# Radial thermal diffusivity of toroidal plasma affected by resonant magnetic perturbations

**Ryutaro Kanno**[1,2], **Masanori Nunami**[1], **Shinsuke Satake**[1,2], **Hisanori Takamaru**[3] **and Masao Okamoto**[3]

[1] National Institute for Fusion Science, Toki 509-5292, Japan

[2] Department of Fusion Science, the Graduate University for Advanced Studies, Toki 509-5292, Japan

[3] Department of Computer Science, Chubu University, Kasugai 487-8501, Japan

E-mail: `kanno@nifs.ac.jp`

**Abstract.**
We investigate how the radial thermal diffusivity of an axisymmetric toroidal plasma is modified by effect of resonant magnetic perturbations (RMPs), using a drift kinetic simulation code for calculating the thermal diffusivity in the perturbed region. The perturbed region is assumed to be generated on and around the resonance surfaces, and is wedged in between the regular closed magnetic surfaces. It has been found that the radial thermal diffusivity $\chi_r$ in the perturbed region is represented as $\chi_r = \chi_r^{(0)}\{1 + c\,\langle \|\delta B_r\|^2\rangle\}$. Here $\langle \|\delta B_r\|^2\rangle^{1/2}$ is the strength of the RMPs in the radial directions, $\langle\cdot\rangle$ means the flux surface average defined by the unperturbed (i.e., original) magnetic field, $\chi_r^{(0)}$ is the neoclassical thermal diffusivity, and $c$ is a positive coefficient. In this paper, dependence of the coefficient $c$ on parameters of the toroidal plasma is studied in results given by the $\delta f$ simulation code solving the drift kinetic equation under an assumption of zero electric field. We find that the dependence of $c$ is given as $c \propto \omega_{\rm b}/\nu_{\rm eff}\, m$ in the low collisionality regime $\nu_{\rm eff} < \omega_{\rm b}$, where $\nu_{\rm eff}$ is the effective collision frequency, $\omega_{\rm b}$ is the bounce frequency and $m$ is the particle mass. In case of $\nu_{\rm eff} > \omega_{\rm b}$, the thermal diffusivity $\chi_r$ evaluated by the simulations becomes close to the neoclassical thermal diffusivity $\chi_r^{(0)}$.

PACS numbers: 52.25.Fi, 52.65.Pp

*Keywords*: collisional transport, thermal diffusion, resonant magnetic perturbation, toroidal plasma, diffusion process, stochastic analysis, $\delta f$ simulation, Monte Carlo method

## 1. Introduction

To understand fundamental properties of plasma transport in a perturbed magnetic field is important for control of a fusion plasma by application of resonant magnetic perturbations (RMPs). In this paper, we investigate how the radial heat transport phenomenon in an axisymmetric toroidal plasma is modified by effect of RMPs. Presuppositions in the study are as simplified as possible for the sake of visible prospect. 1) The plasma particles are assumed to be confined in a tokamak field disturbed partly by the RMPs. The perturbed region is generated on and around the resonance surfaces, and is wedged in between the regular closed magnetic surfaces. In the perturbed region, there is no magnetic field line connected to the divertor. Here the perturbed magnetic field is assumed to be fixed and time evolution of the RMP field is neglected. The ratio of the gyroradius $\rho$ to the width of the perturbed region $\Delta_{\mathrm{RMP}}$ is assumed to satisfy $\rho/\Delta_{\mathrm{RMP}} \ll 1$, and thus the scales of $\rho$ and $\Delta_{\mathrm{RMP}}$ are well separated from each other. 2) The Coulomb collision is assumed to be represented as the collisions between plasma particles of the same species, where the plasma particles are monoenergetic. Then, the species of the plasma particles is assumed to be ion. 3) Electric field, MHD activities, neutrals, and impurities are neglected. Under the above presuppositions, in this paper, effect of the RMPs on the radial heat transport phenomenon is investigated, and a model formula of the radial thermal diffusivity is derived from the investigation.

Diffusion of plasma particles in coordinate space results from Coulomb collisions, which cause small deflections of the velocity vector of a plasma particle [1]. Here the plasma particles are assumed to be included in an axisymmetric toroidal plasma having nested flux surfaces. After a collision time, i.e., after a plasma particle is exposed sufficiently to the Coulomb collision, a sample path of the guiding center in the coordinate space, $\boldsymbol{X}(t)$, is given by a diffusion process: $\mathrm{d}\boldsymbol{X}(t) = \boldsymbol{v}\,\mathrm{d}t \to \boldsymbol{V}(\boldsymbol{X}(t))\,\mathrm{d}t + \sigma(\boldsymbol{X}(t)) \cdot \mathrm{d}\boldsymbol{W}(t)$, as shown in [2]. Here $\boldsymbol{v}$ is the velocity of the guiding center, $\boldsymbol{V}(\boldsymbol{x})$ is the mean velocity at a position $\boldsymbol{x}$ in the coordinate space, $\boldsymbol{W}(t)$ is a Brownian motion, and $t$ denotes time. The coordinate system is set to describe reference surfaces which consist of the nested flux surfaces. The diffusion coefficient $D^{ij}(\boldsymbol{x})$ is represented as $D^{ij} = \sigma^i_k \delta^{k\ell} \sigma^j_\ell$ by $\sigma = (\sigma^i_j(\boldsymbol{x}))$ in the equation of the random motion, where the indexes $i, j, k, \ell$ indicate a component of a tensor (i.e., $i, j, k, \ell = 1, 2, 3$). Diffusive transport phenomena in an axisymmetric toroidal plasma having nested flux surfaces are treated in the neoclassical theory [1]. We can estimate the neoclassical transport coefficients by calculating energy integral of $D^{ij}$ [3, 4].

When the magnetic field, in which the plasma particles are confined, is disturbed partly by RMPs, the neoclassical theory is no longer applicable to the transport phenomena because the nested flux surfaces are destroyed (or ergodized) in the perturbed region. On the other hand, it is known that the theory of field-line diffusion (hereafter, the FLD theory) [5, 6], which is the standard theory of diffusive transport phenomena in a chaotic structure of magnetic field lines in the collisionless limit, is not useful for explaining the heat transport phenomenon in the perturbed region after a collision time [7, 8], where the perturbed region is supposed to be bounded radially on both sides by the regular closed magnetic surfaces. How is the neoclassical transport modified? What is a parameter of the toroidal plasma explaining the transport properties? Answering the questions is the subject of this paper. For the purpose, based on results of drift kinetic simulations including effects of Coulomb collisions and RMPs, the modeling of the radial heat transport is considered. Before execution of the simulations, in order to avoid a haphazard way to approach the subject, we narrow candidates for the key parameters explaining properties of the heat transport phenomenon.

This paper is organized as follows. The candidates for the key parameters are discussed in section 2. In section 3, dependence of the radial thermal diffusivity on the candidates given in section 2 is studied in results of drift-kinetic simulations, and a model formula of the thermal diffusivity is derived. Finally, in section 4, summary and discussions are given.

