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Single Particle and Fluid Picture for the Ponderomotive Drift in Nonuniform Plasmas

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Synopsis

General expressions for the ponderomotive force as a second-order average force acting on a single charged particle is derived in the form of a potential force. The result is valid for arbitrary spatial structure of the ambient static magnetic field. Sum of the resulting drift flux and the second-order magnetic flux due to the induced magnetic moments of the particles is found to agree with the second-order averaged flux calculated by the fluid model.

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§1. INTRODUCTION

Ponderomotive force has received considerable attention as one of the most important nonlinear effects due to large amplitude oscillating fields. It is an average second-order force which acts on a plasma as a result of spatially non-uniform oscillating fields and yields a modification of the average plasma profile, thereby causing a quasilinear modification of the stability characteristics of the original high frequency oscillating fields. The most well-known example of such effects is the excitation of a low frequency wave by the ponderomotive force due to a couple of high frequency waves, one of which is then destabilized accompanied by the excitation of the low frequency wave (parametric excitation)[1].

The ponderomotive force is also receiving interest in connection with rf stabilization of the flute mode in a mirror machine[2], as it can also cause a non-ambipolar particle flux across a static magnetic field.

Both single particle and fluid calculations have been carried out to derive a general expression for the ponderomotive forces[3] and their inter-relations were discussed[4)5)]. However, there seems to exist a controversy concerning the reasoning for the equivalence of the single particle and fluid theories of the ponderomotive drift across the static magnetic field. Moreover theses calculations are restricted to the case of uniform static magnetic field. In the present paper, we confirm the above equivalence by a straightforward vectorial analysis which is valid even for a nonuniform static magnetic field, and thereby clarify the relation between the single particle picture and fluid-based picture on the ponderomotive drift. Using this calculation, we also show that the ponderomotive force acting on a single particle is a potential force independent of the spatial structure of the ambient magnetic field.

In §2, we first derive a general expression for the ponderomotive force acting on a single particle and show that it can always be expressed as a potential force. In

§3, the difference between this expression and the expression obtained by the fluid model and the resulting difference in the averaged flow across the magnetic field are discussed. In §4, we calculate the second-order magnetizing flow based on the single particle picture and confirm that this magnetizing flow can account for the above discrepancy in the averaged flow. In the last section, brief summary and discussion of the important results are elucidated.

Throughout this paper, we, for simplicity, ignore the thermal motion of the particles, but no assumption is made concerning the spatial structure of the magnetic field and the plasma density profile apart from the fact that the oscillating field amplitude is sufficiently weak that the second-order perturbation analysis be justified and that the Larmor radii of the particles are sufficiently small that the guiding center approximation can be used.

§2 Ponderomotive Force on a Single Particle as a Potential Force

We consider a plasma in the presence of electromagnetic fields given by

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \mathbf{E}_1(\mathbf{r},t) \\ \mathbf{B}(\mathbf{r},t) &= \mathbf{B}_0(\mathbf{r},t)+\mathbf{B}_1(\mathbf{r},t) \end{aligned} \tag{1}$$

where $\mathbf{B}_0$ is the static magnetic field in the absence of the oscillating fields and $\mathbf{E}_1$ and $\mathbf{B}_1$ are oscillating electric and magnetic fields which are assumed to be of the form

$$\begin{aligned} \mathbf{E}_1(\mathbf{r},t) &= \overline{\mathbf{E}}_1(\mathbf{r})\exp(-i\omega t)+\overline{\mathbf{E}}_1^{\,*}(\mathbf{r})\exp(i\omega t) \\ \mathbf{B}_1(\mathbf{r},t) &= \overline{\mathbf{B}}_1(\mathbf{r})\exp(-i\omega t)+\overline{\mathbf{B}}_1^{\,*}(\mathbf{r})\exp(i\omega t) \quad . \end{aligned} \tag{2}$$

They satisfy the Maxwell relations

$$\frac{\partial}{\partial t}\mathbf{B}_1 = -\nabla\times\mathbf{E}_1, \quad \nabla\cdot\mathbf{B}_0 = \nabla\cdot\mathbf{B}_1 = 0 \quad . \tag{3}$$

We assume that the oscillating fields are sufficiently weak that the excursion lengths of the particles due to these fields are small as compared with the scale lengths of the spatial variation of the fields. This permits us to treat the effect of the oscillating fields as a perturbation. For simplicity, we ignore the zero-order electric field, i.e., $\mathbf{E}_0=0$.

