Proceedings of ITC18,2008

Numerical Analysis of Slow-Wave Instabilities in Oversized Sinusoidaly Corrugated Waveguide Driven by Finitely Thick Annular Electron Beam

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There are three kinds of model for beam instability analyses, which are based on solid beam, infinitesimally thin annular beam, and finitely thick annular beam. In high power experiments, the shape of electron beam is annulus having finite thickness. We present slow-wave instability driven by finitely thick annular beam in oversized sinusoidaly corrugated waveguide for K-band operation. Our analysis is based on a new version of self-consistent liner theory considering three dimensional beam perturbations. The dependence of Cherenkov and slow cyclotron instabilities on the annular thickness and guiding magnetic field are examined.

Keywords: thin-walled annular beam, oversized sinusoidaly corrugated waveguide, Cherenkov instability, slow cyclotron instability

1. Introduction

The backward wave oscillator (BWO) can be driven by an axially injected electron beam and is one of high-power microwave sources. In BWOs, a periodically corrugated slow-wave structure (SWS) is used to reduce the phase velocity of the electromagnetic mode close to the beam velocity. In order to increase the operation frequency and/or the power handling capability, oversized SWSs are used. The diameter of oversized SWS is larger than free-space wavelength of output electromagnetic wave by several times or more. In many high power experiments, cold cathodes are used and the shape of electron beam is a thin-walled annulus. In order to confine the electron beam, a guiding magnetic field is applied. In the interactions between the beam and the electromagnetic wave, the cyclotron instability as well as the Cherenkov instability plays an important role [1-2]. Near the cyclotron resonance or with relatively low magnetic field, the beam motion perpendicular to the magnetic field cannot be ignored and more definite study of BWO is required by taking into account vertical perturbation of the beam. A pioneering work can be seen in Ref. [3], considering a coupling between a sheet beam and a microwave circuit. A new version of self-consistent field theory considering three-dimensional beam perturbations are developed based on a solid beam [4-5] and an infinitesimally thin annular beam [6]. For infinitesimally thin annular beam, the sheet boundary is modulated due to the transverse modulation of annular surface. Analyses of infinitesimally thin annular beam need to be based on a different theory from thin-walled annular and solid beam. For a finitely thick annular beam, a numerical code has been developed and eigen modes and slow-wave instabilities are analyses in Ref. [7], in which a dielectric-loaded SWS is used for simplicity. The boundary condition at the beam surface is different from the infinitesimally thin annular beam. Solid beam and thin-walled annular beam are based on the same beam boundary condition, but the number of the boundary is different. Thin-walled annular beam has outside and inside surface, and solid beam has only outside surface.

In this work, we develop a numerical code for a sinusoidaly corrugated waveguide with a finitely thick annular beam and analyze slow-wave instabilities in oversized sinusoidaly corrugated waveguide designed K-band operations in a weakly relativistic region less than 100 kV. The organization of this proceeding is as follows. In Sec. 2, we describe numerical method of thin-walled annular beam. In Sec. 3, dispersion relation of oversized sinusoidaly corrugated waveguide driven by thin-walled annular beam is presented. The dependence of growth rate on the annular thickness and guiding magnetic field are examined. In Sec. 4, conclusion of this paper is given.

2. Numerical method

We consider a periodically corrugated cylindrical waveguide in Fig. 1. The wall radius $R_w(z)$ varies along the axial direction z as $R_0+h\cos(k_0z)$, where average radius R_0=1.57 cm, corrugation amplitude h=0.17 cm, pitch length z_0=0.34 cm and corrugation wave number $k_0=2\pi/z_0$. A guiding magnetic field B_0 is applied

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uniformly in the axial direction. An electron beam is propagating along the guiding magnetic field and the shape of electron beam is finitely thick annulus. The beam is uniformly distributed from inside radius R_{ba} to outside radius R_{bb} with beam thickness $\Delta_p(=R_{bb}-R_{ba})$. The outside and inside regions of the beam are a vacuum. The temporal and spatial phase factor of all perturbed quantities is assumed to be $\exp[i(k_z z+m\theta-\omega t)]$. Here, m is the azimuthal mode number and k_z is the axial wave number. Based on this model, the dispersion relation can be derived self-consistently considering the three-dimensional beam perturbations and boundary condition. For the beam, the relativistic effects are considered.

In the system with magnetized beam such as Fig.2, electromagnetic modes are the hybrid mode of transverse magnetic (TM) and transverse electric (TE) mode due to the perturbed perpendicular motion to the magnetic field. Two letters of EH and HE is used, to designate the hybrid mode. In this paper, TM is dominant in EH mode and TE is dominant in HE mode.

