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Development of neoclassical transport database for LHD: DGN/LHD

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Abstract

In helical systems, the neoclassical transport is one of the important issues in addition to the anomalous transport, because of a strong temperature dependency of heat conductivity and an important role in the radial electric field determination. Thus the development of a reliable tool for the neoclassical transport analysis is necessary for the transport analysis in LHD. We have developed a neoclassical transport database for LHD plasmas, DCOM/NNW. The mono-energetic diffusion coefficients are evaluated based on the Monte Carlo method by DCOM code and the mono-energetic diffusion coefficient database is constructed using a neural network technique. The input parameters for the database are the collision frequency, the radial electric field, the minor radius and the configuration parameters (R_{axis}, beta, etc). Recent increment of heating power raises the plasma temperature in LHD. Because the collision frequency decreases in proportion to $T^{3/2}$, we have to estimate the diffusion coefficient in the more collisionless regime. However, DCOM code requires huge calculation time to obtain the diffusion coefficient in such collisionless regime.

In this paper, we improve the DCOM code to reduce the computation time in order to obtain the mono-energetic diffusion coefficients in the more collisionless regime. As a result the DCOM calculation becomes about 6 times faster than previous version. Also we apply GSRAKE code which solves the ripple-averaged drift kinetic equation to obtain further collisionless regime. Finally we construct a neoclassical transport database DCOM-GSRAKE/NNW for LHD (DGN/LHD). The neoclassical transport analyses of high temperature LHD plasma are done using DGN/LHD.

Keywords: LHD, neoclassical transport, neural network, Monte Carlo method

1. Introduction

In helical systems, neoclassical transport is one of the important issues for sustaining high-temperature plasma. In particular, in the long-mean-free-path (LMFP) regime, the neoclassical transport coefficient increases as collision frequency decreases ($1/\nu$ regime), and neoclassical transport plays an important role as well as anomalous transport by plasma turbulence. Moreover, the neoclassical transport plays an important role in the radial electric field determination in helical systems.

The neoclassical transport coefficient has been evaluated using the Monte Carlo method directly following particle orbits, where the mono-energetic diffusion coefficients are estimated by the radial diffusion of test particles [1-3]. This method has a good property in the LMFP regime except for its long calculation time. Thus, we have developed a Monte Carlo simulation code, the Diffusion Coefficient Calculator by Monte Carlo Method (DCOM) code [4], which is optimized in performance in the vector computer. The DCOM code can calculate the mono-energetic diffusion coefficient without convergence problem even if in LMFP regime, especially with finite beta, a large number of Fourier modes of the magnetic field in the Boozer coordinates must be used.

To evaluate the neoclassical diffusion coefficient of thermal plasma, we must take energy convolution into account. Therefore, it is necessary to interpolate discrete data by the DCOM. In a non-axisymmetric system, the diffusion coefficient shows complex behavior and strongly depends on collision frequency and radial electric field (e.g., $1/\nu$, $\sqrt{\nu}$ and ν regimes). The interpolation based on a traditional analytical theory has a problem with connected regions between two regimes.

We apply the neural network (NNW) [5] method to the fitting of the diffusion coefficient of LHD, which shows a complex behavior in several collisional regimes, i.e., ν, $\sqrt{\nu}$, $1/\nu$, plateau and

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P-S regimes. We used a multilayer perceptron NNW with only one hidden layer, generally known as MLP1. The neoclassical transport database, DCOM/NNW [6], has been constructed with input parameters *r/a*, ν^* and *G*, and D^* can be obtained as an output of the NNW, where ν^* is the normalized collision frequency, *G* is the normalized radial electric field and D^* is the normalized diffusion coefficient respectively.

However, as shown in the above-mentioned, the DCOM code has a problem of the computing time. When we calculate the diffusion coefficient of the finite beta plasma which has a complex magnetic field, because we have to consider a large number of Fourier modes of the magnetic field, the computational cost of the calculating the sum of all Fourier modes of the magnetic field become expensive. And when we calculate the diffusion coefficient in the extremely low collision frequency regime, because we have to trace the particle long time, the DCOM code requires a large computing time. In these cause, DCOM/NNW does not have sufficient data in the finite beta plasma or in the extremely low collision frequency regime.

For overcoming these problems, first, we have improved the DCOM code to reduce the CPU time. We also apply the results of the GSRAKE [7] code to the neoclassical transport database using NNW in the extremely low collision regime. Thus, we reconstruct a new neoclassical transport database, DCOM+GSRAKE/NNW for LHD, DGN/LHD.

