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Intrinsic rotation of magnetic island with finite width

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The direction of island rotation is investigated by using a two fluid equations, which includes effects of poloidal beta and also diamagnetic drifts of both ion and electron. It is found that the island rotates toward the electron diamagnetic drift in all cases in the linear regime. The parameter beta plays an important role in determining the direction of the island rotation in the nonlinear regime. It is also found that when the island width is small comparable to the ion Larmor radius, the real frequency strongly is affected to rotate toward the ion diamagnetic direction.

Keywords: island rotation, neoclassical tearing mode, two fluid model

1 Introduction

Tearing modes are resistive Magnetohydrodynamic (MHD) instabilities which break topology of ideal magnetic field and lead to the formation of helically perturbed structures called magnetic islands by tearing or reconnecting magnetic field line around the resonant surfaces in magnetic confinement devices such as LHD. The linear theory shows that the classical tearing mode is stable when the stability parameter

$$\Delta' = \frac{1}{\tilde{\psi}} \left[\left.\frac{d\tilde{\psi}}{dx}\right|_{x=+0} - \left.\frac{d\tilde{\psi}}{dx}\right|_{x=-0} \right]$$

is negative, which is given by the logarithmic jump of the perturbed magnetic flux around the rational surface[1]. However, the new type of tearing modes which can occur even when classically stable (the stability parameter is negative) due to helically perturbed bootstrap current in recent low collisional plasmas. The metastable (linearly stable but nonlinearly unstable) tearing modes are called neoclassical tearing modes (NTMs). The NTMs are found to limit the achievable beta in high performance discharges and deteriorate plasma confinement leading to plasma disruption. Therefore, much attention has been focused on NTMs both theoretically and experimentally[2, 3, 4].

To understand NTM dynamics theoretically, the modified Rutherford equation is often used as the model equation describing temporal evolution of magnetic island width,

$$\frac{\tau_s}{r_s}\frac{dw}{dt} = r_s\Delta' + \beta_p \left(\frac{\Delta_b w}{w^2 + w_0^2} - \frac{\Delta_p}{w^3} \right), \qquad (1)$$

where $\tau_s = \mu_0 r_s^2/1.22\eta$ is the resistive diffusion time at the magnetic surface of radius r_s, η is the neocalssical resistivity, Δ' is the classical tearing stability parameter which is discussed above, β_p is the poloidal beta at r_s, w_0 is the characteristic island width. The coefficients Δ_b and Δ_p represent destabilizing bootstrap current effect and polarization current effect, respectively. Typical representation of Δ_p is

$$\Delta_p = \frac{L_s^2}{kv_A^2}\omega(\omega - \omega_{*i}),$$

where ω is island rotation frequency, ω_{*i} is ion diamagnetic drift frequency, L_s is the magnetic shear length, k is the wave number of the mode, and $v_A = \sqrt{B_0^2/4\pi n m_i}$ is the Alfven velocity[5]. It clearly shows that when $\Delta_p < 0$ the polarization term plays a stabilizing role on NTM growth. And note that the island growth depends on rotation of island direction. It is of much importance to determine the direction of the island so much works have been investigated by using several models[5, 7, 8, 9]. However, effects of finite beta and also ion and electron diamagnetic drifts are not fully investigated. In this work, we investigated numerically the rotation of magnetic island for various island width including both effects.

2 Model Equations

Since there is no degree of freedom determining the island rotation in conventional MHD models, we have investigated the rotation of the island based on a reduced two fluid model which includes both effects of ion and electron diamagnetic drifts. The model equations used here is a two dimensional slab version of the four-field model[6], which consists a set of four equations that describes temporal evolutions of the magnetic flux ψ, the electrostatic potential ϕ, the perturbed electron pressure p, and the parallel ion velocity $v_\parallel$, i.e.,

$$\frac{\partial U}{\partial t} = -[\varphi,\ U] - \nabla_\parallel J + \nu\nabla_\perp^2 U$$
$$-\frac{\delta\tau}{2}\left([p,\ U] + [\varphi,\ \nabla_\perp^2 p] + \nabla_\perp^2[p,\ \varphi]\right),$$