The equation of motion of a particle of mass m and charge q is

$$m\frac{d\mathbf{v}(t)}{dt} = q\left[\mathbf{E}[\mathbf{r}(t),t]+\mathbf{v}(t)\times\mathrm{B}[\mathbf{r}(t),t]\right] \tag{4}$$

$$\frac{d\mathbf{r}(t)}{dt} = \mathbf{v}(t) \quad . \tag{5}$$

We neglect the zero-order motion (including the cyclotron motion and the associated drift motion) and denote the first-order induced motion due to the oscillating field by $\mathbf{r}_1(t)$ and $\mathbf{v}_1(t)$. They can be written as

$$\mathbf{v}_1(t)= \frac{q}{m}\,\frac{\omega_c}{\omega^2-\omega_c^2}\,\mathbf{b}\times\mathbf{E}_1+\frac{q}{m}\left[\frac{\mathbf{b}\times(\mathbf{b}\times\mathbf{E}_1')}{\omega^2-\omega_c^2}-\frac{\mathbf{b}(\mathbf{b}\cdot\mathbf{E}_1')}{\omega^2}\right] \tag{6}$$

$$\mathbf{r}_1(t)=-\frac{q}{m}\,\frac{\omega_c}{\omega^2-\omega_c^2}\,\frac{\mathbf{b}\times\mathbf{E}_1'}{\omega^2}+\frac{q}{m}\left[\frac{\mathbf{b}\times(\mathbf{b}\times\mathbf{E}_1)}{\omega^2-\omega_c^2}-\frac{\mathbf{b}(\mathbf{b}\cdot\mathbf{E}_1)}{\omega^2}\right] \tag{7}$$

where $\omega_c=qB_0/m$, $\mathbf{b}=\mathrm{B}_0/\mathrm{B}_0$ and the prime denotes the time derivative. In eq.(6), the first term on the right-hand side is the $\mathbf{E}\times\mathbf{B}$ drift and the second-term (the bracketed part) is the polarization drift.

The ponderomotive force $\mathbf{G}$ acting on the particle is the second-order averaged force which can be written as follows:

$$\mathbf{G}=\mathbf{G}_1+\mathbf{G}_2 \tag{8}$$

where

$$\mathbf{G}_1=q\left[\overline{(\mathbf{r}_1\cdot\nabla)\mathbf{E}_1}+\overline{\mathbf{v}_1\times\mathbf{B}_1}\right] \tag{9}$$

$$\mathbf{G}_2=q\left[\overline{\mathbf{v}_1\times(\mathbf{r}_1\cdot\nabla)\mathrm{B}_0}\right] \tag{10}$$

the bar denoting the time average over the oscillating period $2\pi/\omega$. Using the relations

$$\overline{\mathbf{v}_1 \times \mathbf{B}_1} = \overline{\mathbf{r}_1' \times \mathbf{B}_1} = -\overline{\mathbf{r}_1 \times \mathbf{B}_1'} = \overline{\mathbf{r}_1 \times (\nabla \times \mathbf{E}_1)} \tag{11}$$

$$\overline{\mathbf{v}_1 \times (\mathbf{r}_1 \cdot \nabla) \mathbf{B}_0} = -\overline{\mathbf{r}_1 \times (\mathbf{v}_1 \cdot \nabla) \mathbf{B}_0}$$

$$= \frac{1}{2} \left[\overline{\mathbf{v}_1 \times (\mathbf{r}_1 \cdot \nabla) \mathbf{B}_0} - \overline{\mathbf{r}_1 \times (\mathbf{v}_1 \cdot \nabla) \mathbf{B}_0} \right] \tag{12}$$

and taking the x-component of $\mathbf{G}$, we calculate as follows:

$$\frac{1}{q} G_{1x} = q \left[\overline{(\mathbf{r}_1 \cdot \nabla) \mathbf{E}_1} + \overline{\mathbf{r}_1 \times (\nabla \times \mathbf{E}_1)} \right]_x = \overline{(\mathbf{r}_1 \cdot \nabla) E_{1x}} + \overline{y_1 (\partial_x E_{1y} - \partial_y E_{1x})}$$