## 2. Key parameters explaining radial heat transport

Hereafter, the original flux surfaces (i.e., the nested flux surfaces) of the unperturbed magnetic field are used as the reference surfaces in the perturbed magnetic field. Effect of the RMPs on guiding center motion is interpreted as noise on the motion in statistical sense. The noise causes small and random changes of the displacement vector of a guiding center because the magnetic field lines are disarranged by the RMPs. If the effect of the RMPs is represented as $\widetilde{\mathrm{N}}(s) \cdot \boldsymbol{v}\, \mathrm{d}s$ by a random function $\widetilde{\mathrm{N}}(s) = (\widetilde{N}^i_j(s))$, then the guiding center motion is given by a stochastic process $\boldsymbol{X}_{t,\boldsymbol{x}}(s)$ in the coordinate space [8]:

$$\begin{aligned}\mathrm{d}\boldsymbol{X}_{t,\boldsymbol{x}}(s) = &\left\{1 + \widetilde{\mathrm{N}}(s)\right\} \cdot \boldsymbol{v}\, \mathrm{d}s \\ &\to \left\{\boldsymbol{V}(\boldsymbol{X}_{t,\boldsymbol{x}}(s)) + \widetilde{\boldsymbol{V}}\right\} \mathrm{d}s + \left\{1 + \widetilde{\mathrm{N}}(s)\right\} \cdot \sigma(\boldsymbol{X}_{t,\boldsymbol{x}}(s)) \cdot \mathrm{d}\boldsymbol{W}(s) \quad \text{as } s\nu \to \infty,\end{aligned} \tag{1}$$

where $s$ denotes time ($s \geq t$), $\boldsymbol{X}_{t,\boldsymbol{x}}(s)$ is a sample path $\boldsymbol{X}(s)$ satisfying $\boldsymbol{X}(t) = \boldsymbol{x}$, $\nu$ is the collision frequency, and $\widetilde{\boldsymbol{V}} = \widetilde{\mathrm{N}} \cdot \boldsymbol{V}$ is the perturbed mean velocity. Because, in general, magnetic field including RMPs satisfies Maxwell's equations, the RMP field is a smooth function of $t$ and $\boldsymbol{x}$. Then, it is natural that the noise originating from the RMPs, $\widetilde{N}^i_j(s)$, is continuous in time $s$, and that $|\widetilde{N}^i_j(s)|$ is bounded. From equation (1), the contributions of the guiding centers to the collective motion at a position $\boldsymbol{x}$ are given as follows [8]:

$$\begin{aligned}\widehat{V}^i &= \lim_{\epsilon\to 0+} \frac{1}{\epsilon} E\Big[E\Big[X^i_{t,\boldsymbol{x}}(t+\epsilon) - x^i \Big| \mathcal{Z}^{t+\epsilon}_t\Big]\Big] \\ &= V^i(\boldsymbol{x}) + \lim_{\epsilon\to 0+} E\Big[E\Big[\widetilde{V}^i \Big| \mathcal{Z}^{t+\epsilon}_t\Big]\Big] \\ &= V^i(\boldsymbol{x}) + V^j(\boldsymbol{x}) \lim_{\epsilon\to 0+} E\Big[E\Big[\widetilde{N}^i_j \Big| \mathcal{Z}^{t+\epsilon}_t\Big]\Big],\end{aligned} \tag{2}$$

$$\begin{aligned}\widehat{D}^{ij} &= \lim_{\epsilon\to 0+} \frac{1}{\epsilon} E\Bigg[E\Bigg[\Big\{X^i_{t,\boldsymbol{x}}(t+\epsilon) - x^i\Big\}\Big\{X^j_{t,\boldsymbol{x}}(t+\epsilon) - x^j\Big\} \Bigg| \mathcal{Z}^{t+\epsilon}_t\Bigg]\Bigg] \\ &= \lim_{\epsilon\to 0+} \frac{1}{\epsilon} E\Bigg[E\Bigg[\Big\{\delta^i_k + \widetilde{N}^i_k\Big\}\sigma^k_{k_1}\Big\{W^{k_1}(t+\epsilon) - W^{k_1}(t)\Big\} \\ &\qquad \times \Big\{\delta^j_\ell + \widetilde{N}^j_\ell\Big\}\sigma^\ell_{\ell_1}\Big\{W^{\ell_1}(t+\epsilon) - W^{\ell_1}(t)\Big\} \Bigg| \mathcal{Z}^{t+\epsilon}_t\Bigg]\Bigg] \\ &= D^{k\ell}(\boldsymbol{x}) \lim_{\epsilon\to 0+} E\Bigg[E\Bigg[\Big\{\delta^i_k + \widetilde{N}^i_k\Big\}\Big\{\delta^j_\ell + \widetilde{N}^j_\ell\Big\} \Bigg| \mathcal{Z}^{t+\epsilon}_t\Bigg]\Bigg],\end{aligned} \tag{3}$$

and

$$\lim_{\epsilon\to 0+}\frac{1}{\epsilon}E\left[E\left[\left\{X^i_{t,\boldsymbol{x}}(t+\epsilon)-x^i\right\}\left\{X^j_{t,\boldsymbol{x}}(t+\epsilon)-x^j\right\}\right.\right.$$
$$\left.\left.\times\cdots\times\left\{X^k_{t,\boldsymbol{x}}(t+\epsilon)-x^k\right\}\middle|\mathcal{Z}_t^{t+\epsilon}\right]\right]$$
$$=0, \tag{4}$$

where $E$ is the expectation operator given by the stochastic process (1), the indexes $i, j, k, k_1, \ell, \ell_1$ in equations (2) - (4) indicate a component of a vector/tensor (i.e., $i, j, k, k_1, \ell, \ell_1 = 1, 2, 3$), $\widehat{\boldsymbol{V}}$ is the mean velocity affected by the noise, and $\widehat{D}^{ij}$ is the coefficient of the diffusion matrix affected. Here $\lim_{\epsilon\to 0+}(1/\epsilon)E[E[\{W^k(t+\epsilon)-W^k(t)\}\{W^\ell(t+\epsilon)-W^\ell(t)\}|\mathcal{Z}_t^{t+\epsilon}]]=\delta^{k\ell}$. Equation (4) is the contribution of the $\ell_0$th order moment of the displacement, where $\ell_0\geq 3$, and is related to the $\ell_0$th order derivatives of fluid quantities. The $\sigma$-algebra $\mathcal{Z}_t^{t+\epsilon}$ is generated by the set of the sample paths $\{\boldsymbol{X}_{t,\boldsymbol{x}}(s); t\leq s<t+\epsilon\}$ and $E[\,\cdot\,|\mathcal{Z}_t^{t+\epsilon}]$ denotes the conditional expectation with respect to $\mathcal{Z}_t^{t+\epsilon}$ [9, 10], where $\epsilon$ is a real number satisfying $\epsilon>0$. Note that $\lim_{\epsilon\to 0+}E[\,\cdot\,|\mathcal{Z}_t^{t+\epsilon}]$ is a function of $\boldsymbol{x}$ [9, 10], and thus $\widehat{V}^i$ and $\widehat{D}^{ij}$ are functions of $\boldsymbol{x}$. We should give attention to the fact that the perturbed mean velocity $\widetilde{\boldsymbol{V}}$ cannot affect the diffusion coefficient in equation (3), i.e., the contribution of $\widetilde{\boldsymbol{V}}$ to the diffusion coefficient is estimated to be $\mathcal{O}(\epsilon)\to 0$ as $\epsilon\to 0+$ [11]. From equations (2) - (4), we see that the collective motion of the stochastic precess (1) is interpreted as a diffusion phenomenon because of the properties of Brownian motion $\boldsymbol{W}(s)$. If the noise $\widetilde{\mathrm{N}}$ is zero-mean, i.e., $\lim_{\epsilon\to 0+}E[E[\widetilde{N}^i_k|\mathcal{Z}_t^{t+\epsilon}]]=0$, then $\widehat{\boldsymbol{V}}(\boldsymbol{x})=\boldsymbol{V}(\boldsymbol{x})$ in equation (2) and $\widehat{D}^{ij}(\boldsymbol{x})=D^{k\ell}(\boldsymbol{x})\left\{\delta^i_k\delta^j_\ell+\lim_{\epsilon\to 0+}E[E[\widetilde{N}^i_k\widetilde{N}^j_\ell|\mathcal{Z}_t^{t+\epsilon}]]\right\}$ in equation (3).