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$$\frac{\partial \psi}{\partial t} = -\nabla_{\|}\varphi + \eta J + \delta\nabla_{\|}p,$$

$$\frac{\partial p}{\partial t} = -[\varphi,\ p] - 2\beta\delta\nabla_{\|}J - \beta\nabla_{\|}v + \frac{1}{2}\beta\eta(1+\tau)\nabla_{\perp}^2 p,$$

$$\frac{\partial v}{\partial t} = -[\varphi,\ v] - \frac{1}{2}(1+\tau)\nabla_{\|}p + D_v\nabla_{\perp}^2 v,$$

with vorticity $U = \nabla_{\perp}^2\phi$, and z-directed current density $J = \nabla_{\perp}^2\psi$. Usual Cartesian coordinate (x, y, z) is adopted. The validity of two dimensional calculation is justified in low beta plasmas, where the magnetic field is represented by $\boldsymbol{B} = B_0\hat{\boldsymbol{z}} + \nabla\psi \times \hat{\boldsymbol{z}}$, where B_0 is the ambient magnetic field along the z-axis. The normalization used here is $(x,\ y,\ t,\ \psi,\ \phi,\ n,\ v_{\|i}) = (x/a,\ y/a,\ v_A t/a,\ \psi/\varepsilon B_0 a,\ c\phi/v_A B_0 a,\ n/n_0,\ \delta v_{\|i}/v_A)$. Here, a is the minor radius, c is the light speed, ε is the inverse aspect ratio, ν is the viscosity, η is the resistivity, and both equilibrium ion and electron density n_0 are constant due to charge neutrality. We suppose that ion and electron temperatures are constant by incroducing the ratio of them $\tau = T_i/T_e$. And two parameters are introduced as two fluid parameters $\delta = (2\omega_{ci}\tau_A)^{-1}$, which is related to the ion skin depth, and $\beta = [1 + (1+\tau\beta_e)/2\beta_e]^{-1}$, where $\omega_{ci} = \sqrt{4\pi n_0 e^2/m_i}$ is the ion plasma frequency and $\beta_e = 8\pi n_0 T_e/B_0^2$ is the electron beta value, and T_e is the constant electron temperature. We note that the parameters are related to the ion Larmor radius, i.e., $(\rho_i/a)^2 = 2\tau\beta\delta^2$. The operator $[,\ ]$ denotes the Poisson bracket, $[A,\ B] = \nabla_{\perp}A \times \nabla_{\perp}B \cdot \hat{\boldsymbol{z}}$, and total derivative $d/dt = \partial/\partial t + [\varphi,\ ]$ includes only $E \times B$ drift velocity.

3 Numerical Results

3.1 Numerical Settings

The model equations are solved numerically by using pseudo spectral code in two dimensional slab geometry. We impose the zero boundary condition for radial direction x, so all components are automatically set to zero at the radial boundary. A finite-differential method is applied to the direction and the periodic boundary condition is imposed for the poloidal direction y. The domain of numerical simulation is $x = [0, 1]$ and $y = [0, 1]$. The number of grids in a simulation box is chosen to 400×20. Temporal evolution is calculated by using a predictor-corrector method with time step $\Delta t = 10^{-3}$. The pressure and magnetic flux equilibrium profiles are $p_{eq}(x) = 0.25\,(1 - \tanh(x - 0.5))\,/L_p$, and $\psi_{eq}(x) = L_s \ln[\cosh(x - 0.5)/L_s]$. And no equilibrium parts are considered for electrostatic potential ϕ and ion parallel velocity v, i.e., $\phi = \tilde{\phi},\ v = \tilde{v}$. In all simulations, we have $\nu = 10^{-4}$, $\eta = 10^{-4}$, and $D_v = 10^{-4}$.

