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Sergey V. Bazdenkov and T. Sato

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On a ballistic method for double layer  
regeneration in a Vlasov-Poisson plasma

Sergey V. Bazdenkov \* and Tetsuya Sato

National Institute for Fusion Science, Nagoya 464-01, Japan.

**Abstract**

Double layers generated by a current-driven instability are usually alive short time and destroyed by emitting solitons and other nonlinear waves. It is analytically shown that they can be regenerated in one-dimensional Vlasov-Poisson plasma system where the upstream velocity distribution is swiftly heated to restore an unstable condition in the downstream region due to ballistic deformation.

Key-words: double layer, double layer regeneration, ballistic effect, bump-on-tail distribution, nonlinear plasma response.

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\*Permanent address: Russian Scientific Center, Kurchatov Institute, Moscow, Russia.

## 1 Introduction.

A current-driven double layer (CDDL) in a Vlasov-Poisson plasma system is a well-known example of nonlinear wave-particle interactions. The CDDL has been studied extensively during the last two decades (see for example the review<sup>1</sup> and the references therein). Consequently, the physics of CDDL formation is now well understood. The CDDL can be considered as a BGK-like structure<sup>2</sup> that occurs in a nonlinear stage of a plasma instability associated with a shifted Maxwellian distribution. Once having formed, the CDDL then disappears as a result of ion-acoustic wave and soliton irradiation. At this moment the electron velocity distribution takes on a stable form, i.e., a monotonically decreasing function with a plateau formed by an intensive wave-particle interaction. Thus no “new” CDDLs can arise after the velocity redistribution caused by the primary CDDL formation in a closed system, i.e. in the system with periodic or any “passive” boundary conditions<sup>3-8</sup>. One can understand it as a manifestation of the LeChatelier-Braun principle when an initially unstable plasma approaches to a thermodynamical equilibrium with stable (and small) fluctuations only. The usual scenario of the CDDL formation and disappearance can be ascribed to a simple tendency of reducing the cause of instability, i.e., changing the velocity distribution to a stable one. In the process of particle redistribution, of course, any temporal self-organized state such as the CDDL can be realized. The magnitudes and the total number of CDDLs in a bounded plasma depend on the peculiarities of the initial condition as well as on the system length<sup>5,6</sup>. But in the closed system the self-organized state is related only with the unstable initial condition and can not be reproduced after the velocity redistribution.

In an open system with an “active” boundary condition free energy can be pumped into the plasma through the boundaries. Thus CDDLs can be reproduced periodically in the open system (relaxation type of oscillations). Note, however, that it does not mean that any certain CDDL can be maintained during long enough (infinitely large) period of time. The role of the “active” boundary condition is only to reproduce an unstable plasma condition, i.e. to shift the system away from the thermodynamic equilibrium. Then regeneration of CDDLs can become possible. One direct way to make the plasma unstable is to inject an electron beam, hence a current, into the plasma through the boundary. The incoming part of the velocity distribution function at the plasma boundary looks like a shifted Maxwellian, thus it becomes unstable just at the vicinity of the boundary.

Being shifted away from the thermodynamic equilibrium, the system exhibits creation of electrostatic electron shocks and holes as well as CDDLs and ion bursts in the nonlinear stage of the beam-plasma interaction<sup>9-14</sup>. Several types of double layers are considered to take place depending on conditions, namely, monotonic double layers, double layers with a dip on the low potential side, and double layers with a bump on the high potential side. As the nonlinear potential structures disappear, the same kind of plasma instability may reoccur which would eventually lead to reformation of the CDDLs<sup>11,12</sup>. Thus the CDDLs formed through the plasma-beam instability may not be stationary and may be periodically destroyed and reformed. Of course, the electron current in the plasma has to be high enough and the electron beam has to be cold enough in order to produce a strong instability. Injection of an unstable electron beam into the plasma is, however, a rather crude method of free energy pumping.

In the present paper, another, more soft method of free energy pumping is proposed. No external sources for plasma instability are assumed inside the plasma body. Moreover, no unstable boundary conditions, i.e., unstable velocity distribution just at the boundary, are assumed. The property of the unstable plasma, hence, the possibility to form a nonlinear structure inside the plasma, comes to light even in the case where both the initial and boundary conditions look like stable ones. The ballistic deformation of the velocity distribution (see, e.g., Ref 13) can lead to the occurrence of an unstable bump-on-tail distribution in the deep interior of the plasma. The region where the plasma is destabilized by a properly chosen time history of the boundary condition is separated from the boundary region where exists a transit stable plasma with small electric field fluctuations. The source of the free energy in this case is a plasma inhomogeneity caused by the non-stationary boundary condition.

This paper is organized as follows. In Sec. 2 a qualitative analysis of the ballistic deformation of the velocity distribution in the absence of the electric field is presented. The restrictions on the boundary condition are also discussed here. In Sec. 3 effects of the electric field are taken into account self-consistently and the corresponding integro-differential equation is obtained. The expression for a nonlinear plasma response is found in terms of the electric field amplitude series. In Sec. 4 a simplified nonlinear equation for the electric field is obtained for the case of “jumping” boundary condition. The corresponding WKB-like solution is found which demonstrates the growth of electric field oscillations caused by a rapid change in the boundary condition. The conclusions are

summarized in Sec.5.

## 2 Ballistic recreation of unstable velocity distribution.

In an inhomogeneous plasma ( $\nabla f \neq 0$ ) the motion of particles with different velocities leads to deformation of the velocity distribution by the “convective” term,  $\mathbf{v} \cdot \nabla f$ , in the kinetic equation. In some cases a two-stream-like distribution can be formed and persist during a finite interval of time even if the initial distribution is monotonic.

Let us consider a one-dimensional plasma with a monotonically decreasing electron distribution,  $\partial f_e / \partial v < 0$ . We assume that the ions are cold, namely,

$$T_i = \int_{-\infty}^{+\infty} \frac{1}{2} M v^2 d\mathbf{w} \ll T_e = \int_{-\infty}^{+\infty} \frac{1}{2} m v^2 d\mathbf{w}$$

(all notations here and below are conventional). Here  $T_j$  is not a temperature but rather an averaged energy of  $j$ -th component because the distribution is not necessarily Maxwellian. It is well known that a plasma with monotonically decreasing velocity distribution is stable even if a non-zero total plasma current exists (see, e.g., Ref. 15). Thus we are able to assume that only electric field fluctuations with small amplitudes,  $|e\phi| < T_e$ , exist in a nearly thermodynamically equilibrium plasma. Of course the existence of a non-zero current means that the plasma is not really at the thermodynamic equilibrium but only approaches to this equilibrium through the current relaxation. Nevertheless, the process of current relaxation can be considered as a slow enough process on the ion-acoustic time-scale, so that the assumption,  $|e\phi| < T_e$ , is reasonable. In this case, as far as the plasma is stable, only a weak wave-particle interaction in the form of quasilinear (QL) diffusion can be expected to take place predominantly for the thermal electrons. Electrons with higher velocities move practically without scattering by electric field fluctuations, i.e. in the ballistic or “free streaming” manner<sup>13</sup>. Hence, one can neglect the electric field effects, at the first approximation, and employ the reduced kinetic equation for such free-streaming electrons:

$$\frac{\partial f_e}{\partial t} + v \frac{\partial f_e}{\partial x} = 0 \tag{1}$$

Eq.(1) is valid as far as the local current of free-streaming electrons stays below the “chaotic” current  $\sim n_e e (2T_e/m)^{1/2}$ . When and where the current exceeds the threshold, the plasma becomes unstable and the effects of the electric field become unavoidable (see

Sec.3). It is to be emphasized that the plasma is stable initially as well as at the nearest vicinity of the boundary, but becomes unstable because of the ballistic deformation of the velocity distribution (see Fig.1).

