High Performance Analysis of Shielding Current Density in High Temperature Superconducting Thin Film

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The numerical method has been proposed for calculating the shielding current density in a high temperature superconducting (HTS) thin film. Under the thin-plate approximation, the shielding current density can be expressed as

\[ j = \nabla \times \left[ \frac{S}{\varepsilon} e \right], \tag{1} \]

and the time evolution of the scalar function \( S(x, t) \) is governed by the following integral-differential equation [1]:

\[ \mu_0 \frac{\partial}{\partial t} \left[ \int Q(|x - x'|) S(x', t) d^2 x' + \frac{S}{\varepsilon} \right] + \frac{\partial}{\partial t} (\mathbf{B} \cdot e_e) + (\nabla \times \mathbf{E}) \cdot e_e = 0. \tag{2} \]

Here, \( \mathbf{B} \) and \( \mathbf{E} \) denote the applied magnetic flux density and the electric field, respectively, and \( 2\varepsilon \) is the thickness of a HTS thin film. In addition, \( \langle \rangle \) is an average operator over the thickness of the film and the explicit form of the integration kernel \( Q(r) \) is given in [1]. Note that \( Q(r) \) becomes singular for the case with \( r = 0 \). Because of the singularity, improper integrals will appear after the discretization of (2).

In general, (2) is solved together with the \( J \)-\( E \) constitutive relation. As the relation, the following power law is adopted:

\[ E = E_C \left( \frac{J}{J_C} \right) \left( \frac{J}{J_C} \right)^{N}, \tag{3} \]

where \( J_C \) and \( E_C \) denote the critical current density and the critical electric field, respectively, and \( N \) is a constant.

When the initial-boundary-value problem of (2) is discretized by means of the backward Euler method and the finite element method, improper integrals appear as coefficients of the nonlinear system [2]. In order to evaluate the coefficients accurately, the asymptotic form of the integrand is analytically determined and the singularity-subtraction method is applied to the improper integrals. Consequently, the shielding current density can be determined accurately.

As an application of the above method, two types of the contactless methods for measuring the critical current density \( J_C \) have been simulated numerically: the inductive method [3] and the permanent magnet method [4]. The results of computations show that, even when the magnet is located above the sample edge, the maximum repulsive force is roughly proportional to \( J_C \) in the permanent magnet method. In addition, it is found that, if the coil is located above the sample edge, the accuracy of the inductive method is degraded remarkably.