# Investigation of Laser Pulse Propagation in Plasma using the PIC Simulation Method 

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In this paper, the propagation of high power and short laser pulse in plasma medium is investigated. To do this aim, the particle model of plasma and particle-in-cell (PIC) computational method is used. The Maxwell equations are solved on a lattice and the relativistic Lorentz equation, by interpolation of the electromagnetic fields on the particles, leads to the temporal evolution of the phase space of the particles. The sources ( $\rho$ and J) are interpolated self-consistently. The equations are discretized with finite difference method and the Leapfrog differencing method is used to perform the computations for fields. The results are in a very good consistency with the current theories of the wave propagation in plasmas. The energy loss, dispersion and the change in the shape of the pulse due to the interaction with the electrons are shown and discussed. The obtained results can be used in the study of laserplasma interactions especially in fast ignition approach to laser fusion process.

Keywords: Particle-in-cell (PIC), Laser-Plasma Interaction, Particle Simulation, Fast Ignition, Pulse Propagation in Plasma.

## 1. Introduction

A plasma medium can be considered as a large number of charged particles moving in their self-consistent electric and magnetic fields. The physics of such system can be studied by computer modeling simulation. Among the most successful models for computer simulation of plasma are particle models. This model becomes more practical in near-critical plasma. In this region the equations of state are not accessible and therefore the fluid description of the system can be no longer valid.

The particle behavior of the plasma also becomes dominant during the interaction of high power short pulse laser $\left(10^{-5} s\right)$ with the charge particle. For this interaction regime, the electron motion in the presence of intense light wave is highly relativistic. As we wish to model our system in high-temperature, the effects are very weak, so the plasma can be considered as a large number of collisionless particles. Therefore the propagation of the laser pulse in the plasma medium can be investigated by using particle-in-cell (PIC) computer simulation method.

On physical grounds the particle simulation of plasma in the region mentioned above is in focus of interest for study the relativistic laser-plasma interaction in laser fusion experiments especially in fast ignition approach. Many researchers have also studied the phenomena which accrue in laser plasma accelerators and X-ray lasers by PIC
method. Since we are interest in systems containing more than $\mathrm{N} \simeq 3 \times 10^{4}$ particles, the total number of arithmetic operations required to evaluate directly the force on each particle due to all other particles of the system will be of the order of $\eta=10 \mathrm{~N}^{2}$. Calculation of this magnitude is totally impractical for exploring the physics of plasma. For this reason one may view each particle in a simulation as representing many particles of a real plasma namely superparticle. In addition, instead of computing direct interactions between particles we use particle-mesh model in our calculations.

## 2. Particle in cell (PIC) method

In this scheme, particles are defined in continuum space in both position and velocity whereas fields are defined at discrete locations in space. The values of particle and field are advanced sequentially in time. The equations of motion of particle evolve in each time step $\Delta t / 2$, using electromagnetic fields interpolated from the discrete grid to the continuous particle locations. The term for charge and current sources, $\rho$ and J , are interpolated self consistently on the discrete mesh points and the Maxwell equations are then advanced on time step. This procedure will be repeated in the next time step. The electromagnetic fields evolve according to the Maxwell equations:

$$
\begin{align*}
& \frac{\partial \mathbf{B}}{\partial t}=-\nabla \times \mathbf{E}  \tag{1}\\
& \frac{\partial \mathbf{D}}{\partial t}=-\nabla \times \mathbf{H}-\mathbf{J} \tag{2}
\end{align*}
$$

subject to initial conditions:

$$
\begin{align*}
& \nabla \cdot \mathbf{D}=\rho,  \tag{3}\\
& \nabla . \mathbf{B}=0 . \tag{4}
\end{align*}
$$

The particle positions and velocities obey the NewtonLorentz equations of motion:

$$
\begin{align*}
& \frac{d}{d t} \gamma m \mathbf{v}=\mathbf{F}=q(\mathbf{E}+\mathbf{v} \times \mathbf{B}),  \tag{5}\\
& \frac{d \mathbf{x}}{d t}=\mathbf{v} \tag{6}
\end{align*}
$$

where the relativistic factory $\gamma$ is given by

$$
\begin{equation*}
\gamma=\sqrt{\frac{1}{1-(v / c)^{2}}}=\sqrt{1+\left(\frac{u}{c}\right)^{2}}, u=\gamma \tag{7}
\end{equation*}
$$

Eqs. (5) and (6) are discretized using the second-order leapfrog scheme. The leapfrog finite-difference approximations to Eqs. (5) and (6) are

$$
\begin{align*}
& \frac{u^{t+\Delta t / 2}-u^{t-\Delta t / 2}}{\Delta t}= \\
& \frac{q}{m}\left(\mathbf{E}^{t}+\frac{u^{t+\Delta t / 2}-u^{t-\Delta t / 2}}{2 \gamma^{t}} \times \mathbf{B}^{t}\right)  \tag{8}\\
& \frac{x^{t+\Delta t}-x^{t}}{\Delta t}=\frac{u^{t+\Delta t / 2}}{\gamma^{t+\Delta t / 2}} \tag{9}
\end{align*}
$$