$$- \overline{z_1 (\partial_z E_{1x} - \partial_x E_{1z})}$$

$$= \overline{\mathbf{r}_1 \cdot \partial_x \mathbf{E}_1} \tag{13}$$

$$\frac{1}{q} G_{2x} = \frac{1}{2} \left[\overline{\mathbf{v}_1 \times (\mathbf{r}_1 \cdot \nabla) \mathbf{B}_0} - \overline{\mathbf{r}_1 \times (\mathbf{v}_1 \cdot \nabla) \mathbf{B}_0} \right]_x$$

$$= \frac{1}{2} \left[\overline{v_{1y} (\mathbf{r}_1 \cdot \nabla) B_{0z}} - \overline{v_{1z} (\mathbf{r}_1 \cdot \nabla) B_{0y}} - \overline{y_1 (\mathbf{v}_1 \cdot \nabla) B_{0z}} + \overline{z_1 (\mathbf{v}_1 \cdot \nabla) B_{0y}} \right]$$

$$= \frac{1}{2} \left[\overline{(\mathbf{r}_1 \times \mathbf{v}_1)} \cdot \partial_x \mathbf{B}_0 + \overline{(\mathbf{r}_1 \times \mathbf{v}_1)_x} \; \nabla \cdot \mathbf{B}_0 \right]$$

$$= \frac{1}{2} \overline{(\mathbf{r}_1 \times \mathbf{v}_1)} \cdot \partial_x \mathbf{B}_0 \tag{14}$$

where ∂_x represents the partial derivative with respect to x, and x_1, y_1, z_1 are the components of $\mathbf{r}_1$ and we used the relation (3). We now calculate both sides of the identity

$$\overline{\mathbf{r}_1 \cdot \partial_x m \mathbf{v}_1'} = \overline{m \mathbf{v}_1' \cdot \partial_x \mathbf{r}_1}$$

using the equation of motion as follows:

$$\overline{\mathbf{r}_1 \cdot \partial_x m \mathbf{v}_1'} = q \overline{\mathbf{r}_1 \cdot \partial_x (\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0)}$$

$$= q \left[\overline{\mathbf{r}_1 \cdot \partial_x \mathbf{E}_1} + \overline{\mathbf{r}_1 \times (\partial_x \mathbf{v}_1)} \cdot \mathbf{B}_0 + \overline{\mathbf{r}_1 \times \mathbf{v}_1} \cdot \partial_x \mathbf{B}_0 \right]$$

$$\overline{m \mathbf{v}_1' \cdot \partial_x \mathbf{r}_1} = q \left[\overline{(\partial_x \mathbf{r}_1) \cdot (\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0)} \right]$$

$$= q \left[\overline{\mathbf{E}_1 \cdot \partial_x \mathbf{r}_1} - \overline{(\partial_x \mathbf{v}_1 \times \mathbf{r}_1)} \cdot \mathbf{B}_0 \right]$$

Equating the right-hand sides of the above two equations, we get

$$\overline{\mathbf{E}_1\cdot\partial_x\mathbf{r}_1}-\overline{\mathbf{r}_1\cdot\partial_x\mathbf{E}_1}=\overline{\mathbf{r}_1\times\mathbf{v}_1}\cdot\partial_x\mathbf{B}_0=\frac{2}{q}G_{2x}. \qquad (15)$$

Combining eqs. (14) and (15), we obtain

$$G_x=G_{1x}+G_{2x}=\frac{q}{2}\ \overline{\partial_x(\mathbf{r}_1\cdot\mathbf{E}_1)} \quad . \qquad (16)$$

Similar calculations for G_y and G_z finally yield

$$\mathbf{G}=\nabla\overline{\left[\frac{q}{2}\mathbf{r}_1\cdot\mathbf{E}_1\right]} \qquad (17)$$

which proves that the ponderomotive force acting on a single particle is a potential force. We specifically note here that the above derivation is valid for arbitrary spatial structure of the ambient magnetic field and that if it is spatially uniform, i.e. ω_c=constant, then the ponderomotive force $\mathbf{G}$ due to a plane-polarized perpendicular electric field ($\mathbf{b}\cdot\mathbf{E}_1=0$) changes its sign or direction at $\omega_c^2=\omega^2$.

