Formation of Forward and Reverse Shock Waves in a Magnetized Plasma: Two-Dimensional Particle Simulations

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(Received 19 November 2012 / Accepted 9 September 2013)

Interactions of exploding and surrounding plasmas are studied with two-dimensional (2D) electromagnetic particle simulations, for a case in which the initial velocity of the exploding plasma is perpendicular to an external magnetic field. It is confirmed that essentially the same phenomena (the penetration of exploding ions, the formation of a strong magnetic-field pulse, and the generation of forward and reverse shock waves) as in the previous one-dimensional (1D) simulations occurred in the 2D simulation. Further, modified two-stream instabilities excited by relative cross-field motion between ions and electrons are investigated with attention to their effects on the 2D structure of the strong magnetic-field pulse and on ion reflection by the pulse.

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Keywords: collisionless shock, modified two-stream instability, whistler wave, particle simulation

DOI: 10.1585/pfr.8.2401155

1. Introduction

Shock waves can be generated from strong disturbances, such as solar flares and supernova explosions, and are believed to play an essential role in producing high energy particles. Particle simulations have demonstrated that a large-amplitude magnetosonic shock wave can accelerate ions, electrons, and positrons through various nonstochastic mechanisms caused by strong electromagnetic fields near a shock front [1,2]. These studies were concerned with acceleration processes after formation of a shock wave.

Shock formation process from a strong disturbance in a collisionless plasma has also been an important issue in plasma physics [3,4] because it would be quite different from that in collisional fluids. In Ref. [5], interactions of two plasmas, surrounding and exploding plasmas, and resultant generation of large-amplitude waves in an external magnetic field have been investigated with theories and simulations. It has been shown that a strong magnetic-field pulse is produced near the initial boundary of the two plasmas, from which the forward and reverse shock waves are created. Furthermore, the generation of large-amplitude Alfvén waves and acceleration of electrons in these waves [6] have been reported. These theories and simulations [5,6] are concerned with one-dimensional (1D) effects, and multidimensional effects on the wave evolution have not been investigated.

In this study, by using two-dimensional (2D) electromagnetic particle simulations, we investigate the interactions of exploding and surrounding plasmas for a case in which the initial velocity of the exploding plasma \( v_0 \) is perpendicular to the external magnetic field \( B_0 \). After describing the simulation model and parameters in sec. 2, we verify the previous 1D result [5] with the 2D simulation in sec. 3. We confirm the theoretical predictions for plasma motions and show that forward and reverse shock waves are created after the formation of a strong magnetic-field pulse. In sec. 4, we study the 2D structure due to modified two-stream instabilities excited by relative cross-field motion between ions and electrons through interaction with whistler waves. Multidimensional effects on ion reflection by the strong magnetic-field pulse are also discussed.

2. Simulation Model and Parameters

We use a 2D (two spatial coordinates and three velocity components) relativistic electromagnetic particle code with full ion and electron dynamics to study the interactions of exploding and surrounding plasmas and the resultant formation of shock waves. A uniform external magnetic field is in the \( z \) direction \( B_z = (0, 0, B_0) \), and the simulation plane is \( (x, z) \) with a size \( L_x \times L_z = 8192 \Delta_g \times 512 \Delta_g \), where \( \Delta_g \) is the grid spacing. The total number of simulation particles is \( N = 1.1 \times 10^9 \).

At \( t = 0 \), we have an exploding plasma with a fluid velocity \( v_0 = (v_0, 0, 0) \) in the region \( x < b \) and a surrounding plasma at rest in the region \( x > b \). The values of \( b \) and \( v_0 \) are set to be \( b = 3300 \Delta_g \) and \( v_0 = 12v_A \). The initial density ratio of exploding to surrounding plasmas is \( n_{E0}/n_{S0} = 2 \). The ion-to-electron mass ratio is \( m_i/m_e = 200 \). The light speed is \( c/(\omega_{pe}A_g) = 4.0 \), and the electron and ion thermal velocities in the upstream region are \( v_{Te}/(\omega_{pe}A_g) = 0.5 \) and \( v_{Ti}/(\omega_{pe}A_g) = 0.035 \), respectively, where \( \omega_{pe} \) is the electron plasma frequency averaged over the entire region. The magnetic field strength is \( |Q|/\omega_{pe} = 0.5 \); hence, the
Alfven speed is \( v_A/(\omega_{pe}A_z) = 0.14 \).