An electron beam surface is modulated as beam is propagating. For finitely thick annular beam, the transverse moderation appears as the surface electric charge at the fixed boundary as shown in Figs.2. Thin-walled annular beam has two surfaces because there is a vacuum region inside the beam.

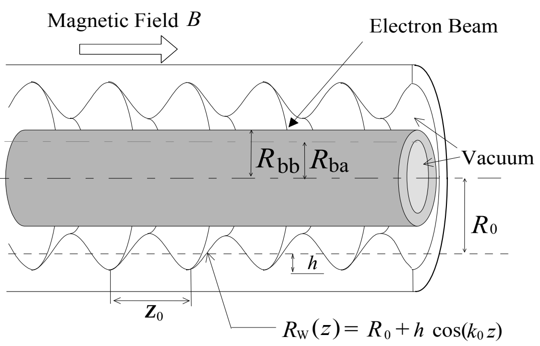

Fig. 1 Model of analysis. The wall of cylindrical waveguide is corrugated sinusoidally.

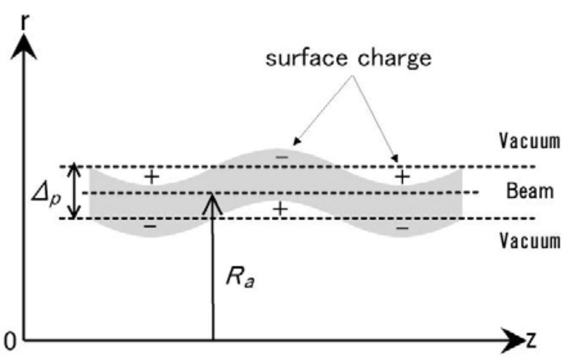

Fig. 2 Beam surface of finitely thick annular beam.

Since the SWS is spatially periodic with z_0, the fields in SWS are expressed by a sum of spatial harmonic series, so-called Floquet's series. The eigen functions for the cylindrical system are the Bessel functions, i.e., J_m and N_m, which have been used in the Floquet's series for non-oversized BWO cases. The electromagnetic modes are volumetric waves having the strong field near the axis. For the oversized BWO, the electromagnetic field is localized near the SWS wall as shown in Fig. 3. All spatial harmonics are evanescent wave in the radial direction. If the spatial harmonics are expressed by J_m and N_m, they have extremely large imaginary number. This causes serious problems in numerical calculations. To avoid this difficulty, the expressions of spatial harmonics should be the modified Bessel functions, i.e., I_m and K_m. We improve the new self-consistent analysis by replacing the Bessel functions to the modified Bessel functions.

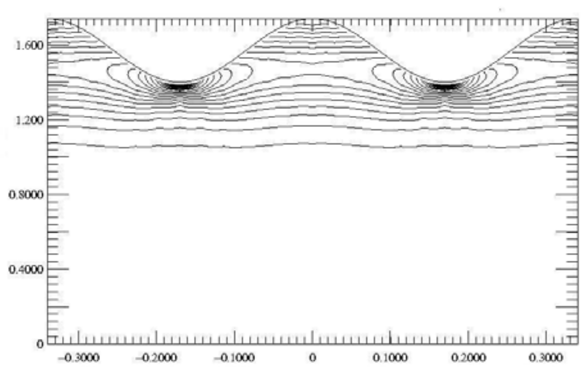

Fig. 3 The electric field distribution of TM_{01} mode in the K-band SWS at the π point.

3. Numerical result

We analyzed the dispersion relation of oversized sinusoidaly corrugated waveguide driven by a finitely thick annular beam. Figure 4 shows the dispersion curves of EH_{01} mode with the beam energy 80 keV, current 200 A, beam thickness Δ_p=0.1 cm and B_0=0.4 T. Four beam modes exist on the axially streaming beam. They are fast and slow space charge modes, fast and slow cyclotron modes. The slow space charge and slow cyclotron modes couple to EH_{01} mode, resulting in the Cherenkov and slow cyclotron instabilities. The growth rate of slow cyclotron instability is shown Fig. 5 for m=-1, 0 and 1. The nonaxisymmetric instabilities are almost the same as the axisymmetric instability. Since the perturbation is assumed to be $\exp[i(k_z z+m\theta-\omega t)]$, the electromagnetic wave propagate helically. The rotating direction is rightward (leftward) in the laboratory frame of reference with positive (negative) m. For oversized BWO, the growth rates are not affected by the rotational direction.

