Effect of Nuclear Plus Interference Scattering on Fast-Ion Slowing-Down Distribution Functions in Thermonuclear Plasmas

Hideaki MATSUURA, Daisuke SAWADA and Yasuyuki NAKAO

Department of Applied Quantum Physics and Nuclear Engineering,
Kyushu University, 744 Motooka, Fukuoka 819-0395, Japan
(Received 5 December 2011 / Accepted 19 March 2012)

The effect of the nuclear plus interference (NI) scattering on the slowing-down distribution functions of energetic-ions, e.g., protons produced by $^3\text{He}(d,p)^4\text{He}$ reactions in a $^3\text{He}$ plasma and an energetic deuterium beam injected into a DT plasma, are examined using the Boltzmann-Fokker-Planck model. As a result of the acceleration of the slowing-down process of energetic ions due to NI scattering, the magnitude of the energetic component is reduced compared with the case when NI scattering is ignored. The degree of the reduction and its energy range are quantitatively evaluated for various plasma conditions.

Keywords: nuclear plus interference scattering, fast-ion slowing-down distribution function, Boltzmann-Fokker-Planck equation

DOI: 10.1585/pfr.7.2403076

1. Introduction

It is well known that fast ions slow down via Coulomb and non-Coulombic, i.e., nuclear plus interference (NI) [1], scattering and create knock-on tails in fuel-ion distribution functions. The effect of knock-on tail formation on the plasma burning characteristics [2] and its application to plasma diagnostics [3, 4] have been examined. NI scattering accelerates the slowing-down process of fast ions and enhances the fractional energy deposition from fast to bulk ions [5]. To date, many studies concerning the effect of NI scattering on the plasma burning characteristics have been conducted. However, most of the previous discussions have focused on the effect on distortion of the “bulk” component in fuel-ion distribution functions and also on the energy transferred from fast to bulk ions. When bulk ions are knocked up to the high-energy region, fast ions are also expected to lose a large fraction of their energy. The slowing-down distribution function is expected to change its shape due to NI scattering. When NI scattering is ignored, the energy per unit time transferred from fast to bulk ions is underestimated (the slowing-down time is overestimated), and the relative magnitude of the suprathermal to bulk component is reduced compared with the case when NI scattering is considered.

We recently examined the correlation between the slowing-down time and the magnitude of the equilibrium slowing-down distribution function of fast alpha particles at low-temperature, high-density operation points in force-free helical reactors. We showed that at these points, the slowing-down time becomes shorter owing to the enhanced “Coulomb scattering” frequency, and the magnitude of the energetic component of the alpha particle distribution function decreases compared with that at normal operation points [6], which may reduce the instability excitation probability caused by fast alpha particles in the Alfvén velocity range.

The slowing-down time of fast ions also decreases owing to NI scattering. In this case, the loss fraction of fast ions during the slowing-down process may also be reduced. It is important to determine the reduction in the fast-ion slowing-down distribution function due to NI scattering. In this paper, using the Boltzmann-Fokker-Planck (BFP) simulation [7], the effect of NI scattering on the fast-ion slowing-down distribution function is evaluated. Protons produced by $^3\text{He}(d,p)^4\text{He}$ reactions and a deuterium beam injected into plasma are considered as the fast ions. A noticeable reduction in the fast ion population in a confined plasma is shown. The effect of the reduction on the fast-ion loss process is discussed.

2. Analysis Model

The BFP equation for fast ions is written as

$$\frac{\partial f_i}{\partial t} = \sum_j \left( \frac{\partial f_j}{\partial t} \right)_{\text{Coulomb}} + \sum_n \left( \frac{\partial f_n}{\partial t} \right)_{\text{NI}} + \frac{1}{v^2} \frac{\partial}{\partial v} \left( \frac{v^3 f_i}{2\tau_i^c(v)} \right) + S(v) - \frac{f_i}{\tau_i^c(v)},$$

(1)

where $f_i(v)$ is the velocity distribution function of fast ions. The first term on the right-hand side of Eq. (1) represents the Coulomb collision term [8]. The summation is taken over the primary fuel-ion species and electrons, i.e., deuterons, $^3\text{He}$ (tritons), and electrons in a $^3\text{He}$ (DT) plasma and $^3\text{He}$ (tritons), and electrons in a $^3\text{He}$ (DT) plasma.
plasma. In this study, for simplicity, the background ions and electrons are assumed to be Maxwellian and to have the same temperature. The second term on the right-hand side of Eq. (1) accounts for NI scattering [7] of fast ions by background (main-fuel) ions \( n \) [9]. We consider NI scattering collisions between \( 1 \) protons and deuterons, \( 2 \) protons and \( ^3\)He in \( ^2\)He plasma and \( 1 \) deuterons and deuterons, \( 2 \) deuterons and tritons in (deuterium-beam injected) DT plasmas. The cross sections of NI scattering are taken from the work of Perkins and Cullen [9].