3. Evolution of 1D Averaged Quantities

In this section, we compare the 2D simulation results with the previous 1D theories and simulations. A theory of characteristic curves for plasma motions is presented in Ref. [5], where the effects of \( E_z \) induced by the cross field-motion of exploding ions are analyzed for the time scale \( |\Omega_e|^{-1} < t < |\Omega_i|^ {-1} \), where \( \Omega_e \) and \( \Omega_i \) are electron and ion gyro-frequencies, respectively. It is shown that the trajectories of particles that are initially at the boundary \( x = b \) are written as

\[
\begin{align*}
   x_{eb} &= v_dt - (v_0 - v_d) \sin(-\Omega_it)/\Omega_i + b, \quad (1) \\
   x_{sb} &= v_dt + v_d(1 + r_{sy}^2/2) \sin(-\Omega_it - r_{xy})/\Omega_i \\
            &= + b + v_d r_{xy}/\Omega_i, \\
   x_{eb} &= v_df + b, \quad (3)
\end{align*}
\]

with \( v_d = cE_z/B_z \) and \( r_{xy} = E_x/E_y \). Here, \( x_{eb} \) and \( x_{sb} \) are for exploding and surrounding ions, respectively. As for electrons, the two kinds of electrons, which are initially separated in the coordinate space, will not mixed up later if the drift approximation is valid. This boundary is denoted by \( x_{eb} \).

The upper panel in Fig. 1 shows the theoretical characteristic curves given by eqs. (1)-(3), for which we have set \( E_z/B_0 = 0.25 \), \( r_{xy} = 0.1 \) (these values are consistent with the simulation result), and \( \Omega_i = \Omega_{i0} \), which is calculated using the external magnetic field strength. In the early phase, \( x_{eb} \) moves faster than \( x_{sb} \), indicating that the exploding ions penetrate in the surrounding plasma, and the exploding and surrounding ions overlap in the region \( x_{eb} < x < x_{eb} \). The electric field \( E_z \) induced by the cross-field motion of exploding ions quickly accelerates the surrounding ions in the \( y \) direction. Because of \( \nu \times B \) force, \( \nu_y \) of the surrounding ions is converted to \( v_y \), and \( \nu_x \) of the exploding ions is converted to \( v_x \). As a result, the two characteristic curves \( x_{eb} \) and \( x_{sb} \) subsequently intersect again.

The lower panel in Fig. 1 shows the trajectories of 2D simulation particles; we followed the orbits of particles that were initially in the regions \( b - A_z < x < b \) and \( b < x < b + A_z \) for exploding and surrounding plasmas, respectively, and plotted their averaged orbits. A comparison of the two panels in Fig. 1 shows that the trajectories of 2D simulation particles can be explained by eqs. (1)-(3), although the time and space scales in the lower panel are different from those in the upper panel; this is because the 2D simulation particles experience the magnetic field that is gradually intensified, which is not considered in the theory.

We now consider the evolution of magnetic field \( B_z \) averaged along the \( z \) direction, \( B_z \). Figure 2 shows \( B_z \) as a function of \( x \) and \( t \), where \( x_{sb} \), \( x_{eb} \), and \( x_{eb} \) are the same as in the lower panel in Fig. 1. In the early stage, \( \Omega_{g0} \), \( B_z \) is intensified in the region \( x_{eb} < x < x_{eb} \), and a strong magnetic-field pulse is formed. At \( \Omega_{g0} \), the pulse splits into two pulses, which then develop into shock waves, one going forward and the other backward. This is caused by the ion reflection. The surrounding (exploding) ions are reflected forward (backward), which produces the forward (reverse) shock wave. The ion phase-space plots will be discussed later.

It is thus confirmed that essentially the same phenomena in the 1D simulation (the penetration of exploding ions, overtaking of surrounding ions, intensification of the magnetic field, and formation of two pulses) have been observed in the 2D simulation.
left- and right-hand sides of the electron boundary \( \Omega \). We assume that ions are effectively unmagnetized while electrons are magnetized. Then, a linear dispersion relation for a system with two ion beams interacting with plasma and fusion research: regular articles volume 8, 2401155 (2013)

\[
\text{whistler modes are written, in the electron rest frame, as}
\]

\[
\frac{\omega^2}{\Omega_e^2} = \frac{\omega_{pe}^2}{1 + \frac{4k^2}{\omega_{pe}^2}} \left( \frac{\omega_{pa}}{(\omega - k_i V_{ei})^2} + \frac{\omega_{pb}^2}{(\omega - k_i V_{bo})^2} \right)
\]

where the subscripts a and b denote different ion groups (surrounding and exploding ions), and \( \theta \) is the propagation angle of whistler modes. Equation (4) predicts that if \( V_{ea} > V_{eb} \), waves excited by a-ions have greater growth rates than those by b-ions. The condition for the destabilization is

\[
\theta < \theta_{\text{max}} = \arctan[(m_i/m_e)^{1/2} V_{ia}/2 V_{ei}].
\]