§3. Relation to the Ponderomotive Force on a Fluid Element

The equation of motion for the cold plasma fluid element is given by

$$m\left[\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}\right]=q\left[\mathbf{E}+\mathbf{u}\times\mathbf{B}\right] \qquad (18)$$

where $\mathbf{u}(\mathbf{r},t)$ is the fluid velocity. Since the linearized version of eq. (18) is exactly of the same form as that of eq. (3), we have

$$\mathbf{u}_1(\mathbf{r},t)=\frac{q}{m}\frac{\omega_c}{\omega^2-\omega_c^2}\ \mathbf{b}\times\mathbf{E}_1+\frac{q}{m}\left[\frac{\mathbf{b}\times(\mathbf{b}\times\mathbf{E}_1')}{\omega^2-\omega_c^2}-\frac{\mathbf{b}(\mathbf{b}\cdot\mathbf{E}_1')}{\omega^2}\right]=\mathbf{v}_1 \quad . \qquad (19)$$

The displacement vector $\mathbf{R}(\mathbf{r},t)$ of the fluid is given by the relation[4)]

$$\frac{\partial}{\partial t}\mathbf{R}+(\mathbf{u}\cdot\nabla)\mathbf{R}=\mathbf{u}\ ,\quad \frac{\partial}{\partial t}\mathbf{R}_1=\mathbf{u}_1\ , \qquad (20)$$

where $\mathbf{R}_1$ and $\mathbf{u}_1$ represent the first order oscillating parts.

The ponderomotive force acting on the fluid element is the second-order averaged force derived from eq. (18):

$$\mathbf{F}= -m\,\overline{(\mathbf{u}_1\cdot\nabla)\mathbf{u}_1}+q\overline{\mathbf{u}_1\times\mathbf{B}_1} \quad . \tag{21}$$

The first term on the right-hand side can be written as

$$\begin{aligned} -m\,\overline{(\mathbf{u}_1\cdot\nabla)\mathbf{u}_1} &= m\overline{(\mathbf{R}_1\cdot\nabla)\mathbf{u}_1'} \\ &= q\overline{(\mathbf{R}_1\cdot\nabla)\left[\mathbf{E}_1+\ \mathbf{u}_1\times\mathbf{B}_0\right]} \\ &= q\overline{(\mathbf{R}_1\cdot\nabla)\mathbf{E}_1}+q\left[\overline{(\mathbf{R}_1\cdot\nabla)\mathbf{u}_1}\right]\times\mathbf{B}_0+q\overline{\mathbf{u}_1\times\left[(\mathbf{R}_1\cdot\nabla)\mathbf{B}_0\right]} \quad . \end{aligned} \tag{22}$$

Substitution of eq. (22) into eq. (21) yields

$$\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3 \tag{23}$$

where

$$\mathbf{F}_1=q\left[\overline{(\mathbf{R}_1\cdot\nabla)\mathbf{E}_1}+\overline{\mathbf{u}_1\times\mathbf{B}_1}\right] \tag{24}$$

$$\mathbf{F}_2=q\overline{\mathbf{u}_1\times\left[(\mathbf{R}_1\cdot\nabla)\mathbf{B}_0\right]} \tag{25}$$

$$\mathbf{F}_3=q\overline{\left[(\mathbf{R}_1\cdot\nabla)\mathbf{u}_1\right]\times\mathbf{B}_0} \quad . \tag{26}$$

Obviously, $\mathbf{F}_1=\mathbf{G}_1$ and $\mathbf{F}_2=\mathbf{G}_2$, so that we have the relation

$$\mathbf{F} = \mathbf{G}+\mathbf{F}_3 \quad . \tag{27}$$

We now discuss the drift across the magnetic field $\mathbf{B}_0$ due to the ponderomotive force. First, in the single particle picture, the drift across $\mathbf{B}_0$ due to ponderomotive force is given by

$$\overline{\mathbf{v}_2}=\mathbf{u}_{\mathbf{G}\times\mathbf{B}}=\frac{1}{q}\,\frac{\mathbf{G}\times\mathbf{b}}{B_0} = \frac{(\mathbf{F}_1+\mathbf{F}_2)\times\mathbf{b}}{qB_0} \quad , \tag{28}$$

