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Charging of Positively Charged Dust Particle in Weak Magnetic field

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The absorption cross-section of the charge particle to the spherical dust particle with positive charge in a weak magnetic field is investigated analytically and numerically. The closest radius of electrons becomes larger than that in the absence of magnetic field due to the Lorentz force, indicating the absorption cross-section smaller. In order to investigate the parameter dependence of the electrostatic force and the magnetic field on the electron orbits, the closest radius of an electron is approximated by the linear dependence of the strength of magnetic field. From this expression of the closest radius one can obtain the absorption cross-section and can calculate the charge state of the dust particle.

Keywords: dust, charging, magnetic field, absorption cross-section

1. Introduction

The generation, growth and transport of dust particles in fusion plasmas are one of the interesting topics. One of the remarkable points is related with absorption of radioactive tritium, which is one of the fusion fuels of the D-T fueled fusion [1, 2]. After operation of plasma discharges, the treatment, collection and disposal of the radioactive dust particles are one of key issues from the viewpoint of the safety.

In order to study behavior of the dust particle in fusion plasmas the charge state of the dust particle in magnetic field is one of the essential issues. In this study absorption cross-section of a spherical dust particle with positive charge by an electron/ion is studied in weak uniform magnetic field. The absorption cross-section in the absence of magnetic field was expressed by the OML (Orbit Motion Limited) theory [3, 4]. The OML theory, where energy and angular momentum of a charged particle are conserved in an infinite Debye length limit, has been widely used to charging of a dust particle in space plasmas as well as laboratory plasmas. An orbit of a charged particle (an ion or an electron) heading to a charged dust particle at rest along the magnetic field is analyzed analytically and numerically. Because of the Lorentz force in the presence of magnetic force, the charged particle with the same sign as the dust charge approaches closer to the dust than the orbit without magnetic field, indicating larger absorption cross section. On the other hand the charged particle with the opposite sign of dust charge leaves further the dust, indicating the absorption cross section smaller than that in the absence of magnetic field. The dust particle immersed in relatively low temperature plasma is charged negatively because of large mobility of electrons. On the other hand, in the case of the hotter plasma than few hundreds eV, the dust particle is charged positively due to the strong thermionic emission [5]. In the previous research the charging of the negatively charged dust particle in the weak magnetic field was analyzed [6], where we clarified 1) the magnetic field effectively affects the orbit of an electron compared to an ion, 2) the charge state of the floating dust particle with a radius of 1 mm increases from 6.63×10^5 to 6.86×10^5 for the 1 eV ions and electrons in the magnetic field of 10 G and for the higher energy of plasmas with 10 eV, dust charge is found to increase from 6.63×10^6 to 6.70×10^6 due to the effects of magnetic field.

2. Dynamics of Dust in Weak Magnetic Field

The charged particle orbits of j-th species ($j = e, i$) in the magnetic field are analyzed in the cylindrical coordinates (ρ, θ, z), where the uniform magnetic field B_0 is applied to the axial z- direction. The unmovable point dust particle is located at the origin (O) with the charge q_d. The charged particle with the charge q_j starts to move from the initial position ($\rho = b_{in}$, $\theta = 0$, $z = z_{in}$) with the velocity $(\upsilon_\rho = \upsilon_\theta = 0, \upsilon_z = \upsilon_{j,in})$. The equations of motion of the j-th charged particle in this axisymmetric system are,

radial direction:

$$m_j\left[\frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2\right] = q_j E_r \frac{\rho}{r} + q_j B_0 \rho \frac{d\theta}{dt} \qquad (1)$$

and axial direction:

$$m_j \frac{d^2 z}{dt^2} = q_j E_r \frac{z}{r} \qquad (2)$$

Here m_j is the mass of the charged particle, r is the radial

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particle position from the origin (O) $(r^2 = \rho^2 + z^2)$ and E_r is the radial electrostatic field due to the charged dust particle. The azimuthal motion is determined from the conservation of canonical angular momentum P_θ:

$$\frac{d\theta}{dt} = \frac{1}{m_j \rho^2}(P_\theta - \frac{q_j B_0}{2}\rho^2). \tag{3}$$

