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Effects of MHD-activity-induced low-*n* error magnetic fields on the neoclassical viscosities in helical plasmas

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Effects of the perturbed magnetic field with low toroidal mode numbers (n) are considered. One cause of this type of perturbation, which has recently been studied in tokamaks, is MHD-activities. In helical/stellarator, this low-n perturbation is sometimes artificially added for island diverters. In viewpoint of the neoclassical viscosities, these perturbed magnetic fields affect on both of bounce center drifts of toroidally trapped and ripple-trapped particles. However, in usual neoclassical analyses in helical/stellarator devices assuming periodic magnetic field strength, these effects had not been studied. For future studies in helical/stellarator devices, a method to use bounce-averaged drift kinetic equation for the toroidally trapped particles is proposed.

Keywords: neoclassical transport, neoclassical viscosity, drift kinetic equation, rotational stabilization of RWM, island diverter

1. Inroduction

Neoclassical analyses in helical plasmas often assume the toroidally periodic magnetic field strength of $B=\Sigma B_{mn}\cos(m\theta-nN\zeta)$ [θ, ζ : poloidal and toroidal angles, N: the toroidal period number] and thus effects of the low-n ($n<N$) error magnetic fields in a more general expression $B=\Sigma B_{mn}\cos(m\theta-n\zeta)$ had not been investigated. However, in recent tokamak studies [1-3], this kind of low-n error magnetic field component induced by MHD-activities is considered to be important since it causes various additional neoclassical effects relating to the rotational stabilization of the resistive wall mode and island physics. The toroidal viscosity caused by additional bounce-averaged bounce center motions has been mainly investigated in these studies in tokamaks. When the low-n modes exist in helical and stellarator configurations, it affects not only on the toroidally trapped (berely trapped) particles but also on the ripple-trapped (deeply trapped) ones. It also should be noted that this type of low-n error fields is sometimes added artificially for island diverters. Although these effects for the viscosity are already covered by a recently proposed basic framework for the neoclassical transport in general non-symmetric toroidal plasmas [4], the "full torus" calculation including the low-n modes will be huge if we adopt the numerical procedures (such as variational methods and Monte Carlo methods) described in Ref.[4]. Practically usable methods to obtain the viscosity coefficients have still remained as future theme. Even in our previous study deriving and testing various analytically approximated formulas [5], the bounce-averaged effects due to the low-n modes are not included. Therefore we recently started to study an extension of the analytical approximation methods for the drift kinetic equation in helical and stellarator configurations to include these additional drift effects [6]. This understanding for the trapped particles' dynamics will be useful not only for studies of mean flows [1-3] but also for studies investigating a relation of the neoclassical transport with the zonal flow [7-8].

As an important implicit basis of the neoclassical transport analysis, we assume here existences of nested closed magnetic flux surfaces [4]. This assumption means that only resonant modes of $m-n\psi'/\chi'=0$ in the Fourier expansions of $1/B^2$ in the Boozer coordinates and of B^2 in the Hamada coordinates are forbidden. Here, χ' and ψ' are radial derivatives ('=d/ds with the arbitrary label of flux surfaces s) of the poloidal and toroidal magnetic fluxes, respectively, and (m,n) are the poloidal and toroidal Fourier modes in the expansions. We shall define the flux coordinates (s,θ,ζ) there to make the safety factor to be positive $q\equiv\psi'/\chi'>0$. Non-resonant low-n modes of $|m-nq|\approx1$, and nearly resonant modes $m-nq\approx0$ in other functions still can exist without breaking the flux surfaces. For the non-bounce-averaged guiding center drift effects such as the parallel viscosity force determining the parallel plasma flows, we already derived analytical formulas which is applicable to arbitrary Fourier spectra of the magnetic field including this type of low-n error fields. Therefore we shall focus on the bounce-averaged bounce-center drifts of the toroidally trapped and ripple-trapped particles in so-called $1/\nu$ collisionality regime. In contrast to the non-averaged effects, in which all of Fourier coefficients B_{mn} are