3.2 Linear Reslults

At first, we examine the island rotation by performing linear calculation. A perturbed quantity $A(x, y, t)$ in the slab coordinates is assumed to varies as

$$\tilde{A}(x, y, t) = \sum_m A_m(x) \exp \mathrm{i}(my - \omega t) + c.c.,$$

where m is the poloidal wave number. Then model equations are reduced to

$$\frac{\partial \tilde{U}}{\partial t} = \frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{J}}{\partial y} - \frac{\partial \tilde{\psi}}{\partial y}\frac{\partial J_{eq}}{\partial x} + \nu\nabla_{\perp}^2\tilde{U} + \delta\tau\left(-\frac{\partial p_{eq}}{\partial x}\frac{\partial \tilde{U}}{\partial y} - \frac{\partial^2 p_{eq}}{\partial x^2}\frac{\partial^2 \tilde{\varphi}}{\partial x \partial y}\right),$$

$$\frac{\partial \tilde{\psi}}{\partial t} = \frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{\varphi}}{\partial y} + \eta\nabla_{\perp}^2\psi + \delta\left(-\frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{p}}{\partial y} + \frac{\partial p_{eq}}{\partial x}\frac{\partial \tilde{\psi}}{\partial y}\right),$$

$$\frac{\partial \tilde{p}}{\partial t} = \frac{\partial p_{eq}}{\partial x}\frac{\partial \tilde{\varphi}}{\partial y} + 2\beta\delta\left(\frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{J}}{\partial y} - \frac{\partial J_{eq}}{\partial x}\frac{\partial \tilde{\psi}}{\partial y}\right) + \beta\frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{v}}{\partial y} + \frac{\beta\eta}{2}(1+\tau)\nabla_{\perp}^2(p_{eq} + \tilde{p}),$$

$$\frac{\partial \tilde{v}}{\partial t} = \frac{1+\tau}{2}\left(\frac{\partial \psi_{eq}}{\partial x}\frac{\partial \tilde{p}}{\partial y} - \frac{\partial p_{eq}}{\partial x}\frac{\partial \tilde{\psi}}{\partial y}\right) + D_v\nabla_{\perp}^2\tilde{v}.$$

In Fig.1, the dependence of real frequency of $m = 1$ island is illustrated as a function of τ for various δ. Here, β value is chosen to 0.01, and the ratio of the shear scale length L_s to the density scale length is chosen to 2/3. The real frequency is normalized by the electron diamagnetic drift velocity, $\omega_{\star(e)} = \delta m p_0'$. Note that the island is rotating with pure electron diamagnetic drift velocity when normalized Ω is unity. It is found that the real frequency reduces with respect to τ in small δ case, but in large δ case the dependence is less effective and the island is almost rotating in the direction of electron diamagnetic drift.

Figure 2 shows the dependence of real frequency with respect to δ for various β. Here, $\tau = 1$ and $L_s/L_p = 2/3$ is chosen. It is found that the real frequency increases with respect to δ, indicating that effects of finite Larmor radius weaken island rotate in the direction of the ion diamagnetic drift. The dependence is same as beta increases. Totally the direction of island rotation points the electron diamagnetic drift.

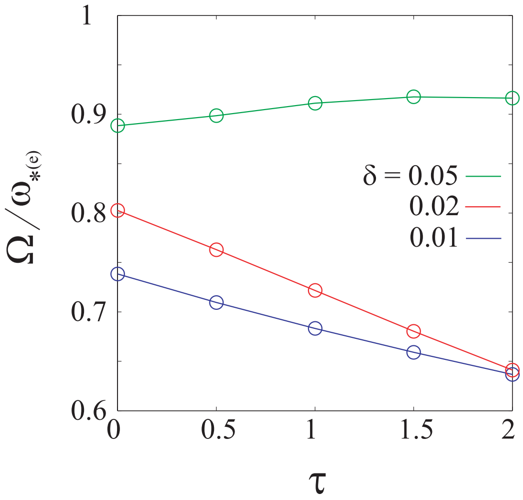

Fig. 1 The dependence of real frequency as a function of τ for various δ. Other parameters are $\beta = 0.01$ and $L_s/L_p = 2/3$.