2. Reconstruction of the neoclassical transport database for LHD

2.1 Improvement of DCOM code

DCOM code evaluates a monoenergetic local diffusion coefficient, *D*, using the Monte Carlo method. In the simulation, monoenergetic *N* particles are released from initial minor radius position r_0, where the particle randomly distribution in the poloidal and toroidal coordinates, and in the pitch angle space. The test particle orbits are monitored by solving the equations of motion in Boozer coordinates. The DCOM code has to calculate the summation of all Fourier modes of the magnetic field in the Boozer coordinate described as, $B=\sum_{\mathrm{m,n}} B_{\mathrm{m,n}}(\psi)\cos(m\theta-n\zeta)$, whenever time step advances. When we evaluate the diffusion coefficient in the finite beta plasma which has a complicated magnetic configuration, we have to consider a large number of Fourier modes of the magnetic field and a large CPU time is required.

Therefore, we assume that the particles are subjected to forces from the magnetic field at initial position instead of from the magnetic field at the position where the particles moved. This enables us to use 2D-spline interpolation for expression of the magnetic field. By using 2D-spline table of only one magnetic surface for calculating the magnetic field, we can calculate the magnetic field quickly without calculating the summation of Fourier modes as before. Here, we call the DCOM(F) which calculates the magnetic field by adding up the Fourier modes and the DCOM(S) which calculates the magnetic field using 2D-spline table.

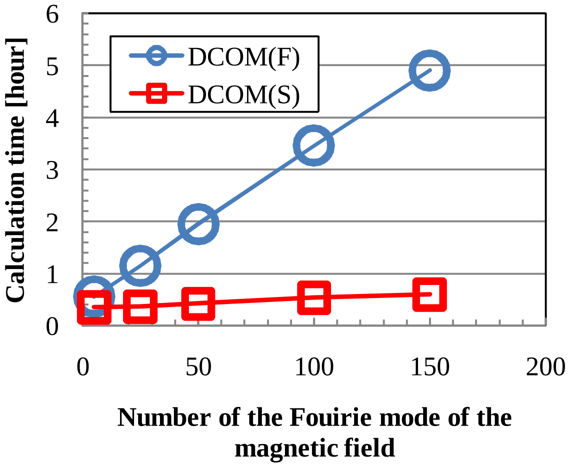

Fig. 1 Calculation time as a function of the number of the Fourier mode of magnetic filed.

In Fig. 1, the calculation time as a function of the number of the Fourier mode of magnetic field are shown. When calculating the diffusion coefficients by the DCOM(F), the necessary CPU time increases as the number of the Fourier mode of the magnetic field increases. In contrast, when calculating by DCOM(S), the CPU time is almost constant even if the magnetic field is complicated. Figure 1 shows that the CPU time reduces to about 1/6 when the number of Fourier mode is 50 and to about 1/10 when the number of Fourier mode is 150. Even when calculating the diffusion coefficient in the finite plasma needing a large number of the Fourier modes, the DCOM(S) can expeditiously calculate.

2.2. Combine the results of the DCOM code with the results of GSRAKE code

In the LMFP regime, a necessary computing time of the DCOM code increases in inverse proportion to the collision frequency. Especially, in extremely low collision regime, the CPU time of over 500 hours is required for calculating the diffusion coefficient on the super computer, the SX-8. Consequently, the data concerning in the extremely low collision frequency regime are insufficient in the DCOM/NNW.

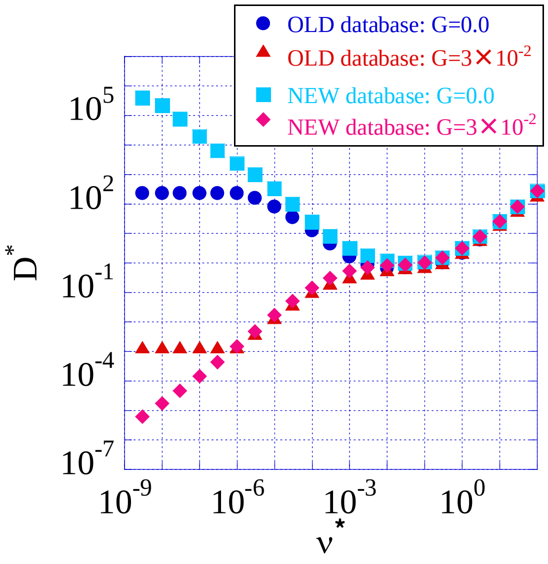

Fig. 2 The normalized diffusion coefficient, D^* as a function of the normalized collision frequency, ν^*, which are outputs of old database, DCOM/NNW and the outputs of new database, DGN/LHD.