With $\gamma^{t}=\left(\gamma^{t-\Delta t / 2}+\gamma^{t+\Delta t / 2}\right) / 2$.
The instability of above equations can be verified through the Boris's algorithm. Based on this algorithm the term, $u^{t+\Delta t / 2}$ can only appear in one side of Eq. (7). The general finite-difference forms of Eqs. (1) and (2) can be written as :

$$
\begin{align*}
& \delta_{t} D_{i}=\delta_{j} H_{k}-\delta_{k} H_{j},  \tag{10}\\
& \delta_{t} B_{i}=-\delta_{j} E_{k}+\delta_{k} E_{j}, \tag{11}
\end{align*}
$$

Where $\mathrm{i}, \mathrm{j}$ and k denote the indices of an orthogonal right-handed set of coordinates. $\delta_{q}$ denotes some finite-difference operator with respect to the variable $q$. In order to perform the numerical implementation of Eqs.
(9) and (10) in PIC codes we use a centre difference for the differentials $\delta$, and place the fields on the mesh. In this manner $\mathrm{D}, \mathrm{E}$ and J are defined at the midpoints of the segments connecting mesh nodes. The centre difference forms of Eqs. (9) and (10) on a uniform mesh become:

$$
\begin{align*}
& \frac{D_{i}^{t}-D_{i}^{t-\Delta t}}{\Delta t}=\frac{H_{k, x_{j}+\Delta x_{j} / 2}^{t-\Delta t / 2}-H_{k, x_{j}-\Delta x_{j} / 2}^{t-\Delta t / 2}}{\Delta x_{j}} \\
& -\frac{H_{j, x_{k}+\Delta x_{k} / 2}^{t-\Delta t / 2}-H_{j, x_{k}-\Delta x_{k} / 2}^{t-\Delta t / 2}}{\Delta x_{k}}-J_{i}^{t-\Delta t / 2}, \\
& \frac{B_{i}^{t+\Delta t / 2}-B_{i}^{t-\Delta t / 2}}{\Delta t}=\frac{E_{k, x_{j}+\Delta x_{j} / 2}^{t}-E_{k, x_{j}-\Delta x_{j} / 2}^{t}}{\Delta x_{j}} \\
& +\frac{E_{j, x_{k}+\Delta x_{k} / 2}^{t}-E_{j, x_{k}-\Delta x_{k} / 2}^{t}}{\Delta x_{k}} \tag{13}
\end{align*}
$$

To complete the discrete model we require prescriptions for obtaining the charge density at mesh points from the distribution of superparticles and for obtaining the forces at superlattice positions from the mesh-defined electric fields. The charge density at mesh point $p$ is given by the total charge in the cell surrounding mesh point $p$ divided by the cell volume:

$$
\begin{equation*}
\rho_{p}^{n}=\frac{q N_{s}}{4} \sum_{i=1}^{N_{p}} W\left(x_{i}^{n}-x_{p}\right)+\rho_{0} \tag{14}
\end{equation*}
$$

Where $N_{s}, H, \rho_{0}, x_{j}$ and $x_{p}=p H$ are the number of electrons per unit length, the cell width, background density charge, the position of the superparticle $i$ in the cell and coordinate of particle $p$, respectively. The charge assignment function $W$ used in Eq.(14) is :

$$
W(x)=\left\{\begin{array}{cc}
1 & |\mathrm{x}|\left\langle\frac{H}{2}, x=\frac{H}{2}\right.  \tag{15}\\
0 & \text { elsewhere }
\end{array}\right.
$$

## 3. Results and Conclusions

In the present work, we have performed the numerical code based on PIC method to investigate the propagation of laser pulse in a plasma medium. To do so, the envelope spatial shape of the laser pulse has been considered as a Gaussian-shaped profile. We also assumed that the pulse propagates in the z direction.


Fig.1. The spatial shape of the pulse in the plasma medium.
The time width of the pulse is $0.3 \omega_{p}^{-1}$ which for electron density $n=10^{17} / \mathrm{cm}^{3}$ will be of the order of 10 fs and the power of the pulse is $\sim$ TW. Evolution of the pulse in the medium is evaluated by solving the equations of motion and Maxwell equations consistently using initial boundary conditions.


Fig.2. Propagation of laser pulse in plasma for scaled time $10 \omega_{p}^{-1}$.


Fig.3. Same as Fig. 1 for $20 \omega_{p}{ }^{-1}$.


Fig.4. Same as Fig. 1 for $40 \omega_{p}{ }^{-1}$.
The spatial shape of the pulse in the plasma medium is illustrated in figure (1). In figures (2)-(4) the results obtained for scaled times, $10 \omega_{p}^{-1}, 20 \omega_{p}^{-1}$, and $40 \omega_{p}^{-1}$ have been shown. Here $\omega_{p}=\sqrt{n e^{2} / \varepsilon_{0} m_{e}}$ is plasma frequency and $\lambda_{D}=\sqrt{\varepsilon_{0} k T / n e^{2}}$ is the Debye wave length.

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