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A Theoretical Model of Ripple Resonance Diffusion of Alpha Particles in Tokamaks

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A theoretical model of ripple resonance diffusion of fusion-produced α particles in tokamaks is presented. An M-shaped energy dependence of diffusion coefficients around ripple resonance conditions, in which the toroidal precession motion of banana particles resonates to the field strength with ripples, was numerically obtained. The M-shaped dependence comes from both island structure and initial distribution of α particles in a $(N\phi, \psi)$ phase space, where N is the number of toroidal field coils, and $(N\phi, \psi)$ is the coordinate of the reflection point of a banana particle in the toroidal angle and the poloidal flux space. Although the particles have periodic motions and a normalized hamiltonian $C(N\phi, \psi)$ is conserved without collisions, pitch angle scattering by collisions changes constant parameters in C and causes the diffusion whose characteristic step size and time are the island size and a variance time to the separatrix respectively, while the characteristic time just outside of the separatrix is determined by a collisionless process.

Keywords: α particle, tokamak, diffusion, ripple resonance

1 Introduction

The confinement of fusion produced α particles is important to maintain burning plasmas in tokamaks. Although α particles are well confined in axisymmetric fields, it has been shown that the loss of α particles due to magnetic field ripple is dominant in the diffusion process in actual tokamaks [1, 2]. However the understanding of the loss processes in detail is not sufficient.

The pitch-angle scattering of fusion-produced α particles in one bounce motion is small enough to maintain banana orbits. Then collisonless orbits are important for the diffusion process. Since the radial displacement of a banana orbit by ripples depends on the toroidal phase at the banana tip [3], the cumulated radial displacement becomes resonantly large when the difference in the toroidal angles of successive banana tips is a multiple of the toroidal angle of adjacent TF coils (the ripple resonance). This toroidal distance of successive banana tips is determined by a toroidal precession ϕ_p. Yushmanov theoretically analyzed this ripple resonance diffusion by means of the banana-drift kinetic equation without the radial change of the toroidal precession $\partial\phi_\mathrm{p}/\partial r$ [4]. Since the toroidal precession, however, strongly depends on the radial position in an actual tokamak, we investigated the ripple resonance diffusion in a realistic system with the radial change of the toroidal precession by numerical calculations. Then the diffusion coefficients with an M-shaped energy dependence around the ripple resonance energy was represented [8]. In this paper, we analyze the physical phenomena of the ripple resonance diffusion using a theoretical representation of collisionless orbits.

Section 2 shows the numerical calculation of the ripple resonance diffusion. Then the importance of the collisionless orbit is explained. In Sec. 3, we analyze the physical phenomena of the ripple resonance diffusion using the theoretical representation of collisionless orbits. The conclusions are given in Sec. 4.

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2 Ripple Resonance Diffusion

In axisymmetric field, the toroidal canonical momentum,

$$P_\phi = mv_\parallel I/B + e\psi = e\psi_\mathrm{b}, \tag{1}$$

where $v_\parallel$ is the velocity parallel to the magnetic field and $I = RB_\phi$, is conserved and corresponds to the poloidal magnetic flux at a banana tip, ψ_b. However it is not conserved in rippled fields and has displacements depneding on the toroidal angle of the banana tip [3, 5, 6, 7],

$$\Delta\psi = \Delta_\mathrm{b} \sin(N\phi \mp \pi/4), \tag{2}$$

$$\Delta_\mathrm{b} = I\gamma\rho_\mathrm{L} \sqrt{\pi Nq/(\varepsilon|\sin\theta_\mathrm{b}|)}, \tag{3}$$

where γ is the ripple strength, ρ_L is the Larmor radius, θ_b is the poloidal angle of the banana tip, $\mp$ corresponds to the upper and lower banana tip, $\varepsilon = r/R_0$, and each parameter is evaluated at the banana tip. Then the cumulated radial displacement becomes resonantly large when the difference in the toroidal angles of successive banana tips is a multiple of the toroidal angle of adjacent TF coils. This ripple resonance conditon is given by

$$2N\phi_\mathrm{p} = 2k\pi \ \ (k = 0, \pm1, 2, 3, \cdots), \tag{4}$$

where ϕ_p is the toroidal precession during half bounce [6]. Yushmanov theoretically analyzed this ripple resonance diffusion without the radial change of the toroidal precession and diffusion coefficients have peaks at resonance energies [4]. Since the toroidal precession, however, strongly depends on the radial position in an actual tokamak, we investigated the ripple resonance diffusion in a realistic system with the radial change of the toroidal precession and found that it has an M-shaped energy dependence around the resonance energies [8] (Fig. 1). The diffusion coefficient also changes with the ripple strength and the plasma density. The value of the coefficients is proportional to the ripple strength, while the energy range where diffusion is enhanced by the ripple becames wider with increasing ripple strength. Diffusion coefficients are roughly proportional to the collision frequency.