The Coulomb collision between particles of the same species is supposed in this paper, then considering the radial thermal diffusivity affected by the RMPs is reasonable for an understanding of fundamental properties of radial transport phenomena in the perturbed region. For the sake of simplicity, we assume that 1) the noise $\widetilde{\mathrm{N}}=(\widetilde{N}^i_j)$ is zero-mean and symmetric (accordingly diagonalizable), 2) $\widehat{\boldsymbol{V}}=\boldsymbol{V}=0$, and 3) a reference surface is labeled by a minor radius $r$ of the unperturbed magnetic field configuration. From these assumptions and equation (3), the radial diffusion coefficient of the monoenergetic particles is expected to be $\widehat{D^*}=D^*\{1+\widehat{N}^2\}$, where $\widehat{N}$ is the strength of the noise in the radial directions and $D^*$ is interpreted as the radial diffusion coefficient of the monoenergetic particles in the case of $\widetilde{\mathrm{N}}=0$. It is natural that the strength of the noise is proportional to the strength of the RMPs in the radial directions. After energy integral of $\widehat{D^*}$ [3, 4], the radial thermal diffusivity at $r$ is represented as

$$\widehat{\chi}(r)=\chi(r)\left\{1+c\,\langle\|\delta B_r\|^2\rangle\right\}, \tag{5}$$

where $\chi(r)$ is the radial thermal diffusivity without the RMPs and $c$ is a positive coefficient. Here $\langle\|\delta B_r\|^2\rangle^{1/2}$ is the strength of the RMPs at $r$ in the radial directions and is defined clearly later. From equations (1) - (5), we see that the effect of the RMPs on the radial heat transport phenomenon comes down to the modification on only the radial thermal diffusivity under the presuppositions in this paper. The radial heat transport phenomenon in the perturbed region is one of the diffusion phenomena caused by the Coulomb collision.

Equation (5) is confirmed in the previous simulation study of ion heat transport in a perturbed tokamak field in the banana regime [8], but the fact remains that the coefficient $c$ is undefined. Considering an interpretation of equation (5), we speculate on the key parameters explaining the coefficient $c$ as follows.

(G1) The coefficient $c$ is expected to be related with $\omega_{\rm b}/\nu_{\rm eff}$ that is one of the candidates characterizing the time scale of the transport, where $\omega_{\rm b} \sim \sqrt{\epsilon_{\rm t}}\,\omega_{\rm t}$ is the bounce frequency in the unperturbed (i.e., original) magnetic field, $\nu_{\rm eff} \sim \nu/\epsilon_{\rm t}$ is the effective collision frequency, and $\omega_{\rm t} = v_{\rm T}/qR$ is the transit frequency. Here $v_{\rm T}$ is the thermal velocity, $\epsilon_{\rm t}$ is the inverse aspect ratio, $q$ is the safety factor, and $R$ is the major radius. It is known that the guiding center motion describing banana orbits is the characteristic motion most contributing to the collective motion in collisionless tokamaks, and that the trapped orbits are typically interrupted by the Coulomb collision in the high collisionality regime $\nu_{\rm eff}/\omega_{\rm b} \gg 1$ [1]. The strength of the RMPs is sufficiently small compared with the strength of the unperturbed magnetic field, e.g., $\langle \|\delta B_r\|^2 \rangle^{1/2}/|B_{\rm t0}| \lesssim 10^{-2}$, where $|B_{\rm t0}|$ is the strength of the unperturbed magnetic field on the magnetic axis. Accordingly, the effect of the RMPs, which is the modification on the thermal diffusivity characterized by banana motion in the collisionless limit $\nu_{\rm eff}/\omega_{\rm b} \ll 1$ (see equation (5)), is expected to be wiped out by the Coulomb collision in the high collisionality regime. Thus the coefficient $c$ in the perturbed tokamak field should be a function of $\omega_{\rm b}/\nu_{\rm eff}$ satisfying $c \to 0$ as $\omega_{\rm b}/\nu_{\rm eff} \to 0$, where $\omega_{\rm b}$ characterizes the time scale of banana motion. Note the following: there is a possibility that $\omega_{\rm t}/\nu$ instead of $\omega_{\rm b}/\nu_{\rm eff}$ is the parameter characterizing the time scale of the transport. It is checked up by means of drift kinetic simulations in section 3.

(G2) The coefficient $c$ is also expected to be related with the particle mass $m$. It is obvious that a plasma particle becomes sensitive to the field line structure as the particle mass $m$ decreases, because the cross-field drift velocity becomes close to zero if $m \to 0$ [1]. Thus, as the particle mass decreases, the strength of the noise affecting on the banana motion is expected to be enhanced. Note that $\omega_{\rm b}/\nu_{\rm eff}$ (and $\omega_{\rm t}/\nu$) is independent of the particle mass $m$ because of $\omega_{\rm b} \propto 1/\sqrt{m}$ and $\nu_{\rm eff} \propto 1/\sqrt{m}$. Consequently, the coefficient $c$ should be also a function of $1/m$ satisfying $c \to \infty$ as $m \to 0$. What is a parameter including the particle mass in this case? From analogy with the conjecture (G1), one of the candidates characterizing the space scale of the transport is the normalized width of a banana orbit $\Delta_{\rm b}\sqrt{\epsilon_{\rm t}}/qR$ (or $\Delta_{\rm b}/qR$).

(G3) Combining the conjectures mentioned above, the model formula of the radial thermal diffusivity is hypothesized to be

$$\widetilde{\chi}(r) \sim \chi(r)\left\{1 + \hat{c}\,F_1\left(\frac{qR}{\sqrt{\epsilon_{\rm t}}\,\Delta_{\rm b}}\right) F_2\left(\frac{\omega_{\rm b}}{\nu_{\rm eff}}\right) \frac{\langle \|\delta B_r\|^2 \rangle}{|B_{\rm t0}|^2}\right\} \tag{6}$$

if the pair of $\omega_{\rm b}/\nu_{\rm eff}$ and $\Delta_{\rm b}\sqrt{\epsilon_{\rm t}}/qR$ is considered, where $F_1(y)$ and $F_2(y)$ are functions of $y$ and $\hat{c}$ is a positive coefficient. Here $\langle \|\delta B_r\|^2 \rangle$ is normalized by $|B_{\rm t0}|^2$. If equation (6) is the connection formula between the thermal diffusivities given by the neoclassical theory [1] and the FLD theory [5, 6] in the collisionless limit, then it is expected by means of dimensional analysis that $F_1(qR/\sqrt{\epsilon_{\rm t}}\,\Delta_{\rm b}) = (qR/\sqrt{\epsilon_{\rm t}}\,\Delta_{\rm b})^2$, $F_2(\omega_{\rm b}/\nu_{\rm eff}) = \omega_{\rm b}/\nu_{\rm eff}$, and $\hat{c} = \pi$. The neoclassical thermal diffusivity in the collisionless limit is $\chi_r^{\rm NC} \sim \sqrt{\epsilon_{\rm t}}\,\Delta_{\rm b}^2\nu_{\rm eff}$ and the thermal diffusivity predicted by the FLD theory is $\chi_r^{\rm FLD} = \pi qR v_{\rm T}\langle \|\delta B_r\|^2 \rangle/|B_{\rm t0}|^2$.

The following section is devoted to investigate dependence of the coefficient $c$ in equation (5) on parameters of the toroidal plasma by means of a $\delta f$ simulation code KEATS [8, 12]. The conjectures (G1) - (G3) are also examined in results of the $\delta f$ simulations. The simulations are independent of the derivation described by equations (1) - (6) because the radial thermal diffusivity in the simulations is evaluated from the radial heat flux given by a distribution function of guiding centers, not from the displacements of guiding centers, where the distribution function is a solution of the drift kinetic equation. Thus the simulations are suitable to verify equation (6).