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required, the effective range of (m,n) is limited in calculation of the bounce-averaged effects. For e.g., contributions of extremely high frequency modulation along the **B**-field line with $|m-nq| \gg |Nq-L|$ vanish in the bounce averaging and are not important for both of the ripple-trapped and toroidally trapped particles. Here $(L.N)$ are the basic poloidal and toroidal modes of the helical modulation, which is used in a "conventional" model expression with the assumption of the toroidal periodicity [5,7,9,10,11,12]

$$B/B_{00} = 1 + \varepsilon_{\rm T}(\theta) + \varepsilon_{\rm H}(\theta)\cos\{L\theta - N\zeta + \gamma(\theta)\} \quad (1)$$

for the ripple-trapping effects. It should be noted here that the analytical bounce-averaging for the ripple-trapped particles [11,12] assumes that the phase term $\gamma(\theta)$ is a slowly varying small function. Furthermore, in the analytical bounce-averaging for the toroidally trapped particles, we shall treat this high frequency modulation (helical and bumpy ripples) $\varepsilon_{\rm H}(\theta)\cos\{L\theta - N\zeta + \gamma(\theta)\}$ by a ripple-averaging. Also in this "ripple-averaging" of the toroidally trapped particles' motion, this characteristic of $\gamma(\theta)$ is favorable. In existing helical/stellarator devices, this characteristic is not attained in the Hamada coordinates $(s, \theta_{\rm H}, \zeta_{\rm H})$, which are used in some theories for tokamaks [3,4]. The Boozer coordinates $(s, \theta_{\rm B}, \zeta_{\rm B})$ are better for analyses of helical/stellarator devices and thus we assume the use of the Boozer coordinates though we omit the subscript "B" indicating "Boozer". Methods to include the low-n perturbed fields and to truncate nonessential B-field spectra in the bounce-averaged kinetic equation depend on the trapped orbit topologies and thus we consider them separately in the next section. It also is assumed in these discussions that $Nq-L$ is positive and $Nq-L \gg 1$, which is satisfied in general helical/stellarator configurations.

2. Expressions of the Magnetic Field Strength for the Bounce-averaged Motions

For a simplicity, we assume the stellarator symmetry of the B-field strength $B(-\theta,-\zeta)=B(\theta,\zeta)$. In the bounce-averaged drift kinetic equations [3,11,12], parallel drift term [4] $V_{\|} f_{a1} \equiv v_{\|}\mathbf{b}\bullet\nabla_{(\mu=\text{const})} f_{a1}$ vanishes as a result of the averaging $\oint dl/v_{\|}$, and the substantially remaining guiding center drift is that due to $\nabla s \times \mathbf{B}\bullet\nabla v_{\|}$ in the radial drift term [4]

$$\sigma_1^{+} \equiv \mathbf{v}_{da}\bullet\nabla s = -\frac{m_a c}{e_a}\frac{4\pi^2}{V'}\frac{v_{\|}B}{\langle B^2\rangle}\times \left(B_\zeta^{(\text{Boozer})}\frac{\partial}{\partial\theta_{\rm B}} - B_\theta^{(\text{Boozer})}\frac{\partial}{\partial\zeta_{\rm B}}\right)_{(\mu=\text{const})}\left(\frac{v_{\|}}{B}\right) \quad (2)$$

and/or

$$\begin{aligned}\sigma_{Xa} &\equiv -\sigma_1^{+} - \frac{m_a c}{e_a}V_{\|}\left(v_{\|}\widetilde{U}\right) \\ &= \frac{m_a c}{e_a}\frac{4\pi^2}{V'}\frac{v_{\|}}{\langle B^2\rangle B}\times \\ &\left(B_\zeta^{(\text{Boozer})}\frac{\partial}{\partial\theta_{\rm H}} - B_\theta^{(\text{Boozer})}\frac{\partial}{\partial\zeta_{\rm H}}\right)_{(\mu=\text{const})}\left(v_{\|}B\right)\end{aligned} \quad . (3)$$