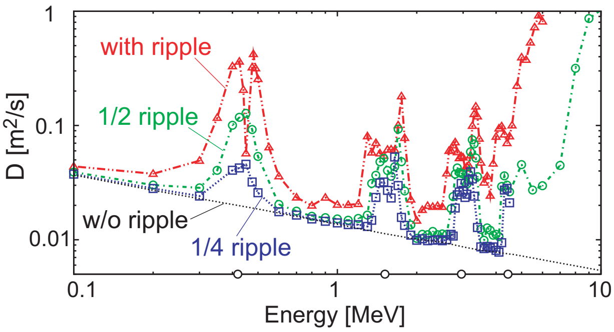

Fig. 1 Energy dependence of the diffusion coefficient changes with the ripple strength. In this configuration, 0.4, 1.5, and 2.9 MeV are the ripple resonance energies without ripples (open circles). Particles start from the same banana tip at any energies. Diffusion coefficients have an M-shaped dependence around each resonance energy. At energies over about 5 MeV, the diffusion coefficient increases with the energy, since this range is nealy stochastic [5].

Next, we investigated collsionless orbits in rippled fields using a mapping model to understand the M-shaped dependence of diffusion coefficients. Then,

$$\left((\psi - \psi_k)/\Delta_p\right)^2 + \cos(N\phi + \Phi_k) = C\,, \qquad (5)$$

$$\Delta_p = \sqrt{2\Delta_b \cos(\Phi_k - \pi/4))/\left(N(\partial\phi_p/\partial\psi)|_{\psi_k}\right)} \qquad (6)$$

is obtained except for the singular point, $\cos(\Phi_k - \pi/4) = 0$, where ψ_k is the resonance surface satisfying Eq.(4),

$$\Phi = N(\phi_b(\psi) + \phi_p(\psi))/2, \qquad (7)$$

ϕ_b is the toroidal length of the banana orbit,

$$\phi_b = 2q\theta_b, \qquad (8)$$

and $\Phi_k = \Phi(\psi_k)$. Equation (5) determines the island width and location. Around the resonance surface $\psi = \psi_k$, an island structure is formed and the width of the island is determined by Δ_p. The integration constant C determines the collisionless orbit and C is conserved in collisionless cases, and $C = 1$ and $-1 < C < 1$ correspond to the separatrix and an orbit inside the separatrix, respectively. The change in the island structure mainly depends on ψ_k, Φ_k and Δ_p. The position of the O-point and the width of the island are specified by (Φ_k, ψ_k) and Δ_p, respectively. When particles outside the separatrix enter inside the saparatrix by collisions, they can jump about the width of the island, leading to the enhancement in the diffusion. Therefore we numerically evaluated the diffusion coefficients starting from each collisionless orbit on the Poincaré map. The results are shown in Fig. 2. C obtained from the guiding center orbit computation is not constant. Then, we introduce a label of each trajectory, $\tilde{C}$, defined by

$$\tilde{C} = \left((\psi - \psi_k)/\tilde{\Delta}_p\right)^2 - 1, \qquad (9)$$

where $2\sqrt{2}\tilde{\Delta}_p$ is the width of the island skimmed from the Poincaré map of the guiding center orbits. Just outside of the separatrix, the diffusion coefficient is large and it becomes small with the distance from the separatrix. On the other hand, inside the separatrix, diffusion coefficients are small. Since the collisionless orbit is important for the ripple resonance diffusion, we consider a theoretical model of the ripple resonance diffuion from the analytical representation of the collisionless orbit, Eq. (5).