The initial conditions ($\rho = b_{in}$, $d\theta / dt = 0$) give the constant value of P_θ:

$$P_\theta = \frac{q_j B_0}{2} b_{in}^2 . \tag{4}$$

The normalized equations of motion become

$$\frac{d^2\bar{\rho}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{\rho}}{\bar{r}^3} + \frac{\mu_j^2}{4}\frac{1-\bar{\rho}^4}{\bar{\rho}^3}, \tag{5}$$

$$\frac{d^2\bar{z}}{d\bar{t}^2} = \frac{\alpha_j}{2}\frac{\bar{z}}{\bar{r}^3}, \tag{6}$$

where the distances, velocity and time are normalized by the impact parameter b_{in}, the initial speed $\upsilon_{j,\,in}$ and $b_{in}/\,\upsilon_{j,\,in}$, respectively. The system in the absence of magnetic field is determined by the parameter α_j,

$$\alpha_j \equiv \frac{q_j q_d}{4\pi\varepsilon_0 b_{in}} / \frac{m_j \upsilon_{j,\,in}^2}{2}, \tag{7}$$

which is the ratio of the electrostatic potential energy at the distance of the impact parameter to the initial kinetic energy and the parameter μ_j indicates the effect of the static magnetic field,

$$\mu_j \equiv b_{in} / \frac{m_j \upsilon_{j,\,in}}{\left|q_j B_0\right|}, \tag{8}$$

which is the ratio of the impact parameter b_{in} to the Larmor radius with respect to the initial speed $\upsilon_{j,\,in}$. The parameter μ_e of the electron is much larger than that of the ion for the case of ions with the sound speed c_s and the thermal speed of the electron υ_{the}:

$$\frac{\mu_e}{\mu_i} = \frac{m_i \upsilon_{i,\,in}}{Z_i m_e \upsilon_{e,\,in}} \simeq \frac{m_i c_s}{Z_i m_e \upsilon_{the}} \simeq \sqrt{\frac{m_i}{m_e}}, \tag{9}$$

where Z_i is the charge state of the ion. This relation indicates the effect of the magnetic field on the ion is much smaller than that of the electron (see Eq. 5).

In high plasma temperature there is a possibility to charge positively due to the thermionic emission from the dust particle. In this study we investigate the electron absorption cross-section of positively charged dust particle. In order to investigate the electron orbit numerically, the start position ($z = z_{in}$) should be determined. In Fig.1 the dependence of the closest radius ($r = r_{min}$) to the dust, where the radial velocity of the charges particle is vanishing ($dr / dt = 0$), on the initial axial position (z_{in}) is shown with the parameters $\alpha_e = -1.0$ and $\mu_e = 0$ and 0.1. In the case without magnetic field ($\mu_e = 0$) the closest radius is obtained from the OML theory as $r_{min} / b_{in} = 0.61803$. The orbit of the closer start to the dust deviates due to the strong Coulomb force of the dust. This figure shows that the initial position should be far from - 1000 b_{in} with the 10^{-3} accuracy. The Coulomb force is proportional to the parameter α_e, so the start point is determined as $z_{in} = -10^3\, b_{in}\, \alpha_e$.

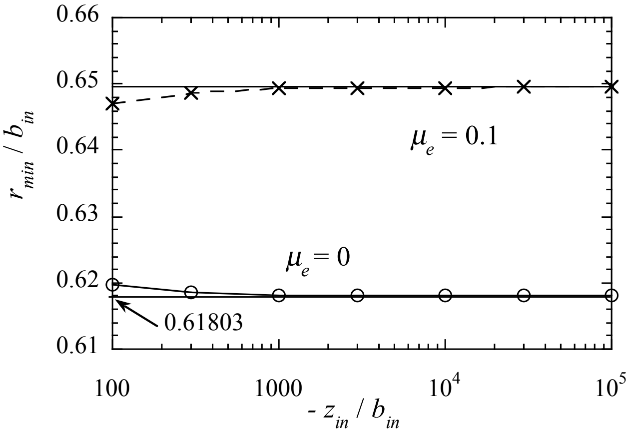

Fig.1 The closest positions as a function of initial axial position (z_{in}), where $\alpha_e = -1.0$ and $\mu_e = 0$ and 0.1.