However, B-field strength modulation along the field line $\mathbf{B}\bullet\nabla B$ also is still implicitly included as the $v_{\|}$ modulation and as a factor determining the positions of trapped particles' reflection points. Procedures carrying out this averaging analytically depend on the trapped orbit topologies; the toroidally trapped orbits in the barely trapped pitch-angle range where $v_{\|} \simeq \text{const}$ along the **B**-field line in a ripple period, and the ripple-trapped orbits in the deeply trapped range where $v_{\|} \neq \text{const}$ in the period. This difference in a characteristic of the parallel drift velocity $v_{\|}$ results in different treatments of B_{mn} components around $m\sim L$, $n\sim N$. For the toroidally trapped particles with the long bounce period over larger poloidal angle ranges, the reflection points are almost determined only by the envelop function $1+\varepsilon_{\rm T}(\theta)+\varepsilon_{\rm H}(\theta)$ in Eq.(1) and the bounce-averaging of $\nabla s \times \mathbf{B}\bullet\nabla v_{\|}$ can be carried out with an approximation of $v_{\|} \simeq \text{const}$ in the ripple period. In this case, the parallel modulation $\mathbf{B}\bullet\nabla B$ in $\nabla v_{\|}$ due to non-axisymmetric $(n\neq 0)$ B_{mn} modes is not essentially important, and the B_{mn} are separated into only two types; axisymmetric modes $n=0$ causing the trapping, and the non-axisymmetric modes as a cause of the bounce-averaged radial drifts. In the tokamak theory assuming $\varepsilon_{\rm H}(\theta) \approx 0$ [2,3], a following expression of B is used to calculate effects of $m\sim nq$ non-axisymmetric modes on the toroidally trapped particles' parallel velocity $v_{\|}$.

$$\begin{aligned}B &= \sum_m B_{m0}\cos(m\theta) \\ &+ \sum_{n\neq 0}\left[\cos(n\zeta_0)\sum_m B_{mn}\cos\{(m-nq)\theta\}\right. \\ &\qquad \left. - \sin(n\zeta_0)\sum_m B_{mn}\sin\{(m-nq)\theta\}\right]\end{aligned} \quad (4)$$

Here, $\zeta_0 \equiv q\theta - \zeta$ is a label of the magnetic field lines. We shall use this method also for toroidally trapped particles in helical/stellarator devices. It should be noted that only non-axisymmetric modes of $|m-nq| < Nq-L$ effectively remain in the field-line integral per the ripple-period in the case of $v_{\|} \simeq \text{const}$, and this remaining poloidal mode (m) range becomes narrower in the integral for longer bounce periods.

On the other hand, we should take into account both of $\nabla s \times \mathbf{B}\bullet\nabla B$ and $\mathbf{B}\bullet\nabla B$ in $\nabla v_{\|}$ for the ripple-trapped particles with $v_{\|} \neq \mathrm{const}$ in the ripple-period. For this calculation, we consider a use of differential $\nabla s \times \mathbf{B} \cdot \nabla J_{\mathrm{r}}$ of the adiabatic invariant J_{r} [11,12], for which an analytically approximated expression for the model field in Eq.(1) is given by

$$J_{\mathrm{r}} \equiv \oint v_{\|} \mathrm{d}l = \frac{16 B_{\zeta}/B_{00}}{N - L/q} \frac{\pi - 2\sin^{-1}\alpha^{*}}{\pi} \left(\frac{\mu B_{00} \delta_{\mathrm{eff}}}{m_a} \right)^{1/2} \times \left\{ E(\kappa) - (1-\kappa^2) K(\kappa) \right\} \quad (\text{for } \kappa^2 < 1) \quad (5)$$