Eq.(1) is solved separately for negative and positive velocity regions. We consider for simplicity a semi-infinite plasma,  $0 \leq x \leq L$ , with the initial condition (spatial homogeneity is assumed)

$$f_e|_{t=0} = f_{e0}(v) \quad (2)$$

and the boundary condition for the incoming part of the velocity distribution function, i.e. for  $v \geq 0$ ,

$$f_e \Big|_{\substack{x=0 \\ v \geq 0}} = F_e(t, v) \quad (3)$$

and for  $v < 0$ ,

$$f_e \Big|_{\substack{x=L \rightarrow \infty \\ v \geq 0}} = G_e(t, v) = f_{e0}(v) \quad (4)$$

Only the left-hand-side (l.h.s.) plasma boundary ( at  $x = 0$  ) is considered to be “active” with an arbitrary incoming velocity distribution function,  $F_e(t, v)$ , while the right-hand-side (r.h.s.) is assumed to be “passive”. This assumption is reasonable until the information from the l.h.s. boundary gets to the r.h.s. one. We assume the r.h.s. boundary to be far enough so that there is a long period of time during which the boundary condition (4) is valid. Another important point is that the boundary conditions have to obey some integral relations between the “incoming” and “outcoming” parts of the velocity distribution function (see Sec.3). Even so, there remains much freedom in the choice of the incoming velocity distribution  $F_e(t, v)$ .

Eqs.(1),(2),(3) and (4) can be solved explicitly as

$$f_e(t, x, v) = \begin{cases} F_e(t - \frac{x}{v}, v) & \text{for } v > \frac{x}{t}, \\ f_{e0}(v) & \text{for } v < \frac{x}{t}. \end{cases} \quad (5)$$

From Eq.(5) one obtains

$$\frac{\partial f_e}{\partial v} = \begin{cases} \left( \frac{\partial F_e}{\partial v} \right)_{t - \frac{x}{v}} + \frac{x}{v^2} \left( \frac{\partial F_e}{\partial t} \right)_{t - \frac{x}{v}} & \text{for } v > \frac{x}{t}, \\ \frac{\partial f_{e0}}{\partial v} & \text{for } v < \frac{x}{t}. \end{cases} \quad (6)$$

Here  $(\cdot)_{t-(x/v)}$  means that the value  $(\cdot)$  be defined at the time  $t - (x/v)$ . It is clear that even for a stable boundary condition with  $\partial f_e/\partial v < 0$  an unstable distribution with  $\partial f_e/\partial v > 0$  can occur at some distance from the plasma boundary if a positive time-derivative  $\partial F_e/\partial t > 0$  takes place during a certain period of time. This conclusion is qualitatively illustrated in Fig.2 where the time histories of  $F_e(t, v)$  and  $f_e(t, x, v)$  are presented. In accordance with the solution (5) the velocity profile at any position,  $x$ , is obtained by scanning the contour  $F_e(t, v) = \text{const}$  (solid line in Fig.2) along a “geodesic” line  $(t - x/v) = \text{const}$  (dashed line in Fig.2) in the  $(t; v)$ -diagram. If there are three or more intersections between a “geodesic” hyperbola and a contour  $F_e = \text{const}$ , then a bump-on-tail distribution takes place at a certain position  $x$  during a finite interval of time ( $t_B < t < t_F$  in Fig.2). It becomes possible to achieve this condition if the incoming part of the distribution is “heated”. A repeated regeneration of the unstable distribution in the plasma interior, therefore, can be organized through a repeated sequence of a rapid “heating” and a slow “cooling” of the incoming particles at the plasma boundary. Regeneration of the unstable distribution inside the plasma does not always guarantee the CDDL reformation. Nevertheless it can be a strong candidate which provokes the CDDL reformation, if the heating is rapid enough and strong enough.

Let us consider one specific example that can demonstrate the underlying idea. We choose the boundary condition at  $x=0$  in the form

$$F_e(t, v) = f_{e0}(v) \theta(t_0 - t) + F_0(v) \theta(t - t_0) \quad (7)$$

where

$$\theta(t) \equiv \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \quad (8)$$

and  $F_0(v)$ , as well as  $f_{e0}(v)$ , is a stable (monotonic,  $\partial F_0/\partial v < 0$ ) distribution function. The incoming distribution functions for  $t < t_0$  ( $f_{e0}(v)$ , dashed line) and for  $t > t_0$  ( $F_0(v)$ , dotted line) are shown in Fig.3.  $F_0(v)$  is assumed to be a broader function than  $f_{e0}(v)$ , this indicating that the transition from  $f_{e0}(v)$  to  $F_0(v)$  at  $t = t_0$  corresponds to “heating”. From Eq.(5) one easily obtains:

$$f_e(t, x, v) = \begin{cases} f_{e0}(v) & \text{for } 0 < v < \frac{x}{t-t_0} \\ F_0(v) & \text{for } v > \frac{x}{t-t_0} \end{cases} \quad (9)$$

The corresponding distribution is presented by the solid line in Fig.3. As is seen, a bump-on-tail-like distribution appears at the position,  $x$ , satisfying

$$t_* - t_0 = \frac{x}{v_*} \quad (10)$$

where  $v_*$  is the solution of the equation

$$f_{e0}(v_*) = F_e(v_*)$$

For  $t > t_*$ ,  $f_e(t, x, v)$  is a monotonically decreasing function, hence, a stable one. Thus an unstable distribution appears where Eq.(10) is satisfied. Let  $\gamma$  be the characteristic growth rate of the instability caused by the beam-like distribution. One can expect that the instability can occur only in the case where  $\gamma(t_* - t_0) \gg 1$ . This means that there is a critical distance from the boundary where the instability and CDDLs could be caused by the ballistic deformation:

$$x \gg \lambda_D \left( \frac{v_*}{v_{Te}} \right) \left( \frac{\omega_{pe}}{\gamma} \right) \equiv x_* \quad (11)$$

Here  $v_{Te} = (2T_e/m)^{1/2}$ ,  $\lambda_D$  and  $\omega_{pe}$  are the electron thermal velocity, the Debye length and the electron plasma frequency, respectively. Generally,  $\gamma \ll \omega_{pe}$  and  $v_*$  can be chosen greater than  $v_{Te}$ . Accordingly, the ballistic deformation method allows us to provoke the CDDL regeneration in a deep interior of the plasma. The optimized time history of the incoming part of the velocity distribution at plasma boundary,  $F_e(t, v)$ , can be found. But the optimization is meaningful only in the case when the effects of the electric field (QL or nonlinear) are taken into account. In the following, therefore, we consider the effects of the electric field.