Because the magnetic field is intensified in the right region, \( \theta_{\text{max}} \) for this region, 78°, is greater than that for the left region, 63°. We have estimated \( \theta_{\text{max}} \) by substituting the observed values of magnetic field, plasma density, and ion speed in Eq. (5). The upper panel in Fig. 4 shows that the excited waves satisfy the condition (5). The value of \( \theta \) of the dominant mode is 65° for the right region (49° for the left region). The theoretical growth rate of the dominant mode, which is calculated from Eq. (4), is \( \gamma = 0.029|\Omega_e| \) for the right region (0.034|\Omega_e| for the left region), which is close to the observed growth rate shown in the lower panel in Fig. 4, \( \gamma = 0.022|\Omega_e| \) (0.031|\Omega_e|). From Eq. (4), we can estimate that \( \theta \) and wavenumber \( k \) for the modes that have maximum growth rates are \( \theta \approx 69° \) and \( k/\omega_{pe} \approx 1.7 \) for the right region, and \( \theta \approx 48° \) and \( k/\omega_{pe} \approx 2.0 \) for the left region. These theoretical \( \theta \)'s of the most unstable modes are close to \( \theta \)'s of the dominant modes observed in the simulation. The observed wavenumbers are smaller.

![Fig. 3 Profiles of z-averaged B_z and n_e, contour map of 2D fluctuation of B_z, and ion and electron phase-space plots (x, p_x) at \( \Omega_{\theta t} = 0.55 \). Pink and green dots represent exploding and surrounding particles, respectively.](image)

![Fig. 4 Power spectra, \( P(k_x, k_z) \), of \( B_z \) in the left- and right-hand sides of \( x_{eb} \) at \( \Omega_{\theta t} = 0.55 \) and time variation of the amplitudes of the dominant modes.](image)
than the theoretical ones; the former is about 58% of the latter for the right region and 41% for the left region. This may be because as \( k \) increases, whistler waves approach electrostatic waves; at \( \Omega_{\text{gi}} \approx 0 \), amplitudes of magnetic fluctuations were quite small at the wavenumbers where the theoretical growth rate becomes maximum, compared to those at smaller wavenumbers. Because the modes with the smaller wavenumbers are also unstable, we observed, at \( \Omega_{\text{gi}} = 0.55 \), large-amplitude magnetic fluctuations at these wavenumbers in the upper panel of Fig. 4.

We now study the nonlinear evolution of the instabilities. Figure 5 (a) shows the \( x \) profiles of \( \bar{B}_z \), \( \bar{E}_x \), and of the magnitudes of the 2D fluctuations of \( B_z \) and \( E_x \) defined by Eq. (6) at \( \Omega_{\text{gi}} = 0.875 \). Electron and ion phase-space plots are also plotted. (b) Variations of \( B_z \) and \( E_x \) along \( z \) at fixed \( x \) positions where \( \sigma_{B_z}^2 \) and \( \sigma_{E_x}^2 \) become maximum, respectively.

\[
\sigma_f^2(x, t) = \frac{1}{F_c} \int_0^{F_c} dz [F(x, z, t) - \bar{F}(x, t)]^2,
\]

where \( \bar{F}(x, t) \) is the \( z \)-averaged \( F(x, z, t) \). We note that the magnitudes of the 2D fluctuations are noticeably great near the boundary of the exploding and surrounding electrons, \( x = x_{\text{eh}} \), and near the positions where \( \bar{B}_z \) has a strong slope. Figure 5 (b) shows the variations of \( B_z \) and \( E_x \) along the \( z \) direction; the values of \( B_z \) and \( E_x \) are at fixed \( x \) positions where \( \sigma_{B_z}^2 \) and \( \sigma_{E_x}^2 \) become maximum, respectively. For comparison, \( B_z \) and \( E_x \) at earlier time, \( \Omega_{\text{gi}} = 0.55 \), are also plotted. The 2D fluctuations at later time (\( \Omega_{\text{gi}} \sim 0.875 \)) have longer wavelength and greater amplitudes. The mechanism for the intensification of 2D fluctuation with longer wavelengths near \( x = x_{\text{eh}} \) is unclear, although it seems that this nonlinear development is caused by strong current flowing there.