The third and fifth terms on the right-hand side of Eq. (1) represent diffusion in velocity space due to thermal conduction and particle transport loss from the plasma respectively. To incorporate the unknown loss mechanism of energetic ions into the analysis, following Bitton’s treatment [10], we simulate the velocity dependence of the energy loss time \( \tau_i^* (v) \) due to thermal conduction and the particle-loss time \( \tau_i^* (v) \) by using a dimensionless exponent \( \gamma \), i.e.,

\[
\tau_i^* (v) = \begin{cases} 
C_{\tau_i^*} \tau_i (P) & \text{when } v < v_0 \\
C_{\tau_i^*} \tau_i (P) (v/v_0)^\gamma & \text{when } v \geq v_0.
\end{cases}
\]

(2)

The chosen high \( \gamma \) value ensures rapid increment of both confinement times in higher energy ranges compared with that in the thermal energy range; thus, very few energetic particles and their energy are evacuated compared with thermal particles. As discussed in Ref. 7, when we look at the distortion of bulk-ion distribution functions under typical plasma conditions, \( \gamma \) would not be a significant parameter if we choose sufficiently large values, e.g., \( \gamma \geq 4 \). For all calculations in this paper, we assume \( \gamma = 4 \). When the distortion of the fast-ion distribution itself is analyzed, however, the energy dependency of the confinement times during slowing down should be taken into account more exactly. To do this, the energy dependency of the confinement time must be clarified beforehand by other analyses, e.g. particle simulations or experiments. We currently do not have enough information on the confinement times and in this study we adopt the loss term following Bitton’s treatment [10]. We cannot discuss the fast-ion loss itself, but we can roughly estimate the impact of NI scattering on the fast-ion slowing-down process. Considering the energy loss mechanisms resulting from both thermal conduction and particles transport loss from plasma, the global energy confinement time is defined as \( 1/\tau_E = 1/\tau_C + 1/\tau_p \). (See Ref. 7.)

The source \([ S (v) ]\) and particle loss \([ L (v) ]\) terms are described so that fast ion generation and transport loss balance each other:

\[
S (v) - L (v) = \frac{S}{4\pi m_p} \delta (v - \nu_0) - \frac{f_i (v)}{\tau_i^* (v)}.
\]

(3)

Here \( \nu_0 \) represents the speed corresponding to the initial kinetic energy of the fast ions. The fast-ion generation rate \( S \) can be determined from the plasma conditions.

3. Results and Discussion

Figures 1 (a) and (b) show the time variation of the distribution functions of a proton that is initially produced at \( t = 0 \) with and without NI scattering, respectively. The initial kinetic energy of the proton is 14.7 MeV. The background ions and electrons are assumed to be Maxwellian with temperatures \( T_i = T_e = 80 \text{ keV} \) and \( ^3\)He densities of \( n_{^3\text{He}} = 2 \times 10^{20} \text{ m}^{-3} \). Energy and particle confinement times of \( \tau_i = 1/2 \tau_p = 3 \text{ s} \) are assumed. A proton slows down more slowly when NI scattering is neglected. The power transferred from protons to bulk ions via Coulomb scattering \( P_{i\rightarrow e}^{\text{Coulomb}} \) and via NI scattering \( P_{i\rightarrow e}^{\text{NI}} \), and that transferred from the proton to electrons via Coulomb scattering \( P_{i\rightarrow e}^{\text{Coulomb}} \) during the slowing-down process are shown in Figs. 2 (a) and (b) with and without NI scattering, respectively. The calculation conditions are the same as in Fig. 1. Initially, a proton loses energy mainly via Coulomb scattering with electrons. The contribution of NI scattering is almost half that of Coulomb scattering with electrons. As the averaged proton energy decreases, the degree of Coulomb scattering with bulk ions gradually increases, and finally the proton distribution function reaches the equilibrium temperature. The effective temperature \( T_{\text{eff}} \),

\[
T_{\text{eff}} \equiv \frac{4\pi m_p}{3\tau_p} \int_0^\infty \nu^4 f_i (\nu) d\nu,
\]

(4)

for each case is also shown on the same plane. As a result of NI scattering, a proton produced with an initial energy of 14.7 MeV loses its energy faster and reaches equilibrium temperature more quickly. The slowing-down time required to reach the equilibrium temperature is roughly estimated as \( \sim 7 \text{ sec} \) when NI scattering is considered and \( \sim 9 \text{ sec} \) when NI scattering is ignored. Under these plasma conditions, if we ignore the effect of NI scattering, the typical slowing-down time is overestimated by almost 30%. From Fig. 2 (a), we also find that the power transferred from the proton to bulk ions via Coulomb scattering becomes almost comparable with that transferred via NI scattering at an effective temperature of \( \sim 1 \text{ MeV} \). When we consider the slowing-down process of energetic ions with energies greater than \( \sim 1 \text{ MeV} \), the effect of NI scattering should be incorporated into the analysis.