Here, $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind, respectively, and the pitch-angle parameter κ^2 in them is defined by $\kappa^2 \equiv \{w - \mu B_{00}(1+\varepsilon_{\mathrm{T}} - \delta_{\mathrm{eff}})\}/(2\mu B_{00}\delta_{\mathrm{eff}})$. Although detailed definitions of the effective ripple-well depth δ_{eff} and the length correction α^* are shown in Ref.[9], approximations of $\delta_{\mathrm{eff}} \cong \varepsilon_{\mathrm{H}}$ and $\alpha^* \cong 0$ can be used in many helical/stellarator devices as stated in Ref.[5]. In contrast to the toroidally trapped cases, the axisymmetric B-field modulation n=0 scarcely has the trapping effects ($\mathbf{B} \cdot \nabla B$) and substantially has only radial drift effects ($\nabla s \times \mathbf{B} \cdot \nabla B$) for the ripple-trapped particles. Although the non-axisymmetric modulations $n \neq 0$ cause both of them, there are two types of the $n \neq 0$ modes; high frequency modulation along the field line $\varepsilon_{\mathrm{H}}(\theta) \cos\{L\theta - N\zeta + \gamma(\theta)\}$, and the low frequency ones $\mathbf{B} \cdot \nabla \cos(m\theta - n\zeta) \approx 0$. If the non-axisymmetric mode satisfies $\nabla s \times \mathbf{B} \cdot \nabla \cos(m\theta - n\zeta) \simeq \mathrm{const}$ in a ripple period, it is appropriate to include it into $\varepsilon_{\mathrm{T}}(\theta)$, but this treatment neglects the $v_{\|}$ modulation due to this mode. In case of the ripple-trapping, we should avoid this neglect as long as contributions of the $n \neq 0$ modes keeps an important characteristic of $\varepsilon_{\mathrm{H}}(\theta)$ as a slowly varying function along the **B**-field line. For this calculation, we shall consider an expression of B

$$\begin{aligned} B = & \sum_m B_{m0} \cos(m\theta) \\ & + \sum_{l=1}^{\infty} \left[\cos\{l(L\theta - N\zeta)\} \sum_m B_{m,\,lN} \cos\{(m - lL)\theta\} \right. \\ & \qquad \left. - \sin\{l(L\theta - N\zeta)\} \sum_m B_{m,\,lN} \sin\{(m - lL)\theta\} \right] \\ & + \sum_m \left[\cos(m\theta_0) \sum_{n \neq 0,\, lN} B_{mn} \cos\{(m/q - n)\zeta\} \right. \\ & \qquad \left. - \sin(m\theta_0) \sum_{n \neq 0,\, lN} B_{mn} \sin\{(m/q - n)\zeta\} \right] \end{aligned} \quad (6)$$

Here, $\theta_0 \equiv \theta - \zeta/q$ in the third term is another label of the magnetic field lines. In previous stellarator theories [5,7,9-12], the first and second terms are used as $1 + \varepsilon_{\mathrm{T}}(\theta)$ and $\varepsilon_{\mathrm{H}}(\theta) \cos\{L\theta - N\zeta + \gamma(\theta)\}$ in Eq.(1), respectively, and the third term is neglected there. Even though it is well-known that $l \geq 2$ in the second term is truncated in Ref.[11], these higher harmonics can be included in the analytically approximated calculation of J_{r} and $\nabla s \times \mathbf{B} \cdot \nabla J_{\mathrm{r}}$ by some methods such as that in Ref.[12], and thus we retain here the $l \geq 2$ terms. Also in this Eq.(6), effective poloidal mode (m) range, which remains in the integral along the field line per ripple-period is limited. Similarly to Eq.(4), only a range of $|m/q - n| < N - L/q$ is important in the first and third terms, and the third term with this truncation has a role as the low frequency modulation along the field line analogous to $1 + \varepsilon_{\mathrm{T}}(\theta)$ in Eqs.(1),(5). In the second term, as discussed in Ref.[9], only a range of $|m - lL| < Nq - L$ should be included to keep $\varepsilon_{\mathrm{H}}(\theta)$ as a slowly varying function compared with $\cos\{L\theta - N\zeta + \gamma(\theta)\}$. Therefore, only for this limited (m,n) region, we can include both of $\mathbf{B} \cdot \nabla \cos(m\theta - n\zeta)$ and $\nabla s \times \mathbf{B} \cdot \nabla \cos(m\theta - n\zeta)$ in $\nabla s \times \mathbf{B} \cdot \nabla J_{\mathrm{r}}$. The trapping effects due to $\mathbf{B} \cdot \nabla \cos(m\theta - n\zeta)$ vanish on the line of $m \cong nq$ in the (m,n) space, and it is appropriate to be included in $\varepsilon_{\mathrm{H}}(\theta) \cos\{L\theta - N\zeta + \gamma(\theta)\}$ for the modes in $m < Nq + (l-1)L$ which do not include this line. The modes in $m > Nq$ being high frequency contributions in $\varepsilon_{\mathrm{H}}(\theta)$ must be separated from this term and be treated in the third term in Eq.(6) to include only $\nabla s \times \mathbf{B} \cdot \nabla \cos(m\theta - n\zeta)$ since $\mathbf{B} \cdot \nabla \cos(m\theta - n\zeta) \approx 0$ near this line $m \cong nq$. Although contributions of $m \sim L \ll nq$ with $n \neq 0$, lN in the third term do not automatically vanish only by the bounce-averaging for the ripple-trapped orbits with $v_{\|} \neq \mathrm{const}$, they are substantially slight toroidal modulations of $\varepsilon_{\mathrm{H}}(\theta)$ and $\gamma(\theta)$, and are not important in the final flux-surface-averaged results. Therefore only $\nabla s \times \mathbf{B} \cdot \nabla \cos(m\theta - n\zeta)$ in the range of $|m - nq| < Nq - L$ is retained in the third term.