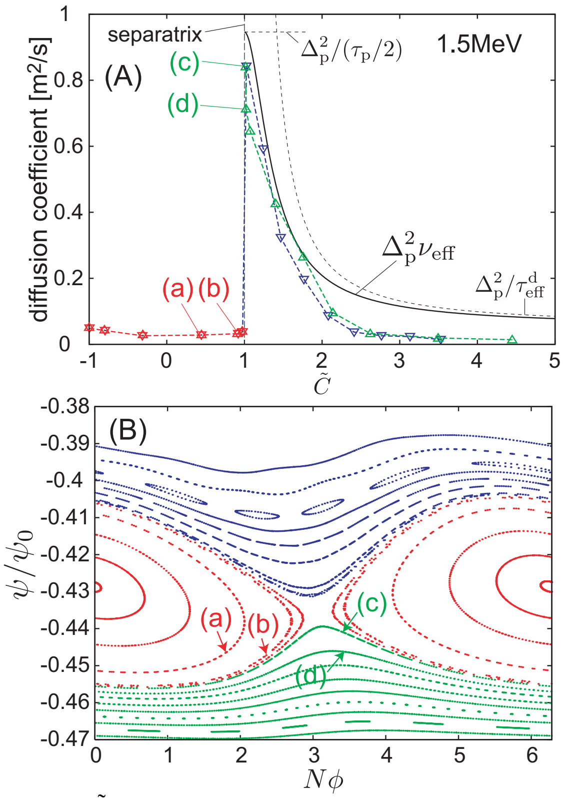

Fig. 2 (A) $\tilde{C}$-dependence of diffusion coefficients at 1.5 MeV from which the neoclassical diffusion is subtracted. $\tilde{C} > 1$, two plots correspond to the trajectory upper and lower side of the island. (B) Initial trajectories of the calculation of each diffusion coefficient in (A). Initial positions are randomly given from a few hundred bounce motions on these collisionless trajectories.

3 Theoretical Model of the Ripple Resonance Diffusion

In this section, we analyze the ripple resonance diffusion by a model consistent with the results of the previous section: the step size of the diffusion is the width of the island, Δ_p, and the characteristic time of the diffusion is the time in which particles reach the separatrix by collision. Here we assume particles are uniformly distributed on a constant C orbit, and hence the characteristic time must be much greater than the bounce time. We evaluate this characteristic time below. As shown in the previous section, the island structure is determined by ψ_k and Φ_k. Since the separatrix is determined by $C = 1$, the characteristic time is the time in which C becomes 1 by collisions.

The energy, W, and the magnetic moment, μ_m, of the particle are conserved in collisionless cases and these are changed by Coulomb collisions. The time evolution of the average and variance of W and $\sin^2\zeta = (v_\perp/v)^2 = \mu_m B/W$ (ζ is the pitch angle) by collisions is represented by the slowing-down time, τ_s, the deflection time, τ_d, and the energy-exchange time, τ_ε, by taking emsemble average. Parameters, ϕ, ψ, Φ_k and ψ_k, which determine the C value of the particle, is changed by the change in W and $\sin^2\zeta$, then a change in C is occured. A change in the position of the particle by collisions brings a change in C,

$$\Delta C_\phi = -\sin(N\phi + \Phi_k)N\Delta\phi, \quad (10)$$

$$\Delta C_\psi = \pm(2/\Delta_p)\sqrt{C - \cos(N\phi + \Phi_k)}\,\Delta\psi, \quad (11)$$

where $\pm$ is corresponds to each case, $\psi > \psi_k$ and $\psi < \psi_k$. When the toroidal angle of the banana tip varies, a phase difference to the map is occured and C is changed. While a change of the resonance surface, ψ_k, and the phase, Φ_k, of the island by collision bring a change in C,

$$\Delta C_{\Phi_k} = [(C - \cos(N\phi + \Phi_k))\tan(\Phi_k - \pi/4) - \sin(N\phi + \Phi_k)]\,\Delta\Phi_k, \quad (12)$$

$$\Delta C_{\psi_k} = \mp(2/\Delta_p)\sqrt{C - \cos(N\phi + \Phi_k)}\,\Delta\psi_k. \quad (13)$$

Since the contribution of the change in ψ and ψ_k to a change in C depends on the width of the island, the ripple strength γ and the spatial change in the toroidal precession, $\partial\phi_p/\partial\psi$, are important. When ϕ_p is changed by collisions, resonance surface ψ_k moves to satisfy the resonance condition (4). Since a change in ψ_k becomes large with the decrease of $\partial\phi_p/\partial\psi$, the contribute of ψ_k depends on $\partial\phi_p/\partial\psi$ while the analysis by Yushmanov [4] is the case, $\partial\phi_p/\partial\psi = 0$. In this paper, we consider the magnetic configulation with $\psi/\phi_p(\partial\phi_p/\partial\psi) \gg \varepsilon^2 S/Nq$, $\varepsilon^2\phi_p/(Nq)^2 \ll N\Delta_p(\partial\phi_p/\partial\psi) < \pi/2\sqrt{2}$ and collision times with $\tau_d/\tau_s \ll (Nq/\varepsilon)^2\Delta_p(\partial\phi_p/\partial\psi)/\phi_p$, $\tau_d/\tau_\varepsilon \ll 1$.