On the other hand, the drift velocity of the fluid element across B_0 due to the ponderomotive force has a term

$$\mathbf{u}_{\mathbf{F}_3\times B} = \frac{\mathbf{F}_3\times\mathbf{b}}{qB_0} \quad , \tag{29}$$

in addition to the drift given by eq. (28). Using the relation (26), we can write this term as

$$u_{F_3 \times B} = -\mathbf{b}\times\overline{\left[(\mathbf{R}_1\cdot\nabla)\mathbf{u}_1\times\mathbf{b}\right]}$$

$$= -\overline{\left[(\mathbf{R}_1\cdot\nabla)\mathbf{u}_1\right]}_{\perp} = \overline{\left[(\mathbf{u}_1\cdot\nabla)\mathbf{R}_1\right]}_{\perp} \qquad (30)$$

where the suffix $\perp$ stands for the component perpendicular to B_0. We note that the right-hand side of eq. (30) is nothing but the perpendicular component of the convection flow velocity.

§4. Magnetizing Drift

In the single particle picture, in addition to the ponderomotive force there exists another second-order effect, which is the induced magnetization due to the oscillating fields[5].

The second-order magnetic moment of a particle induced by the oscillating fields is given by

$$\boldsymbol{\mu}_2 = \frac{q}{2}\,\overline{\mathbf{r}_1\times\mathbf{v}_1} \quad . \qquad (31)$$

Sum of the magnetic moments of the particles constituting the fluid element yields a macroscopic induced magnetization[5],

$$M_2 = \frac{q}{2} n_0 \overline{\mathbf{R}_1\times\mathbf{u}_1} \quad . \qquad (32)$$

When $\mathbf{M}_2$ has a spatial profile, it gives rise to a magnetizing flux

$$\Gamma_2 = \frac{1}{q}\nabla\times M_2 = \frac{1}{2}\nabla\times\left[n_0\overline{\mathbf{R}_1\times\mathbf{u}_1}\right] \quad . \qquad (33)$$

We calculate this flux as follows:

$$\boldsymbol{\Gamma}_2 = \frac{1}{2}\left[\nabla n_0 \times (\overline{\mathbf{R}_1 \times \mathbf{u}_1}) + n_0 \nabla \times (\overline{\mathbf{R}_1 \times \mathbf{u}_1})\right]$$

$$= \frac{1}{2}\left[\overline{\mathbf{R}_1(\mathbf{u}_1 \cdot \nabla n_0)} - \overline{\mathbf{u}_1(\mathbf{R}_1 \cdot \nabla n_0)}\right]$$

$$+\frac{1}{2} n_0 \left[\overline{\mathbf{R}_1(\nabla \cdot \mathbf{u}_1)} - \overline{\mathbf{u}_1(\nabla \cdot \mathbf{R}_1)} - \overline{(\mathbf{R}_1 \cdot \nabla)\mathbf{u}_1} + \overline{(\mathbf{u}_1 \cdot \nabla)\mathbf{R}_1}\right]$$

$$= n_0 \overline{(\mathbf{u}_1 \cdot \nabla)\mathbf{R}_1} + \overline{\mathbf{R}_1 \nabla \cdot (n_0 \mathbf{u}_1)} \ . \qquad (34)$$

The first term on the right-hand side is the convective flux, whose perpendicular component yields the second-order flux due to the drift $\mathbf{u}_{F_3 \times B}$ given by eq. (30).

The second term on the right-hand side of eq. (34) can be calculated by using the first-order continuity equation:

$$\frac{\partial}{\partial t} n_1 + \nabla \cdot (n_0 \mathbf{u}_1) = 0 \qquad (35)$$

which yields

$$\overline{\mathbf{R}_1 \nabla \cdot (n_0 \mathbf{u}_1)} = -\overline{\mathbf{R}_1 n'} = \overline{n_1 \mathbf{u}_1} \ . \qquad (36)$$

This term is a second-order average flux due to the density oscillation and is different from the flux due to the ponderomotive force acting on the fluid element. The latter contributes to $n_0 \overline{\mathbf{u}}_{2\perp}$ in the fluid picture, where, $\mathbf{u}_2$ is the second-order fluid velocity.