### 3 The equation for the electric field.

Let us now consider the full set of 1-D Vlasov-Poisson equations ( $j = e, i$ ):

$$\frac{\partial f_j}{\partial t} + v \frac{\partial f_j}{\partial x} + \frac{e_j}{m_j} E \frac{\partial f_j}{\partial v} = 0 \quad (12)$$

$$\frac{\partial E}{\partial x} = -4\pi e \int_{-\infty}^{+\infty} (f_e - f_i) dv \quad (13)$$



Here a collisionless plasma is considered. It should be remembered that the first few moments of the velocity distribution function have a well-known physical meaning: for example, particle density,

$$n_j = \int_{-\infty}^{+\infty} f_j dv \quad (14)$$

and plasma current density,

$$J = e \int_{-\infty}^{+\infty} (f_i - f_e) v dv \quad (15)$$

From Eqs. (12) and (13) one obtains

$$\frac{\partial E}{\partial t} + 4\pi J = \alpha(t) \quad (16)$$

where the time-dependent function  $\alpha(t)$  can be expressed in terms of the averaged current value and the applied voltage,

$$\alpha(t) = -\frac{1}{L} \frac{d}{dt} (\phi_L - \phi_0) + 4\pi \frac{1}{L} \int_0^L J dx \quad (17)$$

Here  $L$  is the plasma length;  $\phi_L$  and  $\phi_0$  are the potential values at  $x = L$  and  $x = 0$ , respectively.

By using Eqs.(16), (12) and (15) one obtains the equation for the electric field in the following form:

$$\frac{\partial^2 E}{\partial t^2} + (\omega_{pe}^2 + \omega_{pi}^2)E + 4\pi e \frac{\partial}{\partial x} \left[ \int_{-\infty}^{+\infty} (f_e - f_i) v^2 dv \right] = \frac{d \alpha}{d t} \quad (18)$$

Here

$$\omega_{pj}^2 \equiv \frac{4\pi e^2}{m_j} \int_{-\infty}^{+\infty} f_j dv \quad (19)$$

Eq.(18) is a nonlinear integro-differential equation for  $E$  because  $f_j$  depends on  $E$ . Nevertheless, the structure, or constitution, of Eq.(18) obviously reveals the main processes that take place in the Vlasov-Poisson plasma. In order to close this equation one has to express the electron and the ion distribution function in terms of the electric field  $E$  as well as in terms of the initial and boundary conditions. So, let us introduce the initial and boundary conditions in the form ( $j = e, i$ ):

$$f_j |_{t=0} = f_{j0}(v), \quad (20)$$

$$f_j |_{x=0} = F_j(t, v) \quad \text{for } v > 0 \quad (21)$$

$$f_j |_{x=L} = G_j(t, v) \quad \text{for } v < 0 \quad (22)$$

We shall consider, for simplicity, a homogeneous initial distribution function without any breaking of generality. The boundary conditions, Eqs.(21) and (22), define only the incoming parts of the distribution in accordance with the nature of the first order partial differential equation of hyperbolic type.

Eq.(12) consists of two parts: a linear propagator  $\hat{g} = \partial/\partial t + v\partial/\partial x$  and a nonlinear operator  $\hat{N}_j = (e_j/m_j)E(\partial/\partial v)$ , i.e.  $(\hat{g} + \hat{N}_j)f_j = 0$ . One can solve this equation iteratively:

$$f_j = -\hat{g}^{-1}(\hat{N}_j f_j) \quad (23)$$

The inverse propagator  $\hat{g}^{-1}$  includes the boundary conditions and a path-integral along the "geodesic" trajectory. Namely, for  $v > 0$

$$f_j(t, x, v) = F_j(t - \frac{x}{v}, v)\theta(t - \frac{x}{v}) + f_{j0}(v)\theta(\frac{x}{v} - t) - \frac{e_j}{m_j} \int_0^{\min[t, \frac{x}{v}]} \left\{ \frac{\partial}{\partial v}(E f_j) \right\}_{x - v\tau}^{t - \tau} d\tau \quad (24)$$

and for  $v < 0$

$$f_j = f_j(t, x, v) = G_j(t', v)\theta(t') + f_{j0}(v)\theta(-t') - \frac{e_j}{m_j} \int_0^{\min[t, \frac{x-L}{v}]} \left\{ \frac{\partial}{\partial v}(E f_j) \right\}_{x - v\tau}^{t - \tau} d\tau \quad (25)$$

where  $t' \equiv t + (L - x)/v$ . Here  $\{A\}_{t-\tau, x-v\tau}$  means that the value A is taken at the moment of time  $(t - \tau)$  and at the position  $(x - v\tau)$ , i.e., along the geodesic trajectory for the velocity  $v$ . The expressions (24) and (25) can be substituted iteratively into their r.h.s.. In principle, the solution of Eq.(12) can be obtained in the form of the electric field power expansion

$$f_j(t, x, v) = f_j^{(0)} + f_j^{(1)} + f_j^{(2)} + \dots \quad (26)$$

The first few terms have the forms:

$$f_j^{(0)} = \begin{cases} F_j(t - \frac{x}{v}, v)\theta(t - \frac{x}{v}) + f_{j0}(v)\theta(\frac{x}{v} - t) & \text{for } v > 0 \\ G_j(t + \frac{L-x}{v}, v)\theta(t + \frac{L-x}{v}) + f_{j0}(v)\theta(-t - \frac{L-x}{v}) & \text{for } v < 0 \end{cases} \quad (27)$$

$$f_j^{(1)} = \begin{cases} -\frac{e_j}{m_j} \int_0^{\frac{x}{v}} E(t - \tau, x - v\tau) \left\{ \left( \frac{\partial F_j}{\partial v} \right)_{t - \frac{x}{v}} + \frac{x - v\tau}{v^2} \left( \frac{\partial F_j}{\partial t} \right)_{t - \frac{x}{v}} \right\} d\tau & \text{for } \frac{x}{v} < v \\ -\frac{e_j}{m_j} \int_0^t E(t - \tau, x - v\tau) d\tau \left( \frac{d f_{j0}}{d v} \right) & \text{for } 0 < v < \frac{x}{v} \end{cases} \quad (28.1)$$