In Figs. 3 (a) and (b), we show the time variation of the proton distribution function with and without NI scattering, respectively, assuming a continuous source from fusion reactions. The plasma conditions are the same as those in Fig. 1. We assume that protons are produced by both \( ^3\text{He(d,p)}^4\text{He} \) and \( \text{D(d,p)}^3\text{He} \) fusion reactions and that generation begins at \( t = 0 \). During the simulations, the fuel-ion densities, electron temperature and energy (particle) confinement time are kept constant. The slowing-down distribution forms gradually, and at 50 s after the beginning of the simulation, it reaches a near equilibrium state. As shown in Figs. 1 and 2, when NI scattering is ignored, the proton distribution function changes its shape more slowly.
Fig. 1 Time evolution of distribution functions of a proton initially produced at \( t = 0 \) (a) with and (b) without NI scattering. In the calculations, \( T_i = T_e = 80 \text{ keV} \), \( n_D = 2 \times 10^{20} \text{ m}^{-3} \) and \( \tau_e = (1/2) \tau_p = 3 \text{ s} \) are assumed.

Fig. 2 Time evolutions of power transferred from a proton to bulk ions and electrons via Coulomb and NI scattering (a) with and (b) without NI scattering. In the calculations, \( T_i = T_e = 80 \text{ keV} \), \( n_D = 2 \times 10^{20} \text{ m}^{-3} \) and \( \tau_e = (1/2) \tau_p = 3 \text{ s} \) are assumed.

Fig. 3 Time evolution of proton distribution functions (a) with and (b) without NI scattering. A continuous proton source from \(^3\text{He}(d,p)^4\text{He}\) and \( \text{D}(d,p)\text{T} \) fusion reactions is considered. In the calculations, \( T_i = T_e = 80 \text{ keV} \), \( n_D = 2 \times 10^{20} \text{ m}^{-3} \) and \( \tau_e = (1/2) \tau_p = 3 \text{ s} \) are assumed.

toward the equilibrium state. Note that in the equilibrium state, the relative magnitude of the bulk and suprathermal distribution components are also significantly influenced by NI scattering. Figure 4 compares the equilibrium distribution functions (a) with and (b) without NI scattering. When NI scattering is ignored, the magnitude of the distri-
Fig. 4 Equilibrium proton distribution functions (a) with and (b) without NI scattering.

![Equilibrium proton distribution functions](image1)

**Fig. 4** Equilibrium proton distribution functions (a) with and (b) without NI scattering.

Fig. 5 Difference between fast deuterion distribution functions when NI scattering is (a) considered ($f_D^{\text{NI+C}}$) and (b) neglected ($f_D^C$).

![Difference between fast deuterion distribution functions](image2)

**Fig. 5** Difference between fast deuterion distribution functions when NI scattering is (a) considered ($f_D^{\text{NI+C}}$) and (b) neglected ($f_D^C$).

The distribution function is overestimated by $50 \sim 70\%$ in the energy range from 2 to 8 MeV.

Next, we show the result when an energetic deuterion beam is injected into a DT plasma with ion and electron temperatures of 20 keV. Bulk deuterion and triton densities of $n_D = n_T = 2 \times 10^{19} \text{ m}^{-3}$ and energy and particle confinement times of $\tau_E = (1/2)\tau_p = 3 \text{ s}$ are assumed.

In the same manner as in Fig. 4, we evaluated the equilibrium deuterion distribution functions when NI scattering is considered, $f_D^{\text{NI+C}}$, and neglected, $f_D^C$. Figures 5 (a) and (b) show the difference between the distribution functions when NI scattering is considered and neglected, respectively, i.e., $(f_D^C - f_D^{\text{NI+C}})/f_D^{\text{NI+C}}$, as a function of deuterion energy for deuterion beam energies $E_{\text{NBI}}$ of 1 and 2 MeV. When NI scattering is ignored the magnitude of the fast deuterion distribution function is overestimated. The degree of the difference increases as the beam-injection energy increases. This is because the energy transferred to bulk ions via single NI scattering increases with increasing beam-injection energy.

The effect of NI scattering on the magnitude of the fast-ion slowing-down distribution function was evaluated. When NI scattering is ignored, the fast-ion population in the device tends to be overestimated. The loss fraction of fast ions during the slowing-down process and the Alfvéén instability excitation probability are expected to be proportional to the magnitude of the slowing-down distribution function. In the analyses ignoring NI scattering, the fast-ion loss and instability excitation probability may be overestimated. In the present simulation, the particle and energy loss terms are assumed according to the conventional method [10]. The relative intensity of the distribution function would not depend on the loss mechanism as much. When we intend to evaluate the loss fraction and energy loss from the plasma, however, it is important to consider the mechanism. Further detailed analyses, including an understanding of an adequate loss mechanism, would be required.