The low frequency B-field strength modulation along the field line $\varepsilon_{\mathrm{T}}(\theta)$ in Eq.(1) is now extended to include non-axisymmetric component $\varepsilon_{\mathrm{T}}^{(\mathrm{na})}(\theta,\zeta)$ by

$$\begin{aligned} B/B_{00} \cong & 1 + \varepsilon_{\mathrm{T}}^{(\mathrm{as})}(\theta) + \varepsilon_{\mathrm{T}}^{(\mathrm{na})}(\theta,\zeta) \\ & + \varepsilon_{\mathrm{H}}(\theta) \cos\{L\theta - N\zeta + \gamma(\theta)\} \end{aligned}, \quad (7)$$

where the non-axisymmetric component is defined by

$$\begin{aligned} & \varepsilon_{\mathrm{T}}^{(\mathrm{na})}(\theta,\zeta) \\ & \equiv \frac{1}{B_{00}} \sum_{|m-nq|<Nq-L} \left[\cos(m\theta_0) \sum_{n \neq 0, lN} B_{mn} \cos\{(m/q - n)\zeta\} \right. \\ & \qquad \left. - \sin(m\theta_0) \sum_{n \neq 0, lN} B_{mn} \sin\{(m/q - n)\zeta\} \right], \\ & + \frac{1}{B_{00}} \sum_{\substack{|m-nq|<Nq-L \\ m > Nq + (l-1)L}} \left[\cos(m\theta_0) \sum_{n = lN} B_{mn} \cos\{(m/q - n)\zeta\} \right. \\ & \qquad \left. - \sin(m\theta_0) \sum_{n = lN} B_{mn} \sin\{(m/q - n)\zeta\} \right] \end{aligned}$$

and the axisymmetric component $\varepsilon_{\rm T}^{\rm (as)}(\theta)$ is defined by

$$\varepsilon_{\rm T}^{\rm (as)}(\theta) \equiv \frac{1}{B_{00}} \sum_{|m-nq|<Nq-L} B_{m0}\cos(m\theta) \quad .$$