The typical orbit of an electron near the negatively charged dust, which is located at the origin ($\rho = z = 0$), is shown in Fig.2, where $\alpha_e = 1.0$ and $\mu_e = 0.01$. The closest radius in the presence of in the axial magnetic field (solid line in Fig.2) becomes smaller than that in the absence of magnetic field. The orbit of a charged particle in magnetic field is characterized by three-dimensional nature rather than the two dimensional orbit. The particle

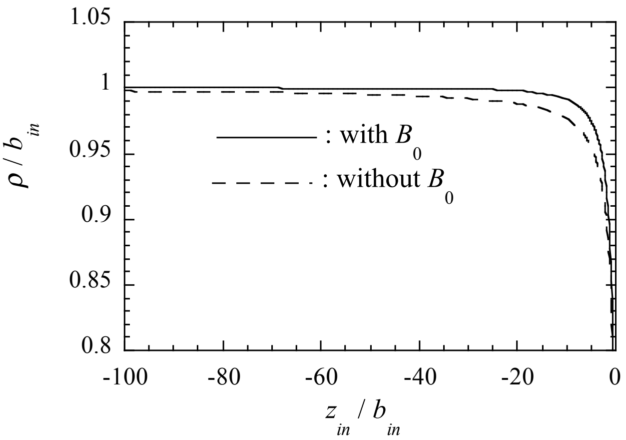

Fig.2 Typical orbit of an electron near the positively charged dust for the case $\alpha_e = -1.0$ and $\mu_e = 0.01$. The solid and dashed lines are the orbits with and without magnetic field, respectively.

has a radial velocity due to the radial electric field. This radial velocity pushes an electron in the azimuthal direction by the Lorentz force. This azimuthal velocity makes the radial force by the axial magnetic field. Thus the magnetic field has the second order effect on the orbit without magnetic field, see Eq. 5. As a result the radial force balance of the particle between the Lorentz force and the centrifugal force determines the radial motion of a charged particle. The radial equation of motion is expressed from Eq. 1,

$$m_j \frac{d^2\rho}{dt^2} = q_j E_r \frac{\rho}{r} + m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt}. \quad (10)$$

where the first term of the RHS is the electrostatic force by the dust particle, the second one is the centrifugal force and the third one indicates the Lorentz force. From the relation of the conservation of the canonical angular momentum (Eqs. 3 and 4), the summation of the centrifugal force and the Lorentz force is expressed as:

$$m_j \rho (\frac{d\theta}{dt})^2 + q_j B_0 \rho \frac{d\theta}{dt} = \frac{1}{m_j \rho^3}(P_\theta^2 - \frac{q_j^2 B_0^2}{4}\rho^4)$$

$$= \frac{q_j^2 B_0^2}{4 m_j \rho^3}(b_{in}^4 - \rho^4). \quad (11)$$

For the case of the orbit of the charged particle with the opp0site sign as the dust charge, its radius ρ is smaller than the initial one (b_{in}) due to the radial electrostatic force, which means the centrifugal force is stronger than the Lorentz force all the time. This deference makes the closest radius larger than that without magnetic field, Fig. 2. On the other hand the charged particle with the same sign of the dust charge approaches to the dust.

For the weak magnetic field, i.e. small μ_e, the closest radius r_{min} is linearly proportional to the strength of magnetic field or μ_e,

$$\bar{r}_{\min}(\alpha_e, \mu_e) = \bar{r}_{\min 0}(\alpha_e) + \gamma_e(\alpha_e)\mu_e. \quad (12)$$

Here γ_e is the constant of proportion, which depends on α_e and $\bar{r}_{\min 0}$ is the closest radius in the absence of magnetic field, which is obtained from the OML theory:

$$\bar{r}_{\min 0}(\alpha_e) = \frac{1}{2}(\alpha_e + \sqrt{\alpha_e^2 + 4}). \quad (13)$$

In Fig.3 the closest radius is shown by the dashed line for the small μ_e, where the parameter α_e = - 1.0, where the linearly approximated line is shown by the solid line. The linear approximation is valid in the range $\mu_e < 0.3$ for α_e = - 1.0. The dependence of the coefficient γ_e on the parameter α_e is investigated numerically, Fig.4. The least square curve (solid line in Fig.4) indicates