$$f_j^{(1)} = \begin{cases} -\frac{e_j}{m_j} \int_0^t E(t-\tau, x-v\tau) d\left(\frac{df_{j0}}{dv}\right) & \text{for } \frac{x-L}{t} < v < 0 \\ -\frac{e_j}{m_j} \int_0^{\frac{x-L}{v}} E(t-\tau, x-v\tau) \left\{ \left(\frac{\partial G_j}{\partial v}\right)_{t'} - \frac{L-x+v\tau}{v^2} \left(\frac{\partial G_j}{\partial t}\right)_{t'} \right\} d\tau & \text{for } v < \frac{x-L}{t} \end{cases} \quad (28.2)$$

$$f_j^{(2)} = -\theta\left(t - \frac{x}{v}\right) \left(\frac{e_j}{m_j}\right)^2 \left(\frac{\partial F_j}{\partial v}\right)_{t-\frac{x}{v}} \int_0^{\frac{x}{v}} E(t-\tau, x-v\tau) E\left(t - \frac{x}{v}, 0\right) \frac{x-v\tau}{v^2} d\tau \\ + \left(\frac{e_j}{m_j}\right)^2 \int_0^{\min[t, \frac{x}{v}]} d\tau \int_{\tau}^{\min[t, \frac{x}{v}]} d\tau' E(t-\tau, x-v\tau) \left[ \left(E \frac{\partial^2 f_j^{(0)}}{\partial v^2}\right)_{\substack{t-\tau' \\ x-v\tau'}} - (\tau-\tau') \left(\frac{\partial}{\partial x} \left(E \frac{\partial f_j^{(0)}}{\partial v}\right)\right)_{\substack{t-\tau' \\ x-v\tau'}} \right] \quad (29.1)$$

for  $v > 0$

$$f_j^{(2)} = -\theta(t') \left(\frac{e_j}{m_j}\right)^2 \left(\frac{\partial G_j}{\partial v}\right)_{t'} \int_0^{\frac{x-L}{v}} E(t-\tau, x-v\tau) E\left(t + \frac{L-x}{v}, L\right) \frac{L-x+v\tau}{v^2} d\tau \\ + \left(\frac{e_j}{m_j}\right)^2 \int_0^{\min[t, \frac{x-L}{v}]} d\tau \int_{\tau}^{\min[t, \frac{x-L}{v}]} d\tau' E(t-\tau, x-v\tau) \left[ \left(E \frac{\partial^2 f_j^{(0)}}{\partial v^2}\right)_{\substack{t-\tau' \\ x-v\tau'}} - (\tau-\tau') \left(\frac{\partial}{\partial x} \left(E \frac{\partial f_j^{(0)}}{\partial v}\right)\right)_{\substack{t-\tau' \\ x-v\tau'}} \right] \quad (29.2)$$

for  $v < 0$

Here  $t' \equiv t + (L-x)/v$ . The zeroth term,  $f_j(0)$ , corresponds to the ballistic or free streaming term that was considered in Sec.2. The first-order term,  $f_j^{(1)}$ , describes the well-known linear response which arises in the linear stability analysis. Indeed, let us consider a Fourier-amplitude expansion of the electric field

$$E(t, x) \propto \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_{\omega k} \exp(ikx - i\omega t) d\omega dk$$

If we assume a stationary boundary condition, then it immediately follows from Eq.(28.1), for example, that

$$f_{j\omega k}^{(1)} \propto -\frac{e_j}{m_j} \left(\frac{df_{j0}}{dv}\right) \frac{E_{\omega k}}{i(\omega - kv)}$$

i.e., a familiar expression<sup>15</sup>. The second-order term,  $f_j^{(2)}$ , as well as all other nonlinear terms,  $f_j^{(n)}$  with  $n > 3$ , describe the QL deformation, nonlinear dispersion effects ( the last term in l.h.s. of Eq.(18) ), nonlinear plasma frequency shift, parametric instability ( the second term in l.h.s. of Eq.(18) with a time-dependent  $\omega_{pj}^2$  ) and so on. Even the ballistic term,  $f_j(0)$ , leads to a parametric instability if the time history of the incoming parts of the velocity distribution is properly chosen.

Another important point is that the boundary conditions depend on the time history of the processes inside the plasma because of the existence of the conservation laws. Indeed, the natural restriction in the form of the total plasma charge conservation law leads to a certain relationship between the outgoing and incoming parts of the distribution functions.

$$\int_{-\infty}^{+\infty} (f_e - f_i) \Big|_{x=0}^{x=L} v dv = 0$$

or equivalently

$$\int_0^{+\infty} (F_e - F_i) v dv - \int_{-\infty}^0 (G_e - G_i) v dv = \int_0^{+\infty} (f_e - f_i)_{x=L} v dv - \int_{-\infty}^0 (f_e - f_i)_{x=0} v dv \quad (30.1)$$

Here the integrals on r.h.s. of Eq.(30) depend on the time history of plasma evolution. So, the boundary conditions, i.e.  $F_j$  and  $G_j$ , are not arbitrary in the exact sense. Another restriction is that the total plasma current at the boundary,

$$I \equiv e \int_{-\infty}^{+\infty} (f_i - f_e) \Big|_{x=0} v dv = e \int_0^{+\infty} (F_i - F_e) v dv + e \int_{-\infty}^0 (f_i - f_e) \Big|_{x=0} v dv \quad (30.2)$$

depends on the peculiarities of the external electric circuit and, hence, on the time history of the plasma potential difference,  $\phi(t, L) - \phi(t, 0)$ , as well as on the time history of the plasma velocity distribution. From this point of view the boundary conditions, Eqs.(21) and (22), have to obey certain integral relations. The fluctuations of the outgoing distribution influence inevitably upon the fluctuations of the incoming one. There is a serious difficulty in the self-consistent analytical consideration, but it does not bring any problem in the case of numerical simulations because the outgoing distributions are known at each time step. As for the plasma destabilization by the ballistic deformation (occurrence of the secondary beam), a strong and abrupt deformation of the boundary condition is desirable. The influence of the CDDL-associated nonlinear fluctuations on the ballistic mechanism of plasma destabilization can not be so important. This is suppressed because the boundary conditions must obey Eqs.(30.1) and (30.2) which contain only the

low-order moment of the distribution, whereby the main part of the information about the outcoming distribution profile is lost.

By using the expansion (26), Eq.(18) can be written in the following form:

$$\frac{\partial^2 E}{\partial t^2} + E\left(\frac{4\pi e^2}{m}\right) \int_{-\infty}^{+\infty} \left(f_e^{(0)} + \frac{m}{M} f_i^{(0)}\right) dv + 4\pi e \frac{\partial}{\partial x} \int_{-\infty}^{+\infty} \left(f_e^{(1)} - f_i^{(1)}\right) v^2 dv = R^{(0)} + R^{(2)} \quad (31)$$

where

$$R^{(0)} \equiv \frac{d\alpha}{dt} - 4\pi e \frac{\partial}{\partial x} \int_{-\infty}^{+\infty} \left(f_e^{(0)} - f_i^{(0)}\right) v^2 dv \quad (32.1)$$

$$R^{(2)} \equiv -E\left(\frac{4\pi e^2}{m}\right) \int_{-\infty}^{+\infty} \left(f_e^{(1)} + \frac{m}{M} f_i^{(1)}\right) dv - 4\pi e \frac{\partial}{\partial x} \int_{-\infty}^{+\infty} \left(f_e^{(2)} - f_i^{(2)}\right) v^2 dv \quad (32.2)$$

Here only the second-order nonlinear terms are taken into account,  $R^{(2)}$ . The l.h.s. of Eq.(31) is a linear integro-differential operator that describes the plasma oscillation and the ion-acoustic wave propagation as well as the corresponding plasma instabilities. It also describes a more wide set of the phenomena ( e.g. parametric instability) because  $f_j^{(0)}$  is a time-dependent function. The expressions for  $f_j^{(1)}$  and  $f_j^{(2)}$  are given in Eqs.( 27), (28) and (29).