Therefore, in extremely low collision frequency regime, we combine the results of the GSRAKE code with the results of the DCOM to construct the neoclassical transport database. GSRAKE is a code which solves general solution of the ripple-averaged kinetic equation. Though GSRAKE can treat few Fourier modes of the magnetic field, a necessary CPU time is small even for calculation in which collision frequency is very small. We supplemented the neoclassical transport database with the GSRAKE results for extremely low collision regime and have improved the neoclassical transport data base.

Because the DCOM and GSRAKE results are discrete monoenergetic diffusion coefficients, it is necessary to interpolate them. We apply the NNW method to the fitting of the diffusion coefficient of LHD. We consider a multilayer perceptron (MLP) NNW model which is the most widely used. We use the MLP1 model which has only one hidden layer. The accuracy of the NNW depends on the number of hidden units. We have to appropriately select the number of hidden units so that the NNW has a good generalization performance. In this research, we evaluate the generalization performance by using information criterion, the Minimum Description Length criterion (MDL criterion) [8] and we assume the number of hidden unit is set to 12.

Figure 2 shows the outputs of the old database, DCOM/NNW, and the outputs of the new database, DCOM+GSRAKE/NNW for LHD (DGN/LHD). At the plasma which has a standard profile of temperature and of density, whose collision frequency, ν^*, are over 10^{-6} in the energy integral, the outputs of old database, DCOM/NNW, are accurate enough. However, in extremely low collision frequency regime, because the computational results of the DCOM don't exist, the outputs of the old database, DCOM/NNW are inaccurate. Because the newly neoclassical transport database, the DGN/LHD, contains the result of the GSRAKE, the outputs of the DGN/LHD are appropriate even in the extremely low collision frequency regime. By using the new neoclassical transport database, DGN/LHD, accurate evaluations of the neoclassical transport in high-temperature plasma and finite beta plasma become possible.

3. Neoclassical transport analysis using the DGN/LHD

We compare the neoclassical transport analysis by the DGN/LHD and by the DCOM/NNW. In this analysis, we consider the hydrogen plasma and $R_{\rm axis}$ = 3.6 m configuration in LHD and β_0=0%. We assume the low-collisional plasma which temperature and density profiles of electron and ion are,

$$T_e = T_i = 4\times(1-(r/a)^2)+1\ [\text{keV}],$$

$$n_e = n_i = 3.5\times(1-(r/a)^2)+1.5[10^{18}/m^3].$$

In these profiles, the energy integral up to ν^*=1×10^{-7} is necessary.

In Fig. 3(a) and 3(b), the electron and ion particle fluxes, Γ_e and Γ_i, at r/a=0.5 are shown as function of radial electric field, E_r. When E_r is from about -20kV/m to 15kV/m, Γ_e calculated using DGN/LHD increases more than using DCOM/NNW and when E_r is from about -3kV/m to 3kV/m, Γ_i using DGN/LHD increases more than using DCOM/NNW. The reason of increases by using DGN/LHD is that DGN/LHD is considering the diffusion coefficient in extremely low collision regime.

Figure 4 shows the ambipolar radial electric field as a function of r/a. There are electron and ion root both using DGN/LHD and using DCOM/NNW. E_r don't make much difference whether by DGN/LHD or by DCOM/NNW. However, as shown in Fig. 5, the thermal conductivities of electron, χ_e, by DGN/LHD and by DCOM/NNW are different. In electron root, χ_e, by DGN/LHD increases to about 1.5 times that of by DCOM/NNW and in ion root, χ_e, by DGN/LHD increases to about 2 times that of by DCOM/NNW.

4. Summary

We have improved the DCOM code to reduce the CPU time. Even in the case with a large number of Fourier modes of the magnetic field, DCOM(S) can calculate the diffusion coefficient at high speed. The GSRAKE results in the extremely low collision regime have been included in the neural network database. The outputs of the new neoclassical transport database, DGN/LHD, which contains the GSRAKE results, are appropriate even in the extremely low collision frequency regime. By using the new neoclassical transport database, DGN/LHD, accurate evaluations of the neoclassical transport in high-temperature plasma and finite beta plasma become possible.