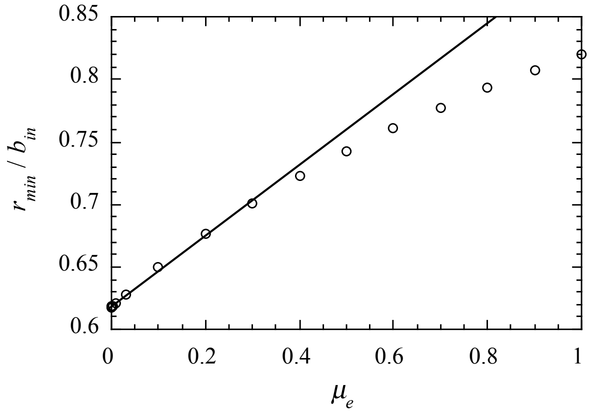

Fig.3 The closest radius as a function of parameter of magnetic field μ_e, where α_e = - 1.0.

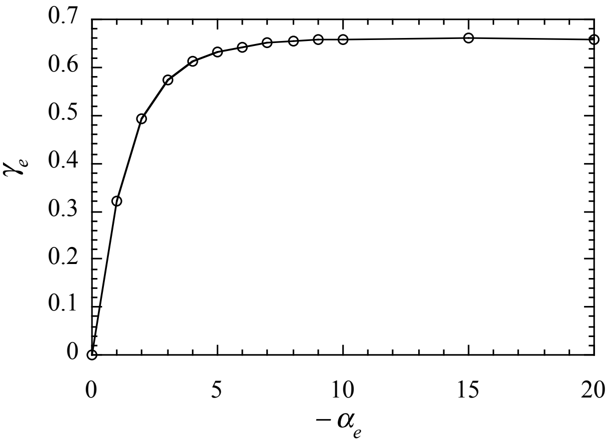

Fig.4 The dependence of the coefficient γ_e on the parameter α_e. The least square and the approximated curves are shown by the solid and the dashed lines, respectively.

$$\gamma_e = 0.658\,[1 - \exp(0.679\,\alpha_e)]. \quad (14)$$

By using Eq. (14), the absorption cross-section is obtained. The un-normalized closest radius is expressed from Eq. 12:

$$r_{\min}(\alpha_e^*, \mu_e^*) = r_{\min 0}(\alpha_e^*)$$

$$+0.658\,[1 - \exp(0.679\,\alpha_e^* / b_{in})]\,\mu_e^*\, b_{in}^2, \quad (15)$$

where

$$r_{\min 0}(\alpha_e^*) = \frac{1}{2}(\alpha_e^* + \sqrt{\alpha_e^{*2} + 4b_{in}^2}), \quad (16)$$

$$\alpha_e^* \equiv b_{in}\alpha_e = \frac{-eq_d}{4\pi\varepsilon_0} / \frac{m_e v_{e,in}^2}{2}, \quad (17)$$

$$\mu_e^* \equiv \mu_e / b_{in} = |eB_0| / m_e v_{e,in}. \quad (18)$$

This closest radius corresponds to the effective finite dust radius R_d to the absorption. From this result, Eq. (15), the absorption cross-section of an electron to the dust is obtained easily.

6. Conclusion

The absorption cross-section of the positively charge particle to the spherical dust particle in a weak magnetic field was investigated analytically and numerically. The closest radius of electrons becomes larger than that in the absence of magnetic field due to the Lorentz force, indicating the absorption cross-section smaller. In order to investigate the parameter dependence of the electrostatic force and the magnetic field on the electron orbits, the closest radius of an electron is approximated by the linear dependence of the strength of magnetic field. From this expression of the closest radius one can obtained the absorption cross-section and can calculate the charge state of the dust particle. These results can be important to analyze the dynamics of the dust particle in plasmas immersed in the magnetic field. The higher order approximation of the effects of magnetic field, the absorption cross-section of electrons to the positively charged dust particle and the velocity distribution of the charged particles are left as future issues. The effects of stronger magnetic field, where the strong Larmor motions are dominant, can be studied by the statistical approach of the particle orbits.

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