As described above, a change in the orbit C by collisions is described by the a change in W and $\sin^2\zeta$ via a change in the parameters, ϕ, ψ, Φ_k, and ψ_k. Then the time evolution of the average and the variance of C is described by collision times by taking the emsemble average. First, we evaluate the time evolution of the average of C by collisions. The average of ΔC_ψ is zero because the change in back and forth banana motion calncels each other. Because we consider the particles outside the separatrix in this model, ΔC_ϕ and the second term of ΔC_{Φ_k} are vanished by taking average over ϕ. Then, the time evolution of average of C is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle\langle C\rangle_\phi\right\rangle = \left(\mp\frac{2\sqrt{C+1}}{\Delta_p}E\left(\sqrt{\frac{2}{C+1}}\right) + C\tan(\Phi_k - \pi/4)N\theta_b\frac{\partial q}{\partial\psi}\right)\frac{\phi_p}{\partial\phi_p/\partial\psi}\frac{1}{\tau_s}, \quad (14)$$

where $\langle\rangle_\phi$ means average over ϕ, $E(k)$ is the complete elliptic integral of the first kind. bThe first term arises from the change in the resonance surface ψ_k and the second term arises from the change in the width of the island by the change in Φ_k. The characteristic time, in which C becomes unity is given by the inverse of Eq. (14),

$$\tau^s_{\mathrm{eff}} = (1 - C)\mathrm{d}t\Big/\mathrm{d}\left\langle\langle C\rangle_\phi\right\rangle. \quad (15)$$

Next, we evaluate the time evolution of the variance of C. With the condition $\psi/\phi_p(\partial\phi_p/\partial\psi) \gg \varepsilon^2 S/Nq$ and $\varepsilon^2\phi_p/(Nq)^2 \ll N\Delta_p(\partial\phi_p/\partial\psi)$, the contribution of the $\left\langle(\Delta C_\psi)^2\right\rangle$ and $\left\langle(\Delta C_{\psi_k})^2\right\rangle$ is much smaller than $\left\langle(\Delta C_{\Phi_k})^2\right\rangle$. Moreover, $\left\langle(\Delta C_\phi)^2\right\rangle/\left\langle(\Delta C_{\Phi_k})^2\right\rangle \sim (\phi_p/q\varepsilon)^2$ is much less than unity. Ignoring τ_ε since $\tau_d \ll \tau_\varepsilon$, the time evolution of the variance of C is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle\left\langle(C - \langle C\rangle)^2\right\rangle_\phi\right\rangle = \left[\left(C^2 + \frac{1}{2}\right)\tan^2(\Phi_k - \pi/4) + \frac{1}{2}\right] \times\left(\frac{NqR_0}{r\sin\theta_b}\right)^2\frac{1}{2}\left\langle\sin^2 2\zeta\right\rangle_b\frac{1}{\tau_d}, \quad (16)$$

where $\langle\rangle_b$ means the average over the bounce time. The main contributions are the changes in the width of the island and in the phase, and these are caused by the change in Φ_k. The particles on C spread to the separatrix, when the variance of C changes 0 to $(C-1)^2$. Because of $\psi/\phi_p(\partial\phi_p/\partial\psi) \gg \varepsilon^2 S/Nq$ and $\varepsilon^2\phi_p/(Nq)^2 \ll N\Delta_p(\partial\phi_p/\partial\psi)$, the change in C comes from the change in ψ and ψ_k is much smaller than the width of the island, then the change in the averege of C is much less than the change in the variance of C. Therfore the change in the average of C can be ignored during C spreads. Then the characteristic time is given by $(C-1)^2$ over the rate of change at C, Eq. (16),

$$\tau^d_{\mathrm{eff}} = (C-1)^2\,\mathrm{d}t\Big/\mathrm{d}\left\langle\left\langle(C - \langle C\rangle)^2\right\rangle_\phi\right\rangle. \quad (17)$$

The average of C changes proportionally to τ_s^{-1} and the variance of C changes proportionally to τ_d^{-1}. However, as mentioned above, the change in C by the change in the variance of C is dominant. Then the effective collision frequendy is $\nu_{\mathrm{eff}} = 1/\tau^d_{\mathrm{eff}}$ and the diffusion coefficient is evaluated by $D \sim \Delta_p^2 \nu_{\rm eff}$. In this diffusion model, diffusion coefficient becomes small with C which corresponds to the distance from the separatrix and the particles inside the separatrix ($-1 < C < 1$) do not contribute to the diffusion. Since, however, at the separatrix, the diffusion coefficient becomes infinity, other characteristic time determines the peak of the diffusion.

