm ax}/q)(R-R_{\rm ax})/R^2$ [20], where q is the safety factor and $q^{-1}=0.9-0.5875(r/a)^2$, and $r=\sqrt{(R-R_{\rm ax})^2+Z^2}$. The RMPs causing resonance with, for example, the rational surfaces of $q=m/n=3/2, 10/7, 11/7$ are numerically given by using the perturbation field $\delta\boldsymbol{B}=\nabla\times(\alpha\boldsymbol{B})$ [6], and order of the strength is $O(|\delta B_{\rm r}/B_{\rm t}|)\sim 10^{-1}$. Here, the function α, which has unit of length, is used to represent the structure of perturbed magnetic field; $\alpha(R,\varphi,Z)=\sum_{m,n}\alpha_{mn}(\psi(R,Z))\cos\{m\theta(R,Z)-n\varphi+\varphi_{mn}\}$, where ψ is a label of magnetic flux surfaces and φ_{mn} is the phase. The

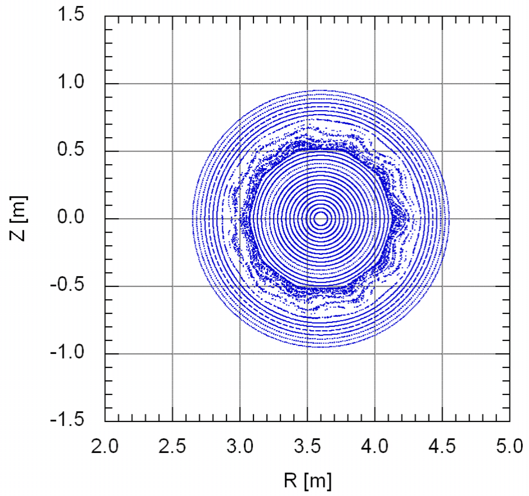

Fig. 1 Poincaré plots of the magnetic field lines on a poloidal cross section for case of $\Delta_{\delta B}=\delta B/\delta B^{(0)}=1$, where $\delta B=|\delta\boldsymbol{B}|$ is the strength of RMPs and $\delta B^{(0)}$ is the strength of the RMPs in the case b) of Fig. 2. The ergodic region is placed between $r/a=0.5$ and 0.7, where $r=\sqrt{(R-R_{\rm ax})^2+Z^2}$ and $R_{\rm ax}=3.6$ m.

Poincaré plots of the magnetic field lines on a poloidal cross section are shown in Fig. 1. The ergodic region appears in $r/a\approx 0.7\sim 1$. In the KEATS code, the number of marker particles is $N_{\rm MP}=16,000,000$.

To investigate effect of the existence of the ergodic region on the transport phenomena, we evaluate the energy flux of ions (protons) $\boldsymbol{q}_{\rm i}$, because the evaluation of electron energy flux is highly time-consuming. The calculation time for ions is about 40 hours in real time to get the result with sufficient numerical-accuracy by using the vector-parallel supercomputer SX-7, and the calculation time for electrons is estimated to be about 40 ($\approx\sqrt{m_{\rm i}/m_{\rm e}}$) times longer than the one for ions if the number of PEs (processing elements) is fixed, where 64 PEs are used in this paper.

The evaluation of the ion energy flux is carried out in the configuration having lower temperature $T_{\rm edge}\sim 100$ eV at a center of the ergodic region. The temperature profile is given as $T_{\rm i}=T_{\rm ax}\{0.02+0.98\exp[-4(r/a)^{2.5}]\}$ with $T_{\rm ax}=250$ eV, which neglects the existence of the ergodic region. The density profile is set homogeneous, $n_{\rm i}=\text{const.}=1\times 10^{19}\ \text{m}^{-3}$. The radial profiles of thermal diffusivities estimated from the KEATS computations are shown in Fig. 2, where from the modeling of the transport given in the previous section the thermal diffusivity can be estimated as $\chi^{\rm i}_{\rm eff}=q_{\rm r}/(n_{\rm i}|\partial T_{\rm i}/\partial r|)$, and $q_{\rm r}$ the radial energy flux evaluated by the KEATS code. Here, the radial mean velocity $V_{\rm r}$ is neglected because of $|V_{\rm r}/v_{\rm th}|\ll 1$. For simplicity, the radial energy fluxes are given by neglecting the existence of the ergodic region, because we have no magnetic coordinate system including several magnetic field structures as the core and ergodic regions. The energy flux $\boldsymbol{q}_{\rm i}$ is averaged over concentric circular shell region in the whole toroidal angles as if there were nested flux surfaces.