As mentioned in the introduction, practically interesting B-field perturbations are low-frequency modulation $|m-nq| \le 1$ with low toroidal modes of $n \ll N$ in both of the MHD-activity induced and artificially added perturbations, and thus $n \ge N$ in $\varepsilon_{\rm T}^{\rm (na)}(\theta,\zeta)$ is not actually important. By these considerations, however, we find that the definition of the "low frequency modulation along the **B**-field lines" differs depending on the pitch-angle (κ^2) range. The treatment of $(n-N)q+L<m<Nq+(l-1)L$ at $n=lN$ differs in two ranges discontinuously; toroidally trapped particles in $\kappa^2>1$, and ripple-trapped particles in $\kappa^2<1$. This difference corresponds to that of bounce- (or ripple-) averaged $\nabla v_{||}$ effects in two pitch-angle ranges $\kappa^2>1$ and $\kappa^2<1$. In contrast to Eq.(5) for $\kappa^2<1$, the ripple-averaged parallel velocity for $\kappa^2>1$ is given by [12],

$$J_{\rm t} \equiv \oint_{\substack{\rm ripple \\ \rm period}} v_{||}{\rm d}l = \frac{16B_\zeta/B_{00}}{N-L/q}\left(\frac{\mu B_{00}\delta_{\rm eff}}{m_a}\right)^{1/2} \kappa E(1/\kappa) \cdot \quad (8)$$

Although J in Eqs.(5),(8) and $\sigma_{\rm I}^{+}$, σ_{Xa} in Eqs.(2),(3) are continuous at $\kappa^2=1$, ∇J and the bounce average of $\sigma_{\rm I}^{+}$, σ_{Xa} are discontinuously change at $\kappa^2=1$. An aforementioned approximation $v_{||} \simeq \text{const}$ in a ripple period at $\kappa^2>1$, in spite of a fact that $v_{||} \ne \text{const}$ at $\kappa^2<1$, is motivated by this discontinuous change. It may be thought that one problem in this definition of "low frequency" in the ripple-trapped pitch-angle range $\kappa^2<1$ is a discontinuous change of our treatment at a boundary regime in the (m,n) space $m \cong Nq+(l-1)L$. In the Fourier expansion of B on the magnetic flux surface coordinates (Boozer coordinates in many practical cases) making the phase function $\gamma(\theta)$ to be a slowly varying small function, however, B_{mn} in this region are small, and thus the discontinuous change is not a serious problem. In addition to it, since the poloidal mode (m) limiters of $|nq-m|<Nq-L$, $|m-lL|<Nq-L$, and so on in Eqs.(4),(7) are actually implemented by low-pass filters in $\partial/\partial\theta$ operation [9], the contributions of the modes do not change discontinuously.

A method to obtain approximated values of $\varepsilon_{\rm H}(\theta)$ for an approximation of

$$\frac{1}{B_{00}}\sum_{l=1}^{\infty}\left[\begin{array}{l} \cos\{l(L\theta-N\zeta)\} \displaystyle\sum_{|m-lL|<Nq-L} B_{m,\,lN}\cos\{(m-lL)\theta\} \\ -\sin\{l(L\theta-N\zeta)\} \displaystyle\sum_{|m-lL|<Nq-L} B_{m,\,lN}\sin\{(m-lL)\theta\} \end{array}\right]$$
$$\cong \varepsilon_{\rm H}(\theta)\cos\{L\theta-N\zeta+\gamma(\theta)\} \quad , (9)$$

which corresponds to retaining only first lowest mode term in the Todoroki's "phase" Fourier series [12], is already described in Ref.[9] and will not be shown here. It only should be emphasized here on this high frequency modulation term $\varepsilon_{\rm H}(\theta)\cos\{L\theta-N\zeta+\gamma(\theta)\}$ that its amplitude $\varepsilon_{\rm H}(\theta)$ is more essential rather than the form of the modulation $\cos\{L\theta-N\zeta+\gamma(\theta)\}$ in calculating the bounce-averaged radial drifts. The higher harmonics of $n=N$ with l=2,3,4,… can contribute to this amplitude $\varepsilon_{\rm H}(\theta)$ and are sometimes non-negligible [9]. The $1/\nu$ component of the ripple-trapped particle distribution function can be obtained only by replacing $\partial\varepsilon_{\rm T}/\partial\theta$ by $\partial\left\{\varepsilon_{\rm T}^{\rm (as)}(\theta)+\varepsilon_{\rm T}^{\rm (na)}(\theta)\right\}/\partial\theta$ in the previous ripple diffusion theories [11,12] and their applications [5,7,9,10]. In the next section, we describe only a method to obtain the $1/\nu$ component in the toroidally trapped particle distribution.