At \( \Omega_{\text{gi}} = 0.95 \), the strong magnetic-field pulse starts to reflect surrounding ions forward and exploding ions backward. Figure 6 (a) shows the profile of \( \bar{B}_z \) and the ion phase space plot \((x, p_x)\) at \( \Omega_{\text{gi}} = 1.125 \), where the color indicates the ion number density including both the surrounding and exploding ions. Because the ion reflection has already started, there are two pulses in Fig. 6 (a). The ions in front of the right pulse \([952 < x/(c/\omega_{pe}) < 995] \) with \( p_{ix} > 0.3 m_i c \) are reflected surrounding ions, and the ions behind the left pulse \([930 < x/(c/\omega_{pe}) < 940] \) with \( p_{ix} < 0.2 m_i c \) are the reflected exploding ions. The strength of ion reflection depends on the position \( z \) because the electromagnetic fields vary along \( z \) direction, as shown in Fig. 5. Figure 6 (b) displays the expanded view of the right pulse region. The upper panel shows the contour map of \( B_z \) in the \((x, z)\) plane for different \( z \) ranges.
and (2) of Fig. 6 at magnetic field, the ion reflection causes the damping of because the reflected ions gain energies from the electro-
ward in the region (1) compared to region (2). Further, compress magnetic field [9], magnetic field lines bend for-
in region (2) is weak. Because the reflected ions tend to
that the ion reflection in region (1) is strong, whereas that
region (2) is weak. Because the reflected ions tend to
compress magnetic field [9], magnetic field lines bend for-
ward in the region (1) compared to region (2). Further,
because the reflected ions gain energies from the electro-
magnetic field, the ion reflection causes the damping of
$B_z$ [9]. Therefore, $B_z$ is weaker in the region (1) than in the
region (2) at $\Omega_{\rho f} = 1.125$, which is slightly after the time
when the ion reflection starts.

Figure 7 shows the forces on ions in the $z$-ranges (1) and (2) of Fig. 6 at $\Omega_{\rho f} = 0.9$. This time is just before the
time when the ion reflection starts. We define the averaged
ion force as

$$F(x, z) = q[E(x, z) + u(x, z) \times B(x, z)/c],$$

where $u$ is the ion fluid velocity,

$$u(x, z) = \int \frac{v f(x, z, v) dv}{\int f(x, z, v) dv}.$$  

We write the $x$ component of $F$ averaged over the $z$-ranges (1) and (2) as $F_{x}^{(1)}$ and $F_{x}^{(2)}$, respectively. This figure clearly
shows that in front of $x_m$, where $x_m$ is the position at which $B_z$ becomes maximum, $F_{x}^{(1)}$ is much stronger than $F_{x}^{(2)}$.

This confirms that the strong ion reflection is caused by
the strong ion force in the $x$ direction, because the change
in the position $z$ of ions during the period from $\Omega_{\rho f} = 0.9$
to 1.125 is negligibly small.

5. Summary
We have studied shock formation processes arising
from the collision of exploding and surrounding plasmas
with 2D electromagnetic particle simulations, for the case
in which the initial velocity of the exploding plasma is per-
pendicular to an external magnetic field. As in the previous
1D simulations, a strong magnetic-field pulse is formed
because of the penetration of the exploding ions, and the
pulse reflects ions, generating forward and reverse shock
waves. The 2D structure of the pulse and its effect on
ion reflection were also investigated with attention to the
evolution of whistler wave instabilities caused by relative
cross-field motion between ions and electrons. Long-time
evolution of 2D structures of forward and reverse shock
waves is a future subject.

Acknowledgments
This work was carried out as part of the collaboration program under Grant No. NIFS12KNSS036 of the National Institute for Fusion Science, and of the joint research program of the Solar-Terrestrial Environment Laboratory, Nagoya University, and was supported in part by a Grant-in-Aid for Scientific Research (C) Grant No. 21540510 from the Japan Society for the Promotion of Science.