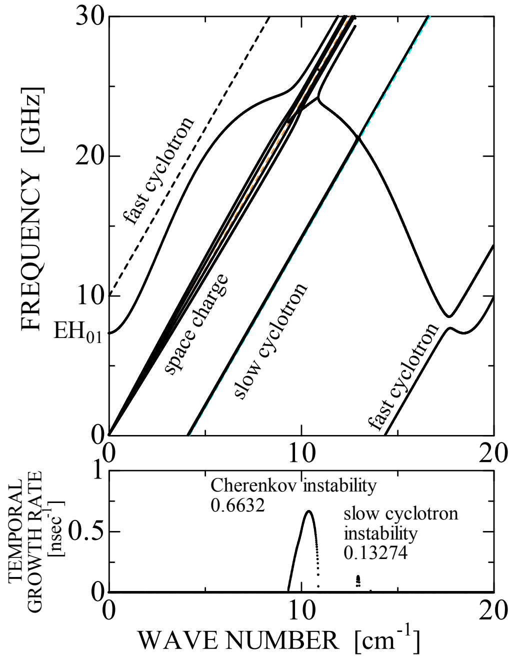

Fig.4 The dispersion curves of axisymmetric mode (m=0) with beam energy 80 keV, current 200 A, outside radius R_{bb} =1.35 cm, inside radius R_{ba}=1.25cm and external magnetic field B_0=0.4 T.

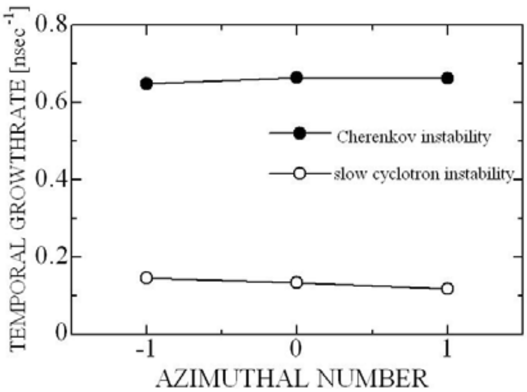

Fig. 5 The temporal growth rate of Cherenkov and slow cyclotron instability versus rotational direction of the electromagnetic field.

Figure 6 shows the dependence of the temporal growth rate on beam thickness Δ_p, for EH_{01} mode. The beam outer radius is fixed to 1.35 cm, and beam inner radius has been changed with a fixed line charge density. The growth rate of Cherenkov instability increases by decreasing Δ_p. The growth rate of slow cyclotron instability also increases by decreasing the beam thickness. But, in the region of $\Delta_p < 0.022$ cm, the growth rate decreases.

In the limit that the beam inner radius is zero, the growth rate of Cherenkov and slow cyclotron instabilities of thin-walled annular beam approaches the growth rate of solid beam, ▲ in Fig.6. In the other limit that $\Delta_p \rightarrow 0$, the corresponding growth rates are those of an infinitesimally thin annular beam model with Δ_p=0 and are depicted by ● in Fig. 6. Two models based on finite and zero Δ_p give almost the same results for the Cherenkov instability. For the slow cyclotron instability, the growth rates are different between two models. This might be caused by the difference of annulus: one has an internal structure between the inner and outer surface and the other is just a sheet without any internal structure.

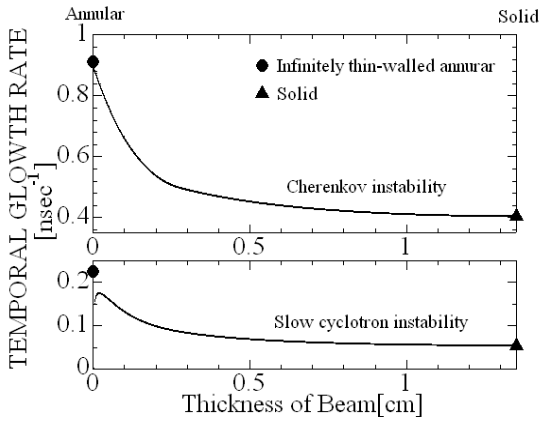

Fig. 6 The dependence of the temporal growth rate on beam thickness.

For EH_{01} mode, the dependence of the temporal growth rate of Cherenkov instability on the guiding magnetic field is shown in Fig.7, and that of slow cyclotron instability in Fig.8. The growth rate of Cherenkov instability hardly changes by the variation of magnetic field. But, in the region of B_0 < 0.18T, the growth rate increases. In this region, the slow cyclotron instability merges into the Cherenkov instability. The dip of growth rate in the vicinity of 1.8T is attributed to the resonant interaction of space charge mode and fast cyclotron mode. Electromagnetic energy excited by the space charge mode is absorbed by the beam due to the fast cyclotron interaction.