## 3. Model formula of thermal diffusivity derived from simulation results

### *3.1. Method of estimating $\chi_r$ and simulation conditions*

In this section, using the drift kinetic simulation code for calculating the radial thermal diffusivity of ion (proton) in a tokamak plasma affected by RMPs, we investigate dependence of the coefficient $c$ in equation (5) on parameters of the toroidal plasma. The simulation conditions are as simplified as possible for the sake of visible prospect. In the simulations, we neglect MHD activities, neutrals, and impurities. In general, effect of an electric field $\boldsymbol{E}$ is important for an understanding of radial transport phenomena in the toroidal plasma. However, in this paper, to focus on the fundamental properties of the radial heat transport affected by the RMPs, the effect of $\boldsymbol{E}$ is also neglected.

The radial heat flux given by a guiding center distribution function is evaluated by the $\delta f$ simulation code KEATS that is a Monte Carlo simulation code based on the drift kinetic equation [8, 12]. The guiding center distribution function $f = f(t, \boldsymbol{x}, \boldsymbol{v}) = f_{\rm M} + \delta f$ evolves with time from the Maxwell distribution $f_{\rm M}$ under effects of the Coulomb collision and the RMPs, where $\delta f = 0$ at $t = 0$. The Coulomb collision in the simulations is given, for the sake of simplicity, by the pitch-angle scattering operator for the collisions with the Maxwell background $f_{\rm M}$, where the operator satisfies the local momentum conservation property and the quadratic collision term $C(\delta f, \delta f)$ is neglected. The Maxwell distribution $f_{\rm M}$ is assumed to be given as a function of $r$ and $v$, i.e., $f_{\rm M} = f_{\rm M}(r, v)$, where $r$ is the label of the original flux surfaces, $v = |\boldsymbol{v}|$ is the speed of a particle, and the zero mean velocity is assumed (i.e., $\boldsymbol{V}_{\rm i} = \boldsymbol{V}_{\rm e} = \boldsymbol{V} = 0$). The subscript “ i ” or “ e ” means a particle species; in the notation of $\mathcal{X}_\alpha$, where $\mathcal{X}_\alpha$ is a physical quantity of the species “ $\alpha$ ”, the subscript $\alpha = $ i means ion and $\alpha = $ e means electron. Hereafter, the radial thermal diffusivities in the toroidal magnetic field with and without RMPs are notated by $\chi_r$ and $\chi_r^{(0)}$, respectively.

The toroidal magnetic configuration used in the simulations is formed by the addition of an RMP field to a simple tokamak field having concentric circular flux surfaces. The perturbed region is bounded radially on both sides by the regular closed magnetic surfaces, for example, as shown in figure 1. The major radius of the magnetic axis is set to $R_{\rm ax} = 3.6$ m, the minor radius of the toroidal plasma is $a = 1$ m, and the strength of the magnetic field on the magnetic axis is $B_{\rm ax} = |B_{\rm t0}| = 4$ T. Note that in subsection 3.3 we change the value of $|B_{\rm t0}|$ for investigating the dependence of $c$ on the banana width. The temperature profile is fixed as $T_\mathrm{i} = T_\mathrm{e} = T(r) = T_\mathrm{ax} - (T_\mathrm{ax} - T_\mathrm{edge})(r/a)$ with $T_\mathrm{ax} = 1.137$ keV and $T_\mathrm{edge} = 0.8T_\mathrm{ax}$, and the density $n_\mathrm{i} = n_\mathrm{e} = n$ is assumed to be a constant, where $r = \sqrt{(R - R_\mathrm{ax})^2 + Z^2}$ is the label of the reference surfaces in the cylindrical coordinates $(R, \varphi, Z)$. The original flux surfaces of the circular tokamak field are used as the reference surfaces in the coordinate space. The unperturbed magnetic field $\boldsymbol{B}_0 = B_{0R}\widehat{R} + B_{0\varphi}\widehat{\varphi} + B_{0Z}\widehat{Z}$ is given by $B_{0R} = -B_\mathrm{ax}Z/qR$, $B_{0\varphi} = -B_\mathrm{ax}R_\mathrm{ax}/R$, and $B_{0Z} = B_\mathrm{ax}(R - R_\mathrm{ax})/qR$ [13], where $\widehat{R}$, $\widehat{\varphi}$ and $\widehat{Z}$ are the unit vectors in the $R$, $\varphi$ and $Z$ directions, respectively, and $q$ is the safety factor given by $q^{-1} = 0.9 - 0.5875(r/a)^2$. The RMPs causing resonance with rational surfaces of $q = k/\ell = 3/2, 10/7, 11/7, \ldots$ are given by a perturbation field $\delta\boldsymbol{B} = \nabla \times (\alpha\boldsymbol{B}_0)$. Here the function $\alpha$ is used to represent the structure of the perturbed magnetic field; $\alpha(R, \varphi, Z) = \sum_{k,\ell} \alpha_{k\ell} = \sum_{k,\ell} a_{k\ell}(r(R, Z)) \cos\{k\theta(R, Z) - \ell\varphi + \varphi_{k\ell}\}$, where $a_{k\ell} = c_{k\ell} \exp\{-(r - r_{k\ell})^2/\Delta r^2\}$ having constants $\{c_{k\ell}\}$, $r = r_{k\ell}$ is the rational surface of $q = k/\ell$, $\Delta r$ is a small parameter controlling the width of the perturbation, and $\varphi_{k\ell}$ is a phase. The square of the strength of the RMPs in the radial directions, $\langle\|\delta B_r\|^2\rangle$, is given by the averaged value of $\|\delta B_r\|^2 = \sum_{k,\ell} |\delta B_r^{(k/\ell)}|^2$ on a reference surface labeled by $r$, where $\delta B_r^{(k/\ell)} = \nabla r \cdot \nabla \times (\alpha_{k\ell}\boldsymbol{B}_0)$. The total magnetic field is $\boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B}$, where the condition $|\boldsymbol{B}_0| \gg |\delta\boldsymbol{B}|$ is assumed. The ratio of the ion gyroradius $\rho_\mathrm{i}$ to the width of the perturbed region $\Delta_\mathrm{RMP}$ is set to $\rho_\mathrm{i}/\Delta_\mathrm{RMP} \lesssim 1/200$ in the simulations in this paper, and thus the drift kinetic equation solver KEATS is applicable to the simulation analysis of the radial heat transport phenomenon, where $\Delta_\mathrm{RMP} \sim |\partial \ln \langle\|\delta B_r\|^2\rangle^{1/2}/\partial r|^{-1}$.

The Poincaré plot of the magnetic field lines on a poloidal cross section in the case of $(c_{k\ell}/a, \Delta r/a, \varphi_{k\ell}) = (6 \times 10^{-3}, 5 \times 10^{-2}, 0)$ is shown in figure 1, and the strength of the RMPs in the radial directions is set to $\langle\|\delta B_r\|^2\rangle^{1/2}/|B_\mathrm{t0}| \sim 1/100$ at the center of the ergodic region, where the perturbed region affected by the RMPs is ergodized in this case. Hereafter $|\delta B_r^{(0)}|$ is defined by the value of $\langle\|\delta B_r\|^2\rangle^{1/2}$ in the case of figure 1. Specific details of the perturbed magnetic field and the particle motion itself are shown in the previous studies [7, 14]. Comparisons between the results of the $\delta f$ simulation and the FLD theory are discussed in detail in [8].