3. Bounce-averaged Solution for the Toroidally Trapped Particles

Another reason for aforementioned choice of the Boozer coordinates is a fact that we cannot fully include all of $\nabla s \times \mathbf{B}\bullet\nabla B$, $\mathbf{B}\bullet\nabla B$ in $\nabla v_{||}$ and neglect some parts of $\mathbf{B}\bullet\nabla B$ in the bounce-averaging as discussed in the previous section. Although the radial drift term $\sigma_{\rm I}^{+}$ given in the Boozer coordinates in a form of $\nabla s \times \mathbf{B}\bullet\nabla(v_{||}/B)$ as shown in Eq.(2) is suitable this analytical approximation, use of σ_{Xa} in Eq.(3) requires more exact calculation of $\mathbf{B}\bullet\nabla v_{||}$ even in the bounce averaged equation. One more reason is that contributions of the boundary regime in the (m,n) space $|m-lL| \simeq Nq-L$ should be small in determining the envelop function $\varepsilon_{\rm H}(\theta)$ in Eq.(9). Therefore, to derive the $1/\nu$ diffusion coefficient, we execute a procedure in Ref.[3] in the Boozer coordinates, in which the Jacobian is given by $\sqrt{g_{\rm B}}=(V'/4\pi^2)\langle B^2\rangle/B^2$ [4], although this previous theory is originally written using the Hamada coordinates ($\sqrt{g_{\rm H}}=V'/4\pi^2$). In this derivation, we allow also existence of helical and bumpy ripple $\varepsilon_{\rm H}(\theta)\cos\{L\theta-N\zeta+\gamma(\theta)\}$ and therefore the ripple-averaged parallel velocity is given by Eq.(8), while the velocity in the axisymmetric limit is used in Ref.[3]. By rewriting Eq.(6) as

$$B/B_{00} = 1+\varepsilon_{\rm T}^{\rm (as)}(\theta) + \sum_{n=1}^{\infty}\left\{A_n(\theta)\cos(n\zeta_0)+B_n(\theta)\sin(n\zeta_0)\right\} \quad , \quad (10)$$

and with a boundary condition at $v_{||}=0$ making $\partial G_{Xa}/\partial\mu$ to be finite,

$$\frac{\partial G_{Xa}}{\partial \mu} = -\frac{c}{e_a \chi' v_{\mathrm{D}}^a} \frac{\oint \frac{|v_{//0}|}{\left(B_{\mathrm{axisymmetric}}\right)^2} \frac{\partial B}{\partial \zeta_0} \mathrm{d}\theta}{\oint \frac{|v_{//0}|}{\left(B_{\mathrm{axisymmetric}}\right)^2} \mathrm{d}\theta} \tag{11}$$

is obtained. The integral period length for $\oint \mathrm{d}\theta$ is determined by the envelope function $1+\varepsilon_{\mathrm{T}}^{(\mathrm{as})}(\theta)+\varepsilon_{\mathrm{H}}(\theta)$, and $|v_{//0}|$, $B_{\mathrm{axsymmetric}}$ in this integral are defined by

$$|v_{//0}| \equiv \frac{4/\pi}{N-L/q}\left(\frac{\mu B_{00}\delta_{\mathrm{eff}}}{m_a}\right)^{1/2} \kappa E(1/\kappa) . \tag{12}$$
$$B_{\mathrm{axisymmetric}} \equiv B_0\left\{1+\varepsilon_{\mathrm{T}}^{(\mathrm{as})}(\theta)\right\}$$