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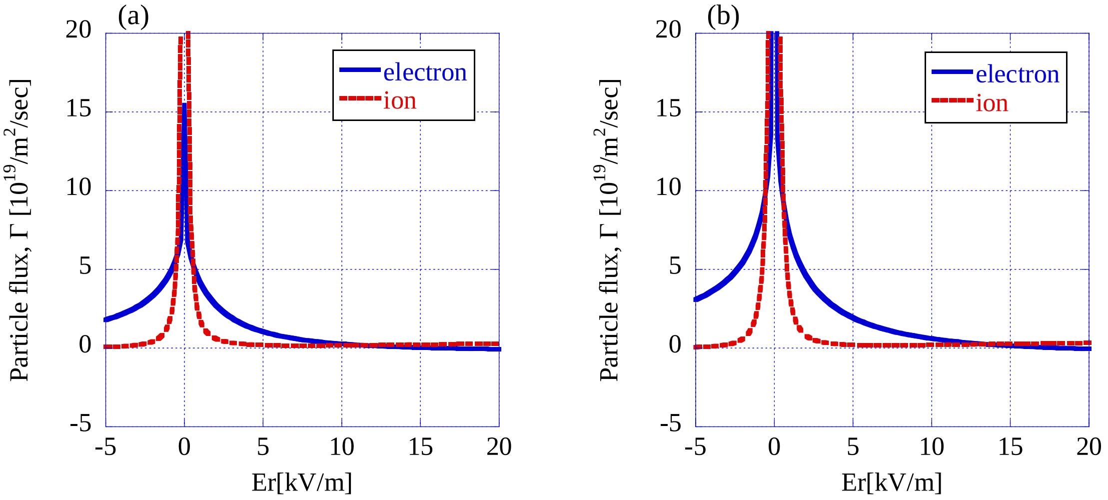

Fig. 3. Particle fluxes, *Γ*, as a function of radial electric field, E_r at r/a = 0.5 by (a) DCOM/NNW and (b) DGN/LHD.

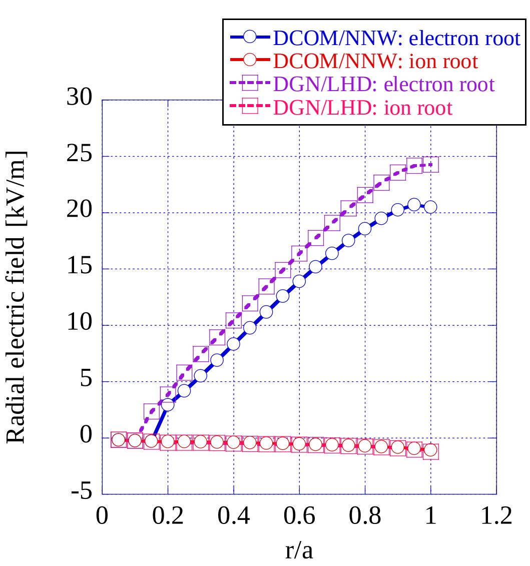

Fig. 4. Radial electric field, E_r, as a function of r/a by DCOM/NNW and by DGN/LHD.

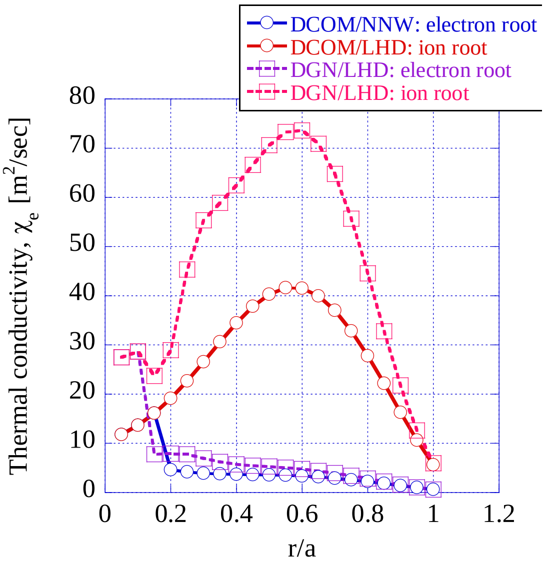

Fig. 5. Thermal conductivity of electron, χ_e, as a function of r/a by DCOM/NNW and by DGN/LHD.