Under the conditions of $n$ = constant, $\boldsymbol{V} = 0$ and $\boldsymbol{E} = 0$, the radial thermal diffusivity of ion is estimated by

$$\chi_r = -\frac{Q_{\mathrm{i}\,r}(r)}{n_\mathrm{i}(r)\partial T_\mathrm{i}(r)/\partial r}, \tag{7}$$

where $Q_{\mathrm{i}\,r}(r)$ is the radial energy flux of ion evaluated by the $\delta f$ simulation code and is interpreted as the radial heat flux under the above conditions. In the simulations, the radial energy flux is evaluated on a reference surface labeled by $r$, and is given by

$$Q_{\mathrm{i}\,r}(r) = \left\langle \nabla r \cdot \overline{\int \mathrm{d}^3 v \, \frac{m_\mathrm{i} v^2 \boldsymbol{v}}{2} \delta f} \right\rangle. \tag{8}$$

Here $\overline{\cdot}$ means the time-average and the averaging time is longer than the typical time scale of $\delta f$ (both the orbit and collision times). The time-averaging is carried out after sufficient exposure to the collisions. The average $\langle \cdot \rangle$ is defined as $\langle \cdot \rangle = (1/\delta\mathcal{V}) \int_{\delta\mathcal{V}} \cdot \, \mathrm{d}^3 x$, where $\delta\mathcal{V}$ is a small volume and lies between two neighboring reference surfaces with volumes $\mathcal{V}(r)$ and $\mathcal{V}(r) + \delta\mathcal{V}$.

We recall the main result in the previous study on the ion heat transport [8]; the radial thermal diffusivity $\chi_r$ depends on $\langle \|\delta B_r\|^2 \rangle$, not on the number of modes of RMPs, nor on details of the magnetic field-line structure. From the simulation results shown in figure 2, we see that the radial thermal diffusivity is represented as $\chi_r = \chi_r^{(0)}\{1 + c\,\langle \|\delta B_r\|^2 \rangle\}$, which is the same form as equation (5), and that this tendency does not change if the collision frequency $\nu$ is changed, where the collision frequency $\nu$ is proportional to the density $n$ and satisfies $\nu/\epsilon_{\mathrm{t}}\omega_{\mathrm{b}} = \nu_{\mathrm{eff}}/\omega_{\mathrm{b}} \lesssim 10^{-1}$ in figure 2. The radial thermal diffusivity for the limit of $\langle \|\delta B_r\|^2 \rangle^{1/2} = 0$ agrees with that given by the neoclassical theory in the banana regime $\nu_{\mathrm{eff}}/\omega_{\mathrm{b}} < 1$, i.e., $\chi_r = \chi_r^{(0)} \approx \chi_{\mathrm{i}\,r}^{\mathrm{NC}} = 1.35\epsilon_{\mathrm{t}}{}^{1/2} T_{\mathrm{i}}/(m_{\mathrm{i}}\Omega_{\mathrm{i}\,\theta}^2 \tau_{\mathrm{ii}}) \sim \sqrt{\epsilon_{\mathrm{t}}}\rho_{\mathrm{i}\,\theta}^2 \nu_{\mathrm{ii}}$ [1], where $\chi_{\mathrm{i}\,r}^{\mathrm{NC}}$ is the neoclassical thermal diffusivity of ion, $\Omega_{\mathrm{i}\,\theta}$ is the ion poloidal gyrofrequency, $\tau_{\mathrm{ii}} = \nu_{\mathrm{ii}}{}^{-1}$ is the ion-ion collision time, and $\rho_{\mathrm{i}\,\theta}$ is the ion poloidal gyroradius.

### *3.2. Dependence of $\chi_r$ on collision frequency*

As shown in figure 2, the thermal diffusivity $\chi_r$ also depends on the collision frequency $\nu$. The dependence of $\chi_r$ on the collision frequency in each case of $\langle \|\delta B_r\|^2 \rangle / |\delta B_r^{(0)}|^2 = 0$ and 1 is shown in figure 3. The thermal diffusivity in the case of $\langle \|\delta B_r\|^2 \rangle / |\delta B_r^{(0)}|^2 = 0$ is explained by the neoclassical theory [8]. In figure 3, we see that the difference between the thermal diffusivities with and without the RMPs is negligibly small in the plateau regime $\nu/\epsilon_{\mathrm{t}}\omega_{\mathrm{b}} = \nu_{\mathrm{eff}}/\omega_{\mathrm{b}} > 1$, and that one of the key parameters explaining the coefficient $c$ is $\epsilon_{\mathrm{t}}\omega_{\mathrm{b}}/\nu = \omega_{\mathrm{b}}/\nu_{\mathrm{eff}}$, rather than $\omega_{\mathrm{t}}/\nu$. Note that the thermal diffusivities in the plateau regime in both the cases of $\langle \|\delta B_r\|^2 \rangle / |\delta B_r^{(0)}|^2 = 0$ and 1 are close to the neoclassical thermal diffusivity in the plateau regime, where the neoclassical thermal diffusivity is given as $\chi_{\mathrm{i}\,r}^{\mathrm{NC}} = \omega_{\mathrm{t\,i}}\rho_{\mathrm{i}}^2\,(3\sqrt{\pi}q^2/4) \approx 0.3$ $\mathrm{m}^2/\mathrm{s}$ in the plateau regime [1]. It is inferred from the results of $\{\chi_r/\chi_r^{(0)}\} - 1$ that the dependence of $c$ on the collision frequency $\nu$ is $c \propto 1/\nu$, as shown in figure 4. Then, the radial thermal diffusivity in the perturbed magnetic field is represented as

$$\chi_r = \chi_r^{(0)} \left\{ 1 + c_1 \left( \frac{\omega_{\mathrm{b}}}{\nu_{\mathrm{eff}}} \right) \langle \|\delta B_r\|^2 \rangle \right\}, \tag{9}$$

where $c_1$ is a positive coefficient: $c = c_1\omega_{\mathrm{b}}/\nu_{\mathrm{eff}}$.

### *3.3. Dependence of $\chi_r$ on banana width*

In the conjecture (G2) in section 2, one of the candidates characterizing the space scale of the transport is the width of a banana orbit $\Delta_{\mathrm{b}}$, where the banana width is given by $\Delta_{\mathrm{b}} \sim \rho_\theta\sqrt{\epsilon_{\mathrm{t}}} \propto \sqrt{m}/|B_{\mathrm{t0}}|$. In order to investigate dependence of the coefficient $c$ on the particle mass $m$, we consider artificial test particles (artificial ions) having the mass of $m = m_{\mathrm{test}} = m_{\mathrm{p}}/10, m_{\mathrm{p}}/100, m_{\mathrm{p}}/1000$ in the simulations, and estimate the radial thermal diffusivities, where $m_{\mathrm{p}}$ is the mass of a proton and the charge number of every artificial ion is set to $Z_{\mathrm{test}} = 1$. From the results of $\{\chi_r/\chi_r^{(0)}\} - 1$ under the condition of $\langle \|\delta B_r\|^2 \rangle / |\delta B_r^{(0)}|^2 = 1$, it is inferred that the dependence of $c$ on the particle mass $m$ is $c \propto 1/m$, as shown in figure 5. Note that the parameter $\omega_{\mathrm{b}}/\nu_{\mathrm{eff}}$ is independent of $m$.