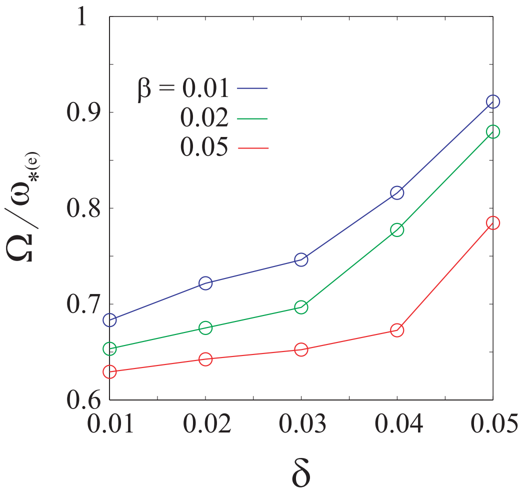

Fig. 2 The dependence of real frequency as a function of δ for various β. Other parameters are $\tau = 1$ and $L_s/L_p = 2/3$.

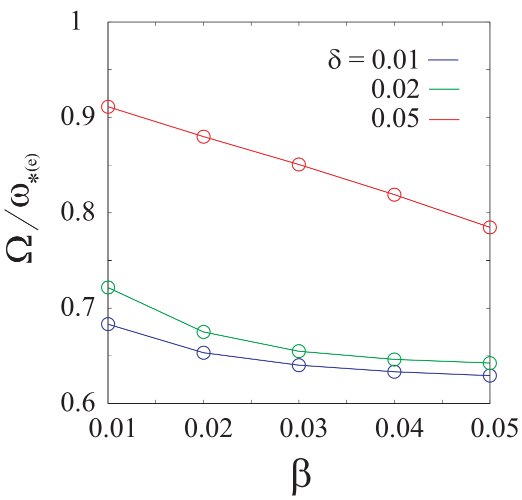

Fig. 3 The dependence of real frequency as a function of β for various δ. $\tau = 1$ and $L_s/L_p = 2/3$.

Figure 3 illustates the dependence of real frequency with respect to β for various δ. Parameters for τ and L_s/L_p are same as used in Fig.2. It is found that the real frequency monotonically decreases with respect to β. The tendency is same for different δ value. As shown in these resuls, the island rotates toward electron diamagnetic drift in the linear regime.

3.3 Nonlinear Results

Next, we examine the island rotation in the nonlinear regime. Here, δ and L_s/L_p is chosen to $\delta = 0.01$, $L_s/L_p = 2/3$. Only $m = 1$ mode grows up and saturate with finite island width because of positive Δ' in this parameter. In Figure 4 the real frequency is illustrated as a function of the island width in the case of $\beta = 0.01$. The island width w is defined as the distance between the separatrix at magnetic neutral surface which is approximately given by $w \simeq 4\sqrt{\tilde{\psi}/\psi_{eq}''}$. The ion Larmor radius is estimated as $\rho_i \simeq 0.001$ in this case. The dotted lines indicate the real frequencies calculated from linear theory. It is found that when the island width is small comparable to the ion Larmor radius, the real frequency drastically decreases to that estimated by the linear calculation. Through the transient phase from $w \simeq 0.04$ to 0.12, the real frequency varies across the line $\Omega = 0$ when it saturates. Therefore, the island finally rotates in the direction of ion diamagnetic drift whereas it is rotating toward electron diamagnetic drift in the linear regime.