#### 4 The case with “jumping” boundary condition.

The expressions (27), (28) and (29) can be significantly simplified in the case of “jumping” boundary conditions:

$$\begin{cases} F_e(t, v) &= F_{e0}(v)\theta(t) + f_{e0}(v)\theta(-t) \\ F_i(t, v) &= f_{i0}(v) \\ G_e(t, v) &= f_{e0}(v) \\ G_i(t, v) &= f_{i0}(v) \end{cases} \quad (33)$$

We assume that only the left, upstream, boundary,  $x = 0$ , is an “active” one (for the electrons), while the downstream boundary,  $x = L$ , is “passive”. This assumption is reasonable for a long enough system where the information from the upstream boundary can not reach to the downstream boundary during the time of observation. We also consider the ions to be cold and massive, so that both boundaries are regarded as “passive”. By using Eq.(33) the expressions (27) and (28) can be written in the following form. For the

electrons,

$$f_e^{(0)} = \begin{cases} F_{e0}(v)\theta(t - \frac{x}{v}) + f_{e0}(v)\theta(\frac{x}{v} - t) & \text{for } v > 0 \\ f_{e0}(v) & \text{for } v < 0 \end{cases} \quad (34)$$

$$f_e^{(1)} = \frac{e}{m} \begin{cases} \left(\frac{dF_{e0}}{dv}\right) \varepsilon_0(t, x, v; \frac{x}{v}) & \text{for } \frac{x}{t} < v \\ (F_{e0} - f_{e0})\delta(t - \frac{x}{v}) \left[ \frac{x}{v^2} \varepsilon_0(t, x, v; t) - \frac{1}{v} \varepsilon_1(t, x, v; t) \right] & \text{for } v = \frac{x}{t} \\ \left(\frac{df_{e0}}{dv}\right) \varepsilon_0(t, x, v; t) & \text{for } 0 < v < \frac{x}{t} \end{cases} \quad (35.1)$$

$$f_e^{(1)} = \frac{e}{m} \left(\frac{df_{e0}}{dv}\right) \begin{cases} \varepsilon_0(t, x, v; t) & \text{for } \frac{x-L}{t} < v < 0 \\ \varepsilon_0(t, x, v; \frac{x-L}{v}) & \text{for } v < \frac{x-L}{t} \end{cases} \quad (35.2)$$

Here the notations were used:

$$\begin{cases} \varepsilon_0(t, x, v; \xi) \equiv \int_0^\xi E(t - \tau, x - v\tau) d\tau \\ \varepsilon_1(t, x, v; \xi) \equiv \int_0^\xi E(t - \tau, x - v\tau) \tau d\tau \end{cases} \quad (36)$$

For the ions,

$$f_i^{(0)} = f_{i0}(v) \quad (37)$$

$$f_i^{(1)} = -\frac{e}{M} \left(\frac{df_{i0}}{dv}\right) \begin{cases} \varepsilon_0(t, x, v; \frac{x}{v}) & \text{for } \frac{x}{t} < v \\ \varepsilon_0(t, x, v; t) & \text{for } \frac{x-L}{t} < v < \frac{x}{t} \\ \varepsilon_0(t, x, v; \frac{x-L}{v}) & \text{for } v < \frac{x-L}{t} \end{cases} \quad (38)$$

Then, Eq.(18) takes the form:

$$\frac{\partial^2 E}{\partial t^2} + (\omega_{pe}^{(0)})^2 \left(1 + \frac{m}{M} + \rho\left(\frac{x}{t}\right)\right) E + \frac{4\pi e^2}{m} \left[ \frac{\partial^2 D_1}{\partial x^2} + \frac{\partial D_2}{\partial x} \right] = C^{(0)} + C^{(2)} + C^{(3)} + \dots \quad (39)$$

where

$$(\omega_{pe}^{(0)})^2 = \frac{4\pi e^2}{m} n_{e0} \equiv \frac{4\pi e^2}{m} \int_{-\infty}^{+\infty} f_{e0} dv \quad (40.1)$$

$$\rho\left(\frac{x}{t}\right) \equiv \frac{1}{n_{e0}} \int_{\frac{x}{t}}^{+\infty} (F_{e0} - f_{e0}) dv \quad (40.2)$$

$$D_1 = \int_{\frac{x}{t}}^{+\infty} \left( \Pi_e + \frac{m}{M} \Pi_i \right) \varepsilon_1(t, x, v; \frac{x}{v}) dv + \int_{\frac{x}{t}}^{\frac{x-L}{t}} \left( \Pi_e + \frac{m}{M} \Pi_i \right) \varepsilon_1(t, x, v; t) dv \quad (40.3)$$

$$D_2 = \int_{-\infty}^{\frac{x-L}{t}} \left( \Pi_e + \frac{m}{M} \Pi_i \right) \varepsilon_1(t, x, v; \frac{x-L}{v}) dv + \frac{\partial}{\partial x} \left[ \int_{\frac{x}{t}}^{+\infty} (Q - \Pi_e) \varepsilon_1(t, x, v; \frac{x}{v}) dv \right] \quad (40.4)$$

$$-2 \left( \int_{\frac{x}{t}}^{+\infty} (F_{e0} - f_{e0}) v dv \right) \left[ \varepsilon_0(t, x, \frac{x}{t}; t) - \frac{1}{t} \varepsilon_1(t, x, \frac{x}{t}; t) \right]$$

$$C^{(0)} = \frac{d\alpha}{dt} + 4\pi e \frac{x^2}{t^3} \left[ F_{e0}\left(\frac{x}{t}\right) - f_{e0}\left(\frac{x}{t}\right) \right] \quad (40.5)$$

$$C^{(2)} = 4\pi e \frac{\partial}{\partial x} \left[ \int_{-\infty}^{+\infty} (f_i^{(2)} - f_e^{(2)}) v^2 dv \right] - E \left( \frac{4\pi e^2}{m} \right) \int_{-\infty}^{+\infty} (f_e^{(1)} + \frac{m}{M} f_i^{(1)}) dv \quad (40.6)$$

We also used the notations:

$$v^2 \frac{d f_{j0}}{d v} \equiv \frac{d \Pi_j}{d v} \quad \text{and} \quad v^2 \frac{d F_{j0}}{d v} \equiv \frac{d Q}{d v} \quad (41)$$

Since  $D_1$  and  $D_2$  are not dependent upon the choice of the arbitrary constants of  $Q$  and  $\Pi_j$ , we can choose the following forms;

$$Q(v) = v^2 F_{e0}(v) + 2 \int_v^{+\infty} F_{e0}(v') v' dv'$$

and

$$\Pi_j = v^2 f_{e0}(v) + 2 \int_v^{+\infty} f_{j0}(v') v' dv'$$

where  $Q(\infty) = P_j(\infty) = 0$ .