We considered when a particle reaches a separatrix, it can jump the width of an island so far. However, it takes half a time in which particle rounds the island on a collisionless orbit (τ_p) to jump the width of an island. Therefore we assume the effective collision frequency is

$$\nu_{\rm eff} = 1/(\tau_{\rm eff}^{\rm d} + \tau_p/2). \tag{18}$$

Here τ_p is evaluated at $\psi - \psi_k = \Delta_p$ in the axisymmetric field,

$$\tau_p = 2\pi\tau_b \Big/ \left[N\Delta_p(\partial\phi_p/\partial\psi)|_{\psi_k} \right] \quad (C \le 1)\ , \tag{19}$$

where τ_b is the bounce time. Then the diffusion coefficient at the separatrix is evaluated, $D \sim \Delta_p^2\nu_{\rm eff} = \Delta_p^2/(\tau_p/2)$. It has a dependence $D \propto W$ and $D \propto \gamma^{1/2}$ since $\Delta_p \propto W^0$, $\Delta_p \propto \gamma^{1/2}$ and $\tau_p \propto W^{-1}$, $\tau_p \propto \gamma^{-1}$. Meanwhile, the dependence of diffusion coefficients becomes $D \propto W^{-3/2}$ and $D \propto \gamma$ when particles are away from the separatrix because of the deflection time $\tau_d \propto W^{3/2}$.

Fig. 2 also shows the comparison between above theoretical model (solid line) and the numerical results. In this magnetic configulation, $\tau_p/2 \sim 9\tau_b$ and the peak of the diffusion coefficients at the separatrix is about $1.0\,{\rm m^2/s}$. and the main contribution to the change of C is the change of the island width determined by Eq. (12). Thus particles of $C > 2(1/\cos\Phi_k)^2 - 1$ can not enter inside the separatrix since the island width is limited and cannot be extended to all region by collisions. However trajectories on large C can be changed to unity by the changes in the width of the island and the phase in the theoretical model. Then Eq. (17) has a finite value at $C = \infty$ and there is difference between the theoretical model and the numerical results at large C in Fig. 2.

If the diffusion coefficients at the separatrix is determined by τ_p, it does not depend on the collision frequency. Fig. 3 shows the dependence of the diffusion coefficients on the plasma densisy (collision frequency). Except for just outside the separatrix, diffusion coefficients are propotional to the plasma density. However the diffusion coefficients just outside the separatrix are independent of the plasma density. Therefore the peak of the diffusion coefficients at the separatrix is considered to be determined by a collisionless process, the physical mechanism for which is left for future work.

4 Conclusions

The diffusion coefficients were found to have an M-shaped energy dependence around resonance energies by the numerical calculation and it was shown to be caused by the collisionless orbit in the phase space. The ripple resonance diffusion was analyezed by the theoretical model: step size of the diffusion is about the width of the island, Δ_p and characteristic time of the diffusion is the time in which particles reach the separatrix by collisions. Concluding remarks are made as follows,

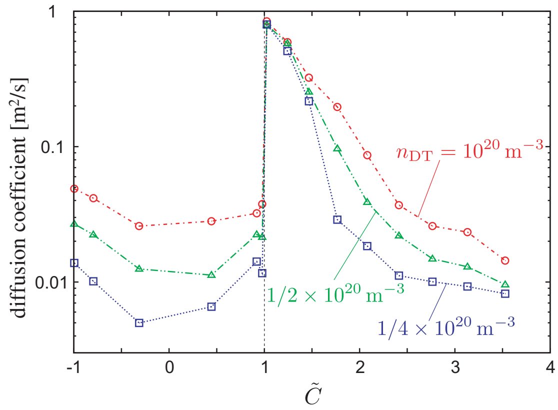

Fig. 3 The dependence of the diffusion coefficients (Fig. 2) on the plasma density. The peak of the diffusion coefficients at the separatrix do not depend on the plasma density.

1. In the magnetic configuration with $\psi/\phi_p(\partial\phi_p/\partial\psi) \gg \varepsilon^2 S/Nq$ and $\varepsilon^2\phi_p/(Nq)^2 \ll N\Delta_p(\partial\phi_p/\partial\psi) < \pi/2\sqrt{2}$, particles enter inside the separatrix, jump the island then cause the large diffusion by collisions via the change in the phase of the island, which has root in the chage in the poloidal angle of the banana tip.
2. The peak of the diffusion coefficient at the separatrix is determined by a collisionless process. Meanwhile, the dependence of diffusion coefficients becomes $D \propto W^{-3/2}$ and $D \propto \gamma$ when particles are away from the separatrix.

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