We therefore confirm that the allowance for the induced magnetizing flux can indeed account for the relation between the single particle picture and the fluid picture concerning the second-order average drift across the ambient magnetic field.

As a final remark, we note that the total average second-order flux due to both the ponderomotive potential force and the induced magnetization calculated by the single particle picture, that is,

$$n_0 \overline{\mathbf{v}}_2 + \boldsymbol{\Gamma}_2 = n_0 \overline{\mathbf{v}}_2 + \overline{n_1 \mathbf{u}_1} + n_0 \overline{(\mathbf{u}_1 \cdot \nabla)\mathbf{R}_1} \qquad (37)$$

can be derived from a more general consideration based on the fluid picture. The average flux due to the fluid picture can generally be written as

$$\overline{n\mathbf{u}} = \bar{n}\ \bar{\mathbf{u}} + \overline{(n-\bar{n})(\mathbf{u}-\bar{\mathbf{u}})} \tag{38}$$

in which the average fluid velocity can be written as[4)]

$$\bar{\mathbf{u}} = \overline{\frac{\partial}{\partial t} R} + \overline{(\mathbf{u}_1 \cdot \nabla)R_1}$$

$$= \overline{<\mathbf{v}>} + \overline{(\mathbf{u}\cdot\nabla)R} \tag{39}$$

where $<\mathbf{v}>$ is the average guiding center velocity of the particles (of charge q) which constitute the fluid element. Substituting eq. (39) and retaining only the second-order terms, we immediately obtain the right-hand side of eq. (37). This clarifies the relation between the single particle picture and fluid picture including both the parallel and perpendicular components of the second-order average flux.

§5. Summary and Discussions

We have derived a general expression for the ponderomotive force as a second-order average force acting on a single charged particle, and have shown that it can be written as a potential force. The derivation is valid for arbitrary spatial structure of the ambient magnetic field. In particular, the result is applicable even to the case in which the wavelengths of the oscillation are comparable to or longer than a spatial scale length of the ambient magnetic field. The resulting drift across the static magnetic field is compared with the drift due to the ponderomotive force acting on a plasma particle as a fluid element and the difference is accounted for as due to the convective flow in the fluid model. The induced second-order magnetization is calculated based on the single particle model and the total flux due to this induced magnetization and the ponderomotive potential force is found to agree with the second-order average flux calculated by the fluid model.

It is clear that the magnetizing flux is divergence-free, but the flux due to the ponderomotive potential force is not divergence-free except for the case of spatially uniform density and uniform ambient magnetic field. This flux aslo depends on the mass and charge of the particle. As a result, a non-ambipolar particle flux across the ambient magnetic field can be produced. Since the interchange flute mode instability arises from the non-ambilolar flux due to the centrifugal force acting on a particle associated with a magnetic field curvature, its rf-stabilization effect is determined by the ponderomotive potential force. On the other hand, the plasma profile modification is governed by the ponderomotive force acting on the fluid element. We note that a second-order static electric field can be produced by the non-ambipolar particle flux, but its effect on the plasma flow is obviously the same for both the single particle and fluid picture.

In the present calculation, we have totally ignored the thermal motion of the particle as well as the zero-order static electric field. Allowance for these effects requires a complicated calculation, but the essential feature concerning the equivalence of the single particle drift and the fluid drift will be unchanged. For actual calculations, some specific form will have to be assumed for the oscillating field. The calculations are underway and will be reported elsewhere. We , however, note that the present calculations can be justified provided that the following conditions are satisfied:

1) The excursion lengths of the particles due to both the static and oscillating electric fields during the one oscillation period $2\pi/\omega$ are small as compared with the characteristic scale lengths (including the wavelengths of the oscillation) of the electric and magnetic fields;
2) the zero-order velocity (including both the thermal velocity and drift velocity due to the zero-order fields) is small as compared with the phase velocity of the oscillation.

These conditions are satisfied in many cases of interest in the present-day plasma physics, so that the present results are applicable to many actual situations of nonlinear wave-plasma interaction. Of course, resonant wave-particle interactions are neglected, but they are not responsible for the ponderomotive drift.

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