The l.h.s. of Eq.(39) describes the propagation and Landau damping of the plasma waves in an unperturbed plasma ( the term with  $D_1$  ) as well as the effects of the ‘‘jumping’’ boundary condition ( the terms with  $\rho(x/t)$  and  $D_2$  ). A plasma instability that would

occur as a result of the ballistic deformation (see Sec.2) is described by the  $D_2$ -term. In the r.h.s. of Eq.(39) the  $C^{(0)}$ -term corresponds to the ballistic effect, while the  $C^{(2)}$ ,  $C^{(3)}$  and so on describe the nonlinear effects.

Eq.(39) is an integro-differential equation because it contains the path-integrals  $\varepsilon_0$  and  $\varepsilon_1$  ( see Eq.(36) ). These integrals depend on the characteristic time- and space-correlation scales,  $t_c$  and  $l_c$ , respectively. In the linear regime with infinitesimally small amplitudes the correlation scales are large enough (  $t_c \propto \gamma^{-1}$ , where  $\gamma$  is the instability growth rate), therefore, there are resonances between the traveling waves,  $\propto \exp(-\omega t + ikx)$ , and the particles, i.e.  $\omega/k = v_{ph}$ . In this case  $\varepsilon_0$  and  $\varepsilon_1$  are the delta-function-like functions in the phase space. In the strongly nonlinear regime with chaotic fluctuations,  $t_c$  and  $l_c$  are small enough,  $t_c \propto \omega_{pe}^{-1}$  and  $l_c \propto \lambda_D$ , so that  $\varepsilon_0 \propto E/\omega_{pe}^{(0)}$  and  $\varepsilon_1 \propto E/(\omega_{pe}(0))^2$ . In this case, no resonances between the particles and waves can exist because of the chaotic phase behavior.

Let us consider a nonlinear regime rather than a linear one because we are interested in regeneration of CDDLs and because after the previous CDDLs have disappeared, strong electric field fluctuations would have survived in the system. As the correlation length is assumed to be small,  $l_c \propto \lambda_D$ , we can rely on a “local” approach, namely  $\partial/\partial x$  acts on the electric field only ,  $\partial E/\partial x \propto E/\lambda_D$ .

Eq.(39) can be written approximately in the following form:

$$\frac{\partial^2 E}{\partial t^2} + \omega_{pe}^2(1 + \rho)E - \omega_{pe}u \frac{\partial E}{\partial x} = C^{(0)} = \frac{d\alpha}{dt} - \frac{2\pi e n_{e0} x}{t} u' \left( \frac{x}{t} \right) \quad (42)$$

Here  $\omega_{pe} \equiv \omega_{pe}^{(0)}$  and  $m/M \rightarrow 0$  is assumed. We used also the notation:

$$u \left( \frac{x}{t} \right) = \frac{2}{n_{e0}} \int_{\frac{x}{t}}^{+\infty} (F_{e0} - f_{e0}) v dv \quad (43)$$

We have not taken into account the nonlinear effects described by the r.h.s. of the Eq.(39) as well as the dispersion effects, i.e. the  $D_2$ -term ( the last one is important on the ion-acoustic time-scale ). We assume a “jumping” boundary condition, Eq.(33). It is natural to demand the total plasma charge conservation in the form:

$$u \left( \frac{x}{t} = 0 \right) = \frac{2}{n_{e0}} \int_0^{+\infty} (F_{e0} - f_{e0}) v dv = 0 \quad (44)$$

It is the only condition that restricts the choice of  $F_{e0}$  ( of course,  $F_{e0}$  has to be stable, i.e. monotonic ). In Eq.(42)  $\rho$  and  $u$  as well as the r.h.s. of the equation, are slowly varying



functions on the  $\omega_{pe}^{-1}$ -time scale. Hence, the WKB-approach is reasonable,

$$E = A(x, t) \exp[-i\omega B(x, t)] + c.c. \quad (45)$$

where A and B obey the following equations:

$$\begin{cases} \left( \frac{\partial B}{\partial t} \right)^2 - u \frac{\partial B}{\partial x} = 1 + \rho \\ \frac{\partial B}{\partial t} \frac{\partial A}{\partial t} + \frac{1}{2} A \frac{\partial^2 B}{\partial t^2} - \frac{1}{2} u \frac{\partial A}{\partial x} = 0 \end{cases} \quad (46)$$

We neglect the forcing term in the r.h.s. of Eq.(46) and assume a nonzero initial amplitude A. In this case Eq.(46) describes the growth of the amplitude A. Indeed, for small  $\rho$  and  $u$  ( this is a usual situation ) one can approximately write  $B \approx t$ , thus the last equation in Eqs.(46) can be transformed to

$$\frac{\partial A}{\partial t} - \frac{i}{2} u \frac{\partial A}{\partial x} = 0 \quad (47)$$

Let us assume that  $F_{e0}$  corresponds to a plateau-like distribution in the velocity interval  $v_{min} < v < v_{max}$ , while outside this interval  $F_{e0} = f_{e0}$  (see Fig.4). In this case  $u(x/t)$  is approximately equal to

$$u\left(\frac{x}{t}\right) \approx \begin{cases} 0 & \text{for } \frac{x}{t} < v_{min} \text{ or } v_{max} < \frac{x}{t} \\ u_0 \text{ (const)} & \text{for } v_{min} < \frac{x}{t} < v_{max} \end{cases} \quad (48)$$

In Fig.4.c the domain for  $u = 0$  is presented. As can be seen, there is a wide enough region where the assumption  $u \approx u_0$  is valid. By using Eq.(48) the solution of Eq.(47) can be obtained in the general form,

$$A = A_0 \left( x + i \frac{u_0}{2} t \right) \quad (49)$$

where  $A_0(x)$  is the initial spatial distribution of the amplitude  $A(t = 0, x)$ . There are many functions  $A_0(x)$  which correspond to the growth of A in Eq.(49). For example,  $A_0(x) = \sin(kx)$ . In this case Eq.(49) can be transformed to

$$A = \frac{1}{2i} \left[ \exp \left( ikx - \frac{ku_0}{2} t \right) - \exp \left( -ikx + \frac{ku_0}{2} t \right) \right]$$

The corresponding growth rate of oscillations is

$$\gamma \approx \left| \frac{k u_0}{2} \right|. \quad (50)$$

This solution exists during a finite period of time  $x/v_{max} < t < x/v_{min}$  at each point  $x$  (see Fig.4). Then, the amplitude amplification factor,  $\beta$ , is approximately equal to

$$\beta \approx \exp \left[ \gamma \left( \frac{x}{v_{min}} - \frac{x}{v_{max}} \right) \right] \approx \exp(\sigma x) \quad (51)$$

where

$$\sigma = \left| \frac{k}{2} \left( \frac{u_0}{v_{min}} - \frac{u_0}{v_{max}} \right) \right| \quad (52)$$

Thus, a strongly nonlinear regime of the plasma oscillations is expected to occur in the deep interior of the plasma, while in the vicinity of plasma boundary only stable “thermal” fluctuations appear.