Dependence of the coefficient $c$ on $|B_{\mathrm{t0}}|$ is also investigated, and it is inferred that $c \propto |B_{\mathrm{t0}}|^2$, as shown in figure 6. Here, in all the cases in figure 6, we change only the values of $\langle||\delta B_r||^2\rangle^{1/2}$ and $|B_{\mathrm{t0}}|$ under the condition that the ratio $\langle||\delta B_r||^2\rangle^{1/2}/|B_{\mathrm{t0}}|$ is fixed. From the results in figures 5 and 6, we see that $(y/z) \propto |B_{\mathrm{t0}}|^2/m$, where $y = \{\chi_r/\chi_r^{(0)}\} - 1$ and $z = (\omega_{\mathrm{b}}/\nu_{\mathrm{eff}})\langle||\delta B_r||^2\rangle/|B_{\mathrm{t0}}|^2$.

Therefore, combining equation (9) with the above results, we have a model formula of the radial thermal diffusivity in the perturbed region:

$$\chi_r = \chi_r^{(0)} \left\{ 1 + c_0 \left( \frac{\omega_{\mathrm{b}} |B_{\mathrm{t0}}|^2}{\nu_{\mathrm{eff}}\, m} \right) \frac{\langle||\delta B_r||^2\rangle}{|B_{\mathrm{t0}}|^2} \right\}, \tag{10}$$

where $c_0$ is a positive coefficient and is independent of $\langle||\delta B_r||^2\rangle^{1/2}$, $\nu$, $m$ and $|B_{\mathrm{t0}}|$. Note that the result in figure 6 is also interpreted as the dependence of $\chi_r$ on $\langle||\delta B_r||^2\rangle$ because the ratio $\langle||\delta B_r||^2\rangle^{1/2}/|B_{\mathrm{t0}}|$ is fixed.

## 4. Summary and discussions

In order to investigate the radial thermal diffusivity of a low-collisional tokamak plasma having a perturbed region generated on and around the resonance surfaces, we apply the drift kinetic equation solver KEATS to the ion heat transport phenomenon in the tokamak field disturbed partly by the resonant magnetic perturbations (RMPs). The simulation conditions are as simplified as possible for the sake of visible prospect. 1) The perturbed region is wedged in between the regular closed magnetic surfaces. Thus, in the region, there is no magnetic field line connected to the divertor. 2) The Coulomb collision is assumed to be represented as the collisions between plasma particles of the same species. 3) Electric field, MHD activities, neutrals, and impurities are neglected. Under these conditions, we evaluate the radial thermal diffusivity of ion from the radial heat flux given by the drift kinetic simulations, and find that the radial thermal diffusivity is represented as

$$\chi_r = \chi_r^{(0)} \left\{ 1 + \tilde{c}_0 \left( \frac{q R_{\mathrm{ax}}}{\sqrt{\epsilon_{\mathrm{t}}}\, \Delta_{\mathrm{b}}} \right)^2 \left( \frac{\omega_{\mathrm{b}}}{\nu_{\mathrm{eff}}} \right) \frac{\langle||\delta B_r||^2\rangle}{|B_{\mathrm{t0}}|^2} \right\}. \tag{11}$$

Here $\chi_r^{(0)}$ is the neoclassical thermal diffusivity, $q$ is the safety factor, $R_{\mathrm{ax}}$ is the major radius of the magnetic axis, $\epsilon_{\mathrm{t}}$ is the inverse aspect ratio, $\Delta_{\mathrm{b}} \sim \rho_\theta \sqrt{\epsilon_{\mathrm{t}}}$ is the banana width, $\rho_\theta$ is the poloidal gyroradius, $\omega_{\mathrm{b}}$ is the bounce frequency, $\nu_{\mathrm{eff}} \sim \nu/\epsilon_{\mathrm{t}}$ is the effective collision frequency, $\nu$ is the collision frequency, $\langle||\delta B_r||^2\rangle^{1/2}$ is the strength of the RMPs in the radial directions, $|B_{\mathrm{t0}}|$ is the strength of the magnetic field on the magnetic axis, $\tilde{c}_0 = \{\Delta_{\mathrm{b}}^2 |B_{\mathrm{t0}}|^2/q^2 R_{\mathrm{ax}}^2 m\} c_0$ is the coefficient related to $c_0$, $m$ is the particle mass, and $c_0$ is a positive coefficient which is independent of $\langle||\delta B_r||^2\rangle^{1/2}$, $\nu$, $m$ and $|B_{\mathrm{t0}}|$. Note that the model formula (11) is derived from only the results of the drift kinetic simulations.

By the simulation result (11), the conjectures (G1) - (G3) in section 3 are almost confirmed. The coefficient $c$ in equation (5) is given by $c = (\omega_{\mathrm{b}}/\nu_{\mathrm{eff}}\, m)\, c_0$. When the collision frequency is in the collisionless limit, the thermal diffusivity in the case of $\langle||\delta B_r||^2\rangle = 0$ satisfies $\chi_r = \chi_r^{(0)} \approx \chi_r^{\mathrm{NC}} \sim \sqrt{\epsilon_{\mathrm{t}}}\, \Delta_{\mathrm{b}}^2 \nu_{\mathrm{eff}}$. A value of the coefficient $\tilde{c}_0$ is expected to be $\tilde{c}_0 \sim \pi$ if the model formula (11) is connected to that predicted by the FLD theory in the collisionless limit. The coefficient $\tilde{c}_0$ evaluated by the simulations is, however, as small as satisfying $0 < \tilde{c}_0 \ll \pi$, i.e., $\tilde{c}_0 \sim 10^{-4}$ under the simulation conditions of this paper. This result is presumed to be caused from that the condition $\Delta_{\rm RMP}/L \ll 1$ is satisfied in the simulations in the cases of the perturbed magnetic field having the ergodic region, as discussed in [8], where $\Delta_{\rm RMP}$ is the width of the perturbed region ($\Delta_{\rm RMP}/a \lesssim 0.3$) and $L$ is the space scale length characterizing the plasma confinement, e.g., $L/a \sim v_{\rm T}\langle\|\delta B_r\|^2\rangle^{1/2}/\nu|B_{\rm t0}|a \gtrsim 1$. The fact remains that the coefficient $c_0$ (or $\tilde{c}_0$) is undefined in the present study, and dependence of the coefficient $c_0$ on parameters of the toroidal plasma will be a topic in the future study.

Finally, we discuss transport properties of electron in the perturbed region. The profile of the radial thermal diffusivity in and around the ergodic region is shown in figure 7, where the electron thermal diffusivity is estimated by the simulations with both the electron-electron and electron-ion collisions. The thermal diffusivity of electron is larger than that in the case of ion only in the perturbed region. As also shown in figure 7, the radial thermal diffusivity of electron estimated by the simulations is extremely small compared with that predicted by the FLD theory, which is the same result as that in the case of ion [8]. Dependence of the electron thermal diffusivity on the strength of the RMPs is shown in figure 8. The radial thermal diffusivity of electron $\chi_{{\rm e}\,r}$ depends on $\langle\|\delta B_r\|^2\rangle$. We see that the electron thermal diffusivity is quite sensitive to the strength of the RMPs as compared with the ion thermal diffusivity, and that the radial thermal diffusivity of electron is also represented as $\chi_{{\rm e}\,r} = \chi_{{\rm e}\,r}^{(0)}\{1 + c\,\langle\|\delta B_r\|^2\rangle\}$, which is the same form as equation (5).

## Acknowledgments

The authors would like to thank Professor Hideo Sugama for his useful suggestions in the preparation of this work. This work was performed with the support and under the auspices of the NIFS Collaborative Research Program NIFS11KNST019.