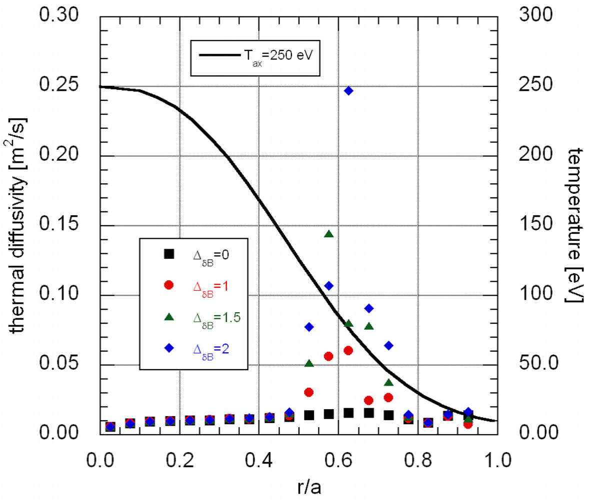

Fig. 2 Radial profile of the ion thermal diffusivity χ_r for a) no RMP (black squares), b) $\Delta_{\delta B} = \delta B/\delta B^{(0)} = 1$ (red circles), c) $\Delta_{\delta B} = 1.5$ (green triangles), and d) $\Delta_{\delta B} = 2$ (blue lozenges), where $\delta B = |\delta \boldsymbol{B}|$ is the strength of RMPs and $\delta B^{(0)}$ is the strength of the RMPs for the case b). Radial profile of temperature is fixed as $T_i = T_{ax}\{0.02 + 0.98\exp[-4(r/a)^{2.5}]\}$ with $T_{ax} = 250$ eV (black solid line), where $r = \sqrt{(R-R_{ax})^2 + Z^2}$ and $R_{ax} = 3.6$ m. The center of the ergodic region is located at $r/a \approx 0.6$.

Here, in the KEATS computations the energy flux is given as [9, 10]

$$\boldsymbol{q}_i(\boldsymbol{x}) = \overline{\int d^3v\, \frac{m_i v^2}{2}(\boldsymbol{v}_\parallel + \boldsymbol{v}_d)\delta f}, \tag{20}$$

where $\overline{\cdots}$ means the time-average, and the averaging time is longer than the typical time-scale of δf (both the orbit and collision times). It is confirmed that the energy flux evaluated by the KEATS code becomes quasi-steady after a sufficient time. The radial profiles of the thermal diffusivities χ_r in Fig. 2 show that the diffusivity in the ergodic region is proportional to the square of the strength of RMPs.

4 Summary

We have been developing the modeling of collisional transport to study the transport phenomena in the ergodic region. For estimation of transport coefficients, we apply the KEATS code to ions in the ergodic region disturbed by resonant magnetic perturbations under the assumption of neglecting effects of an electric field and neutrals, and find that the coefficients are proportional to the square of the strength of RMPs.

For a lower-collisionality case, the transport is strongly affected by the existence of the ergodic region. The strong particle and energy fluxes cause the time-evolution of the background described by the fluid equations. Further simulation study of the transport by solving simultaneously both the kinetic and fluid equations is needed for understanding of the collisionless edge ergodized plasma; the interim report of developing the KEATS code is written in Ref. [9].

Acknowledgements

This work was performed with the support and under the auspices of the NIFS Collaborative Research Program NIFS08KLDD016 and NIFS08KTAD004.

[1] A.B. Rechester and M.N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[2] K.H. Burrell *et al.*, Plasma Phys. Control. Fusion **47**, B37 (2005).
[3] T.E. Evans *et al.*, Nature Phys. **2**, 419 (2006).
[4] N. Ohyabu *et al.*, Nucl. Fusion **27**, 2171 (1987).
[5] I. Joseph *et al.*, J. Nucl. Mater. **363-365**, 591 (2007).
[6] A. Maluckov *et al.*, Physica A **322**, 13 (2003).
[7] A. Maluckov *et al.*, Plasma Phys. Control. Fusion **43**, 1211 (2001).
[8] M. Nunami *et al.*, Research Report NIFS Series No. NIFS-871 (2007).
[9] R. Kanno *et al.*, Contrib. Plasma Phys. **48**, 106 (2008).
[10] R. Kanno *et al.*, Plasma Fusion Res. **3**, S1060 (2008).
[11] W.X. Wang *et al.*, Plasma Phys. Control. Fusion **41**, 1091 (1999).
[12] S. Satake *et al.*, Plasma Fusion Res. **1**, 002 (2006).
[13] S. Satake *et al.*, Plasma Fusion Res. **3**, S1062 (2008).
[14] Y. Feng *et al.*, J. Nucl. Mater. **266-269**, 812 (1999).
[15] A. Runov *et al.*, Nucl. Fusion **44**, S74 (2004).
[16] B. Øksendal, *Stochastic Differential Equations* (Springer-Verlag, Berlin Heidelberg, 2003).
[17] P.C. Clemmow and J.P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading, Mass., 1969).
[18] D.R. Nicholson, *Introduction to Plasma Theory* (John Wiley & Sons, New York, 1983).
[19] A. D. Wentzell, *A Course in the Theory of Stochastic Processes* (McGraw-Hill, New York, 1981).
[20] J. Wesson, *Tokamaks* 3rd ed. (Oxford Univ. Pr., New York, 2004).