Note that $E(\kappa)$ in $0 \leq \kappa \leq 1$ is a monotonically decreasing function in a range of $1 \leq E(\kappa) \leq \pi/2$, and thus the ripple-averaged parallel velocity coincides with the axisymmetric value $|v_{//0}| = v(1-\mu B_{\mathrm{axisymmetric}}/w)^{1/2}$ in the small ripple limit of $\delta_{\mathrm{eff}} \to 0$. Results shown in Refs.[2-3] are those for this $\delta_{\mathrm{eff}} \to 0$ limit with only one axisymmetric B-field modulation B_{10} ($B_{m0}=0$ for $m \geq 2$). As a result of $v_{||} \simeq \mathrm{const}$ in a ripple period and resulting

$$\oint_{\substack{\text{ripple}\\ \text{period}}} v_{||}(\partial B/\partial \zeta_0)\mathrm{d}l \cong$$
$$\frac{N-L/q}{2\pi B_\zeta/B_{00}}\left\{\oint_{\substack{\text{ripple}\\ \text{period}}}(\partial B/\partial \zeta_0)\mathrm{d}l\right\}\left\{\oint_{\substack{\text{ripple}\\ \text{period}}} v_{||}\mathrm{d}l\right\},$$

Eq.(11) has a identical form to the $\partial \varepsilon_{\mathrm{T}}/\partial \theta$ term in the stellarator ripple diffusion theories [11-12]. If at least one of $v_{||}$ and $\partial B/\partial \zeta_0$ is almost constant in the period, this result is obtained. This approximation cannot be used for the ripple-trapped particles, and therefore $\partial J_{\mathrm{r}}/\partial \theta$ including $\partial \varepsilon_{\mathrm{H}}/\partial \theta$ is calculated by using Eq.(5) for the deeply trapped pitch-angle range of $\kappa^2<1$ [11-12]. The approximation of $v_{||} \simeq \mathrm{const}$ for the barely trapped pitch-angle range $\mu \sim w/B_{\mathrm{M}}$ [B_{M}: maximum value of B in the flux surface] is justified by a fact that the $1/\nu$ diffusions without the low-n and nearly resonant B-field modes of $n<N$ and $|m-nq| \leq 1$ are almost determined only by those of ripple-trapped particles in various helical/stellarator configurations [4,5,9,10]. This fact means that the toroidally trapped particles do not have effective bounce-averaged radial drift in spite of the break of the axisymmetry due to the ripples, and only $\partial B/\partial \zeta_0$ due to additional perturbation fields with $|m-nq| \leq 1$ can cause their bounce-averaged drift. In cases with the stellarator symmetry $B(-\theta,-\zeta)=B(\theta,\zeta)$, the contribution of $B_n(\theta)$, which is an odd function of θ, vanish in Eq.(11), and thus $\partial B/\partial \zeta_0$ is given there by $(1/B_{00})\partial B/\partial \zeta_0 = -\sum n A_n(\theta)\sin(n\zeta_0)$. By further pitch-angle integral with the boundary condition $G_{Xa}=0$ at the circulating/trapped boundary $\mu=w/B_{\mathrm{M}}$, the $1/\nu$ diffusion coefficients (the diagonal mono-energetic viscosity coefficient L^* defined in Ref.[4]) can be obtained in a form of

$$L_{(\mathrm{MHD})}^{*} \propto \frac{1}{v_{\mathrm{D}}^a}\int_{\mathrm{trapped}} \mathrm{d}\mu \frac{\sum_{n=1}^{\infty} n^2\left\{\oint \frac{|v_{//0}|}{\left(B_{\mathrm{axisymmetric}}\right)^2} A_n(\theta)\,\mathrm{d}\theta\right\}^2}{\oint \frac{|v_{//0}|}{\left(B_{\mathrm{axisymmetric}}\right)^2}\mathrm{d}\theta} .$$

Though this integral can only be obtained numerically, this estimation is still easier than applications of existing numerical methods for helical/stellarator devices [4].

4. Conclusion

In this paper, we investigated effects of the nearly resonant magnetic field spectra on two types of trapped particles' drifts (toroidally trapped and ripple trapped) by extending and combining the analytical methods for tokamak neoclassical viscosity [2-3] and the ripple diffusions in helical/stellarator devices [5-12]. The analytical bounce-averaging methods for these particles' bounce-center drifts in the $1/\nu$ collisionality regime are proposed.

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