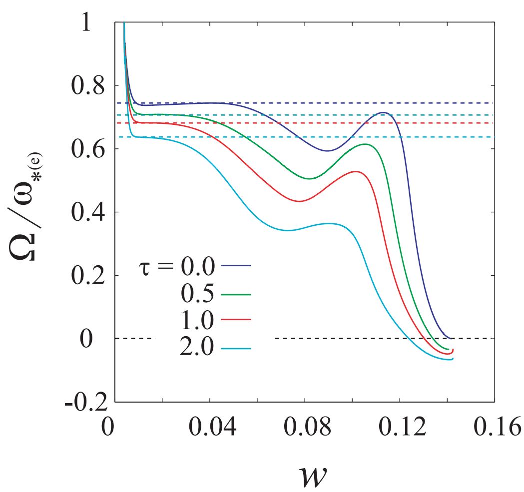

Fig. 4 The dependence of real frequency as a function of the island width for various τ. Other parameters are chosen to $\beta = 0.01$, $\delta = 0.01$, and $L_s/L_p = 2/3$.

Figure 5 shows the real frequency with respect to the island width in the case of $\beta = 0.02$, with $\rho_i \simeq 0.002$. We can also see the reduction of real frequency when the width is around the ion Larmor radius. And it has wide plateau region where linear analysis is enough valid with respect to the width, and finally becomes approximately zero and the island has locked.

Figure 6 illustrates the real frequency as a function of the island width in the case of $\beta = 0.05$. The ion Larmor radius is estimated as $\rho_i \simeq 0.01$. The real frequency has a finite positive value when the island saturates. The island is still rotating in the direction of electron diamagnetic drift. The parameter beta plays an important role in determining the direction of the island.

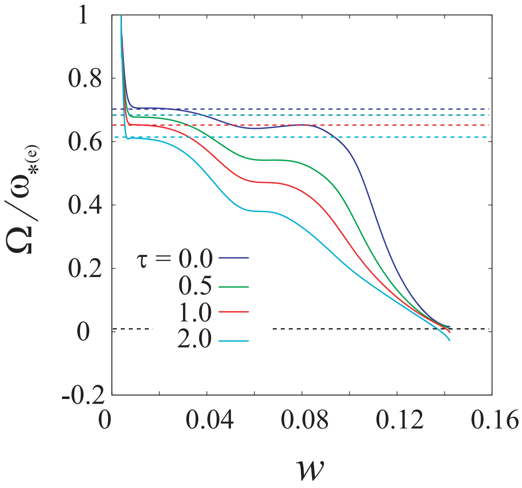

Fig. 5 The dependence of real frequency as a function of the island width for various τ in the case of $\beta = 0.02$. Other parameters are same as used in Fig.4.

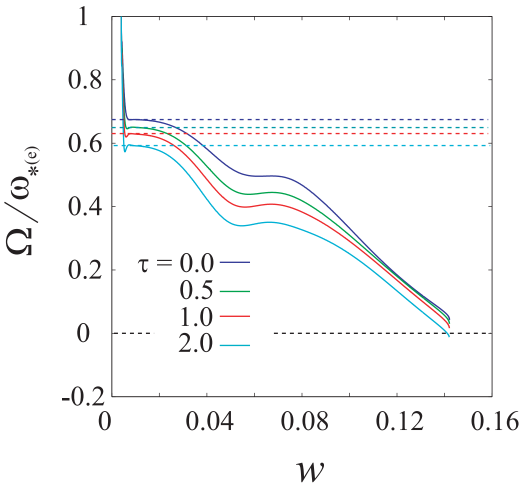

Fig. 6 The dependence of real frequency as a function of the island width for various τ in the case of $\beta = 0.05$. Other parameters are same as used in Fig.4.

We have also investigated dependence of δ on the island rotation. It is found that the growth rate of the island susceptibly depends on δ. In linearly stable region where island remains small comparable to initial noise, island rotates toward the electron diamagnetic drift.

4 Conclusion

We have investigated the intrinsic rotation of magnetic island based on a reduced two-fluid model which includes both effects of ion and electron diamagnetic drifts as well as the ion parallel motion. In nonlinear regime, the direction of island rotation strongly depends on β, whereas it rotates toward the electron diamagnetic drift in the linear regime. It is also found that when the island width is small comparable to the ion Larmor radius, the real frequency is affected to rotate toward the ion diamagnetic direction.

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