## 5 Conclusions.

In a current carrying plasma the presence of strong electric field oscillations can lead to the occurrence of an anomalous resistance of the plasma and, hence, can provoke a current driven double layer formation. But the existence of a nonzero plasma current does not necessarily guarantee that a plasma instability can arise. Free energy of the plasma is dependent upon the detailed structure of the velocity distribution, i.e., not only upon the first moment ( current ), but also upon the higher moments. At the nonlinear stage of the plasma instability, e.g., after the CDDL formation, a strong wave-particle interaction leads to the velocity redistribution in such a manner that the plasma becomes stable. More specifically, after the first generation of CDDLs no new CDDLs can arise because the plasma free energy is exhausted. Thus one needs to introduce some other sources of free energy in order to regenerate the nonlinear structures. In the present paper, pumping of free energy in the form of rapidly varying boundary condition is proposed. The source of the free energy in this case is the plasma inhomogeneity caused by non-stationary boundary conditions together with the ballistic deformation of the distribution function (  $\mathbf{v} \cdot \nabla f$  term in the kinetic equation ). The important feature of the ballistic method of regenerating an unstable distribution is that the growth of electric field oscillations takes place in the deep interior of the plasma, while the transit plasma in the vicinity of the plasma boundary is stable. A nonlinear integro-differential equation is obtained

that describes the electric field evolution. A WKB-like solution is found. This solution clearly demonstrates regeneration of a plasma instability inside the plasma by means of the "jumping" boundary condition with the subsequent ballistic deformation in the velocity distribution. The growth of the electric field oscillations can probably provoke CDDL formation in a current carrying plasma.

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## Figure Captions.

Fig.1. A bounded one-dimensional plasma with the incoming velocity distribution functions  $F_e(t, v)$  at  $x=0$  and  $G_e(t, v)$  at  $x=L$  ( schematically ). Region 1 is a stable plasma with small electric field fluctuations. Region 2 is the unstable plasma with a beam-like velocity distribution formed by the ballistic deformation due to non-stationary boundary conditions ( namely  $F_e(t, v)$  ).

Fig.2. a). The contours  $F_e=\text{const}$  ( solid lines ) and the “geodesic” lines  $(t - x/v)=\text{const}$  ( dashed lines ).

b). The corresponding velocity distribution function at the moment  $t_B < t_1 < t_F$ . The points A, B and C correspond to the intersections A, B and C in Fig.2.a).

Fig.3. The incoming part of the velocity distribution function at the boundary  $x = 0$  for  $t < t_0$  ( dashed line ) and for  $t > t_0$  ( dotted line ) as well as the velocity distribution function  $f_e(t, x, v)$  inside the plasma ( solid line ) for  $t_0 < t < t_* = t_0 + x/v_*$  ( see the text ).

Fig.4. A particular example of a “jumping” boundary condition.

a)  $f_{e0}$  ( solid line ) and  $F_{e0}$  ( dashed line ) profiles;

b) The corresponding  $u(x/t)$  profile (see Eq.(42)).

c) The domain in the  $(x,t)$ -diagram where  $u=0$  ( shaded area ). Straight lines:

1 —  $t = x/v_{max}$  ; 2 —  $t = x/v_{min}$ .

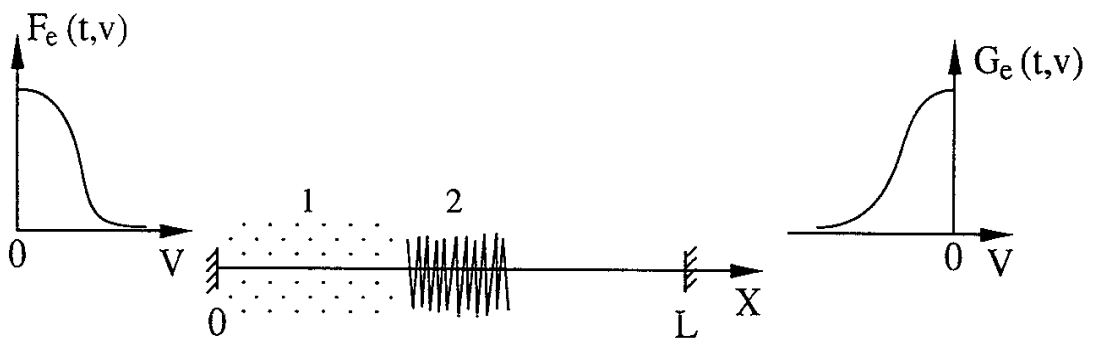


Fig. 1.

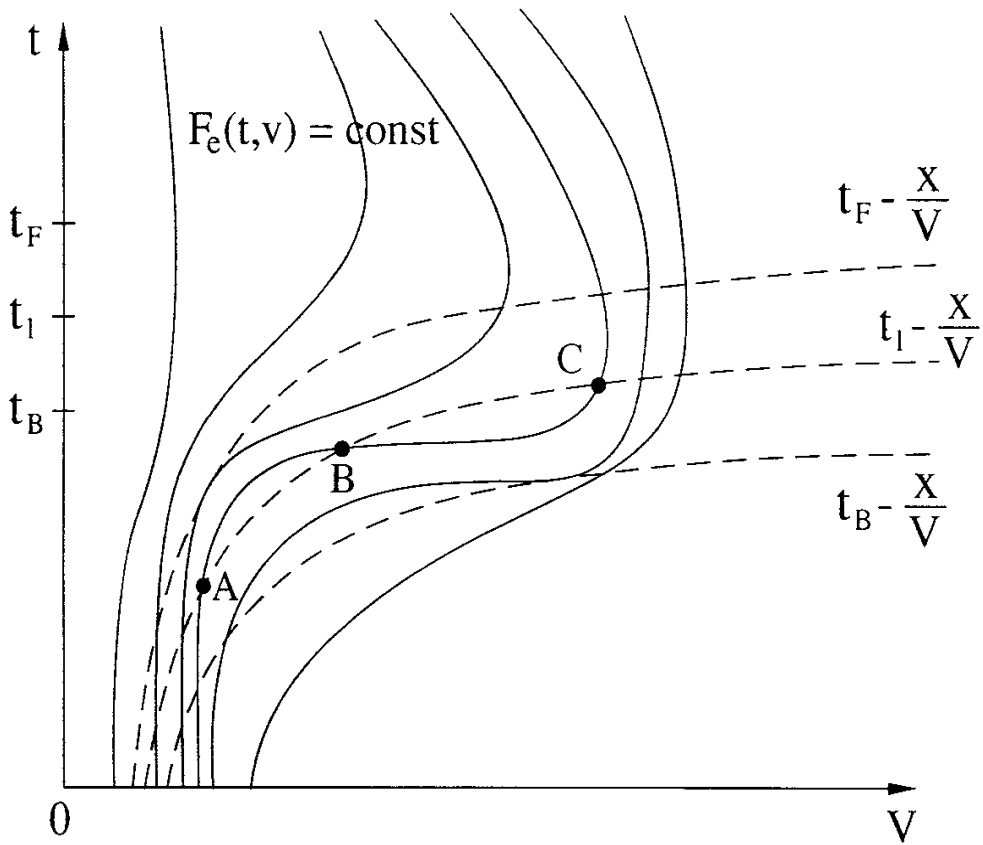


Fig. 2. a.