## References

[1] Helander P and Sigmar D J 2002 *Collisional Transport in Magnetized Plasmas* (Cambridge: Cambridge University Press)
[2] Maluckov A *et al* 2001 *Plasma Phys. Control. Fusion* **43** 1211
[3] Kanno R *et al* 1997 *Nucl. Fusion* **37** 1463
[4] Sugama H and Nishimura S 2002 *Phys. Plasmas* **9** 4637
[5] Rechester A B and Rosenbluth M N 1978 *Phys. Rev. Lett.* **40** 38
[6] Ghendrih Ph *et al* 1996 *Plasma Phys. Control. Fusion* **38** 1653
[7] Maluckov A *et al* 2003 *Physica A* **322** 13
[8] Kanno R *et al* 2010 *Plasma Phys. Control. Fusion* **52** 115004
[9] Øksendal B 2003 *Stochastic Differential Equations* (Berlin: Springer)
[10] Doob J L 1953 *Stochastic processes* (New York: Wiley)
[11] The same result is also found in the equation of fluid motion; see Kanno R *et al* 2006 *Plasma Fusion Res.* **1** 012
[12] Nunami M *et al* 2007 *Research Report NIFS Series* No. NIFS-871
[13] Wesson J 2011 *Tokamaks* 4th edn (New York: Oxford University Press)
[14] Maluckov A *et al* 2003 *Contrib. Plasma Phys.* **43** 198

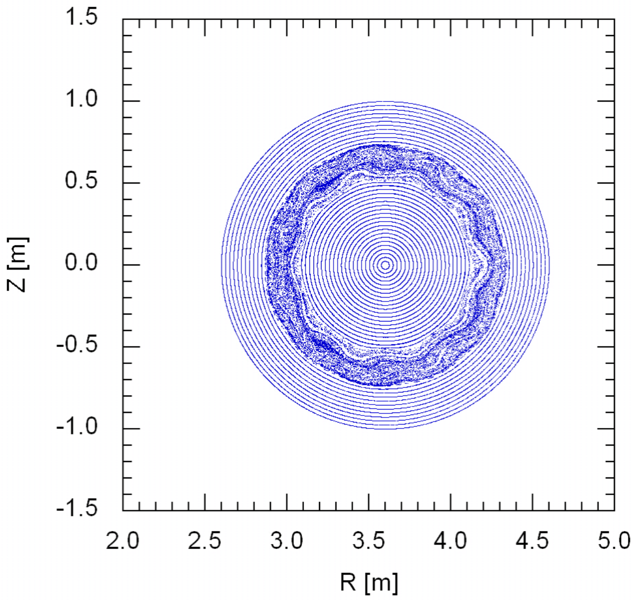


**Figure 1.** Poincaré plot of the magnetic field lines on a poloidal cross section in the coordinate space, where the strength of the RMPs is set to $\langle ||\delta B_r||^2 \rangle / |\delta B_r^{(0)}|^2 = 1$ and the RMPs cause resonance with the rational surfaces of $q = k/\ell = 3/2, 10/7, 11/7$. The ergodic region bounded radially on both sides by the regular closed magnetic surfaces is generated between $r/a \approx 0.5$ and 0.75, where $r = \sqrt{(R - R_{\rm ax})^2 + Z^2}$, $R_{\rm ax} = 3.6$ m and $a = 1$ m.

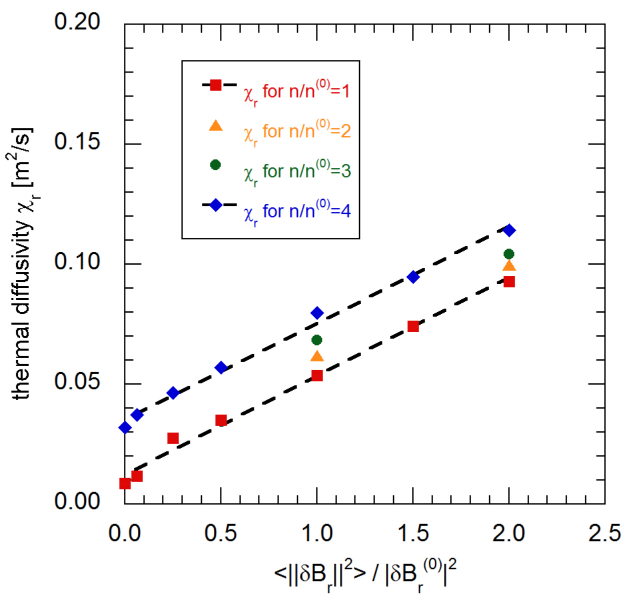


**Figure 2.** The radial thermal diffusivity of ion (proton) at the center of the perturbed region depends on both the strength of the RMPs $\langle ||\delta B_r||^2 \rangle / |\delta B_r^{(0)}|^2$ and the density $n$; $\chi_{\rm ir}$ in each case of (i) $n = n^{(0)} = \text{constant} = 1 \times 10^{19}\ \text{m}^{-3}$ (red squares), (ii) $n = 2n^{(0)}$ (yellow triangles), (iii) $n = 3n^{(0)}$ (green circles) and (iv) $n = 4n^{(0)}$ (blue rhombuses). The regression lines are illustrated as the black dashed lines.

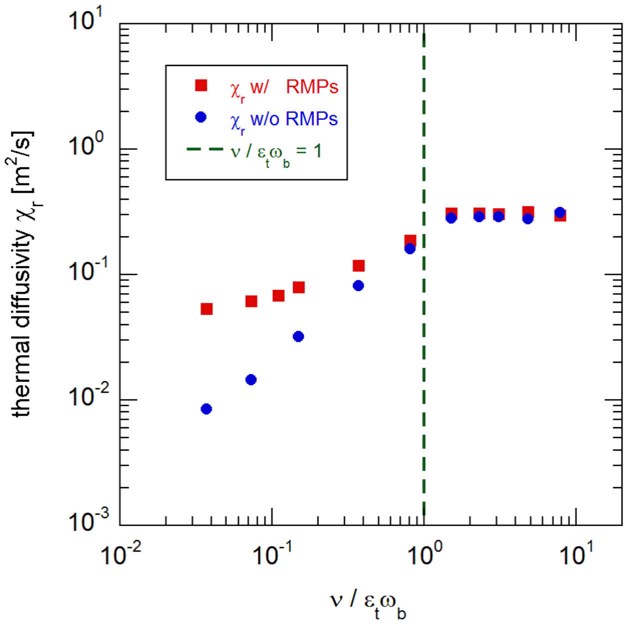


**Figure 3.** The radial thermal diffusivity of ion (proton) at the center of the perturbed region in each case of $\langle ||\delta B_r||^2 \rangle / |\delta B_r^{(0)}|^2 = 1$ (red squares) and 0 (blue circles) depends on the collision frequency $\nu$. The line of $\nu/\epsilon_t\omega_b = 1$ is illustrated by the green dashed line, where $\epsilon_t$ is the inverse aspect ratio and $\omega_b$ is the ion bounce frequency.

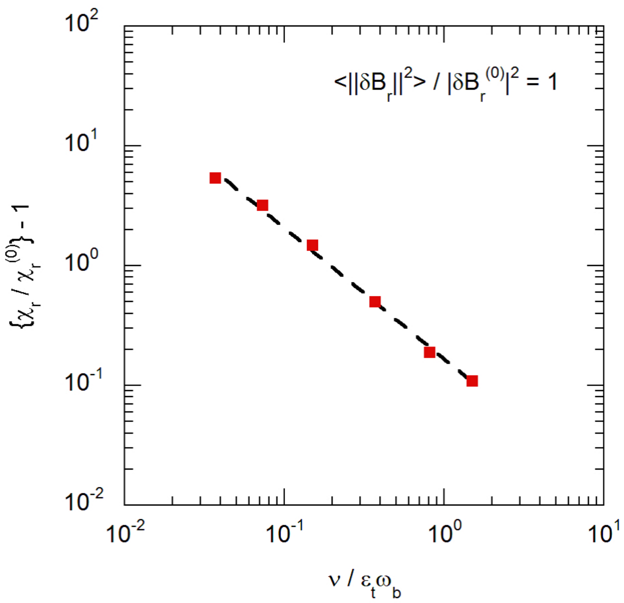


**Figure 4.** The radial thermal diffusivity of ion (proton) at the center of the ergodic region depends on the collision frequency $\nu$, where the strength of the RMPs is fixed as $\langle ||\delta B_r||^2 \rangle / |\delta B_r^{(0)}|^2 = 1$. The regression line is illustrated as the black dashed line. The results in this figure are given from the data in figure 3.