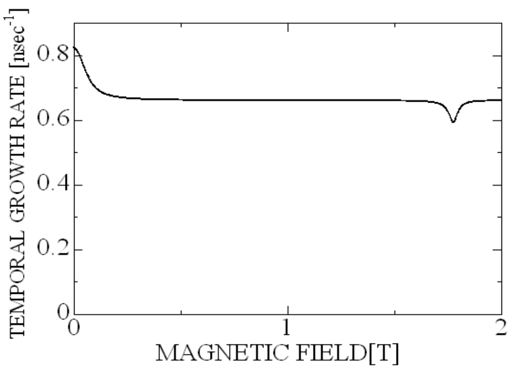

Fig.7 Dependence of Cherenkov instability on the guiding magnetic field. The beam parameter is the same as Fig. 4.

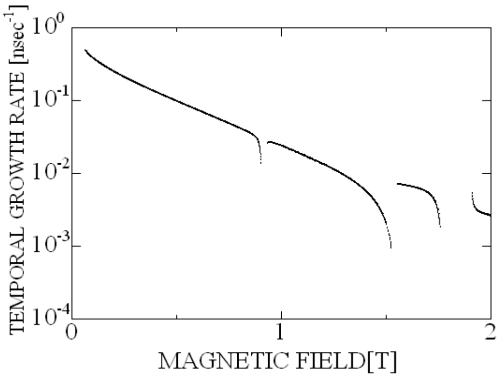

Fig. 8 Dependence of slow cyclotron instability on the guiding magnetic field. The beam parameter is the same as Fig. 4.

The growth rate of slow cyclotron instability decreases by increasing magnetic field. Slow cyclotron mode shifts to the right and fast one to the left in Fig. 4. In the vicinity of 0.9T and 1.8T, the slow and fast cyclotron modes are interacting and the growth rate becomes discontinuous. In the vicinity of 1.5T, the slow cyclotron mode begins to cross EH_{01} mode at the 2π point with a very small growth rate, at point 1 in Fig. 9. Other interaction points 2 and 3 in the forward region appear as Fig. 9. After passing this point, interacting point 1 and 2 merge and disappear as the magnetic field rises. The slow cyclotron mode couples to EH_{01} mode at point 3 only, leading to the discontinuity of growth rate near 1.5 T. In summary, the slow cyclotron mode couples to EH_{01} mode in the backward region of 0-1.5 T and 1.8-2.0 T. From 1.5-1.8 T, the slow instability occurs in the forward region.

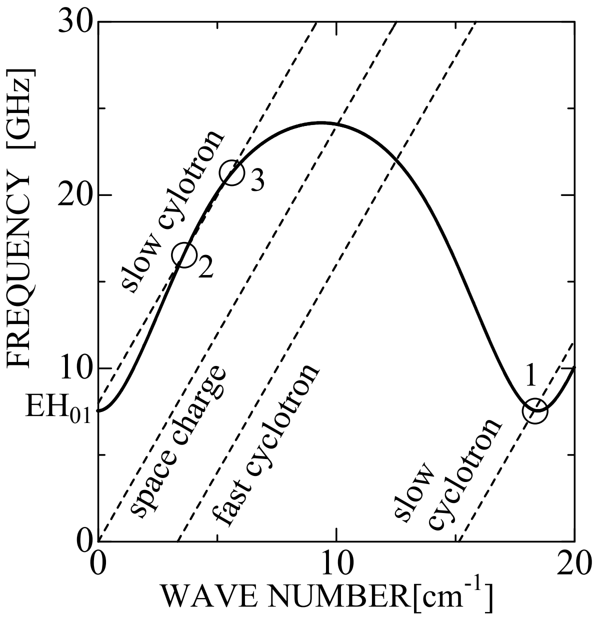

Fig. 9 Dispersion curve of EH_{01} mode. Three beam lines of space charge, fast cyclotron and slow cyclotron modes are plotted for B_0=1.5T.

4. Conclusion

We develop a numerical code for a sinusoidaly corrugated waveguide with a finitely thick annular beam considering three-dimensional beam perturbations. The self-consistent field analysis is improved and the slow cyclotron and Cherenkov instabilities of oversized BWO are numerically examined. Nonaxisymmetric instabilities are excited even in the completely axisymmetric system. The growth rates are almost the same among nonaxisymmetric and axisymmetric instabilities. The slow cyclotron and Cherenkov instabilities depend on the annular thickness. The Cherenkov instability has a week dependence on the guiding magnetic field, while the slow cyclotron instability strongly depends on the magnetic field.

Acknowledgments

This work was partially supported by Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan, and by NIFS Collaboration Research Program.

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