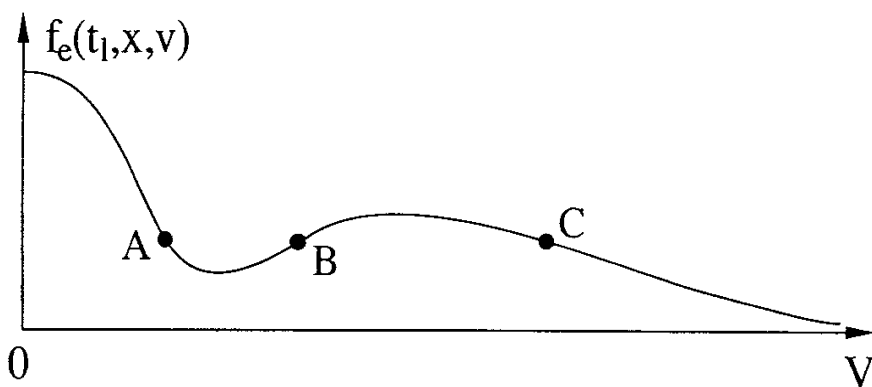


Fig. 2. b.

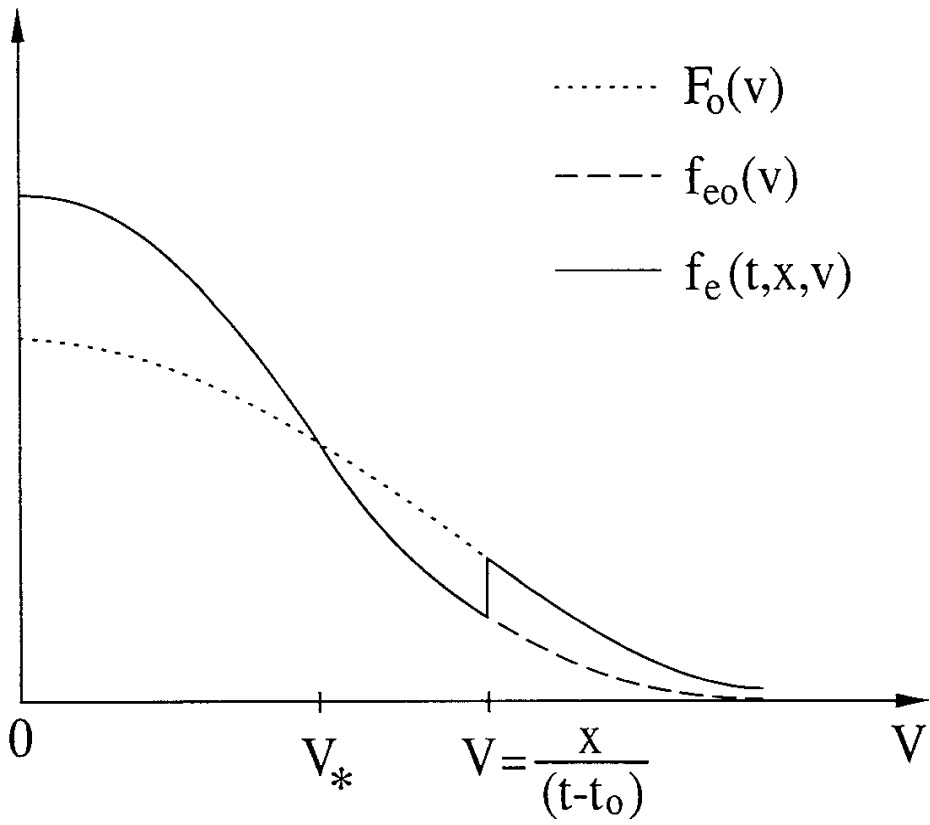


Fig. 3.



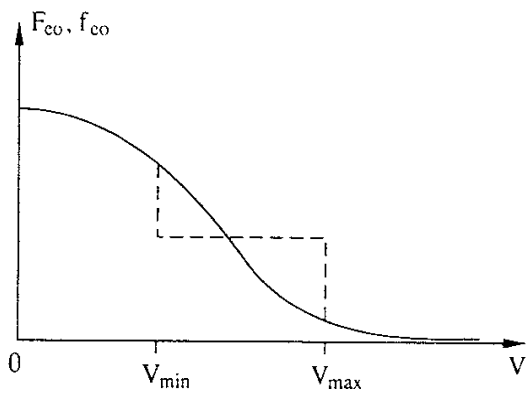


Fig. 4. a.

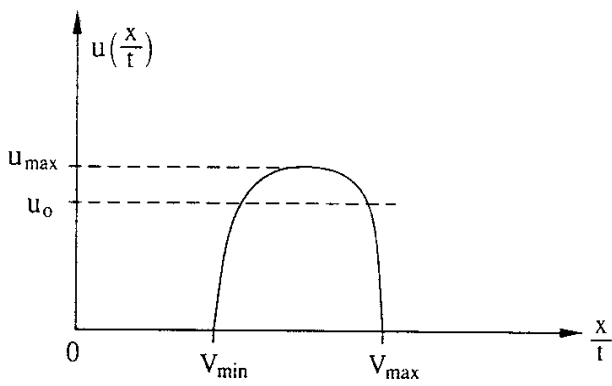


Fig. 4. b.

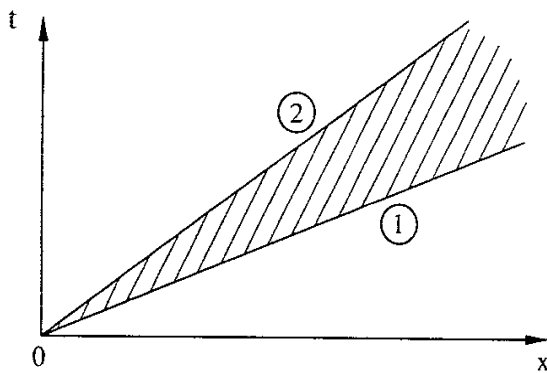


Fig. 4. c.

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