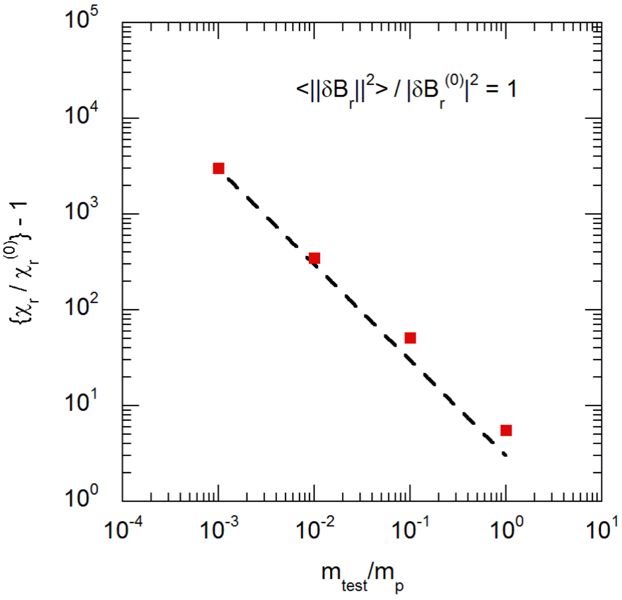


**Figure 5.** The radial thermal diffusivities of artificial ions at the center of the ergodic region depends on the particle mass $m_{\mathrm{test}}$, where the strength of the RMPs is fixed as $\langle||\delta B_r||^2\rangle/|\delta B_r^{(0)}|^2 = 1$, the particle mass is assumed to be $m_{\mathrm{test}}/m_{\mathrm{p}} = 1, 1/10, 1/100, 1/1000$, and $m_{\mathrm{p}}$ is the mass of a proton. The regression line is illustrated as the black dashed line.

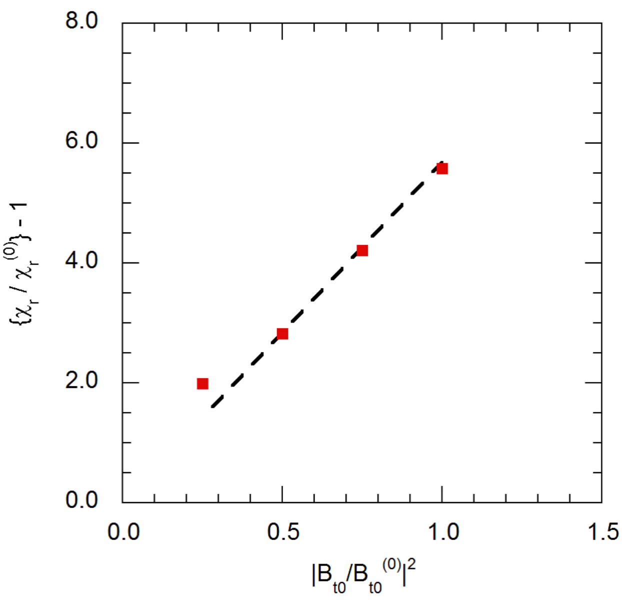


**Figure 6.** The radial thermal diffusivity of ion (proton) at the center of the perturbed region depends on the strength of the magnetic field on the magnetic axis $|B_{\mathrm{t0}}| = B_{\mathrm{ax}}$, where $|B_{\mathrm{t0}}{}^{(0)}| = 4$ T is the strength of the magnetic field on the axis in figure 1. The ratio $\langle||\delta B_r||^2\rangle^{1/2}/|B_{\mathrm{t0}}|$ is fixed in all the cases, where $\langle||\delta B_r||^2\rangle/|\delta B_r^{(0)}|^2 = 1$ at $|B_{\mathrm{t0}}/B_{\mathrm{t0}}{}^{(0)}|^2 = 1$. The regression line passing $(x, y) = (0, 0)$ is illustrated as the black dashed line, where $x = |B_{\mathrm{t0}}/B_{\mathrm{t0}}{}^{(0)}|^2$ and $y = \{\chi_r/\chi_r^{(0)}\} - 1$.

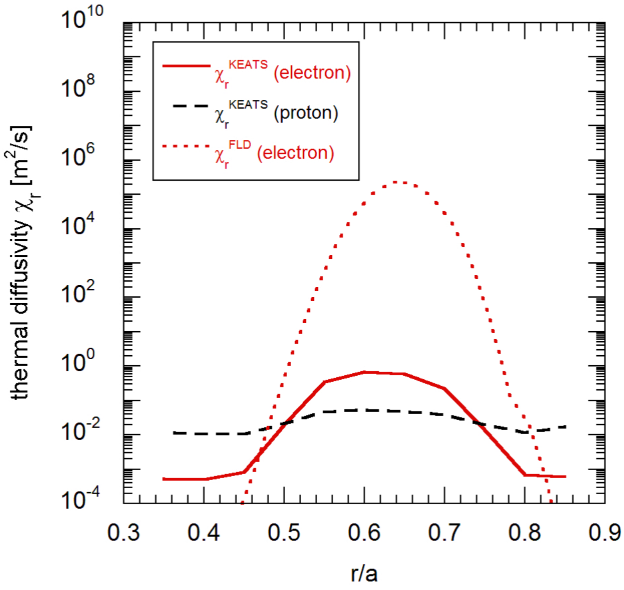


**Figure 7.** Profiles of the radial thermal diffusivities in and around the ergodic region in figure 1, which are given by (i) the simulation in the case of electron (red solid line), (ii) the simulation in the case of ion (black dashed line), and (iii) the theory of field-line diffusion in the case of electron (red dotted line). The ergodic region is generated between $r/a \approx 0.5$ and 0.75.

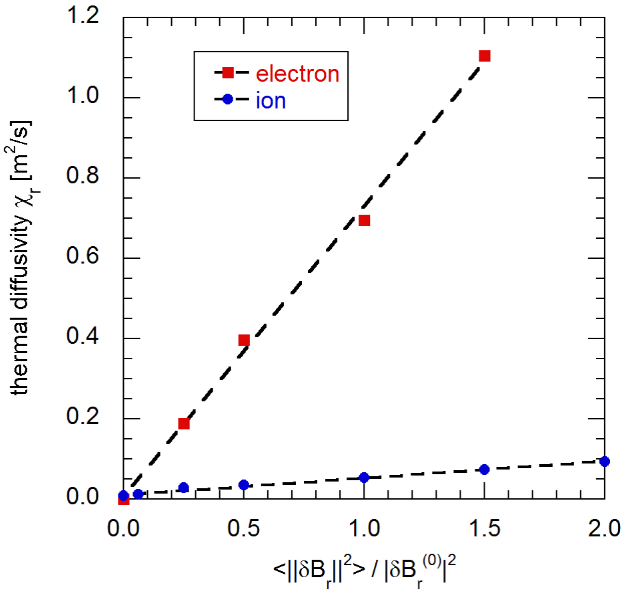


**Figure 8.** The radial thermal diffusivities of electron (red squares) and ion (proton; blue circles) at the center of the perturbed region. The regression lines are illustrated as the black dashed lines.