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On the Lagrangian of the Linearized MHD Equations

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Abstract

Lagrangian for the linearized, ideal and resistive, MHD equations is discussed, by introducing the perturbation of the total pressure. In the resistive MHD equations, the Lagrangian is expressed in terms of the electric displacement vector (time integral of the electric current) as well as the plasma displacement. The NOVA and NOVA-R formulation can be derived by using the obtained Lagrangian.

Key Words: Lagrangian, linearized MHD equation, resistive MHD

In this paper, the Lagrangian for the linearized resistive magneto-hydrodynamic (MHD) equations is derived. The perturbation of the total pressure is included as a dependent variable in addition to the displacement vector and electric displacement vector, in order to separate the operator associated with the continuous spectrum in the ideal limit.[1,2]

We shall consider the MHD equilibrium, satisfying the following magneto-static equations

$$\mathbf{J} \times \mathbf{B} = \nabla P, \quad \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad (1)$$

where P is the equilibrium pressure, \mathbf{B} the equilibrium magnetic field, and \mathbf{J} the equilibrium electric current. The simply nested toroidal magnetic surface is assumed, so that $P=P(\psi)$. Assuming that the perturbed quantities have the time dependence of the form $\exp(qt)$, we can write the linearized MHD equations in the following form [3]

$$q^2 \rho \boldsymbol{\xi} + \mathbf{b} \times \mathbf{J} + \mathbf{B} \times \nabla \times \mathbf{b} - \nabla [\boldsymbol{\xi} \cdot \nabla P + \gamma_s P \nabla \cdot \boldsymbol{\xi}] = 0, \quad (2)$$

$$\mathbf{b} - \nabla \times (\boldsymbol{\xi} \times \mathbf{B}) + \frac{1}{q} \nabla \times (\eta \nabla \times \mathbf{b}) = 0, \quad (3)$$

where $\boldsymbol{\xi}$ is the plasma displacement and \mathbf{b} , is the perturbed magnetic field, γ_s being the ratio of the specific heat, and η the resistivity. The boundary conditions are, assuming the surrounding fixed resistive boundary at $\psi=1$, $\boldsymbol{\xi} \cdot \mathbf{n} = 0$ and $\mathbf{n} \times \eta \nabla \times \mathbf{b} = 0$, where \mathbf{n} is the outward normal on the boundary. In the ideal case ($\eta=0$) the latter condition has no meaning.

For the ideal case, the perturbed magnetic field can be expressed in terms of $\boldsymbol{\xi}$, and the Lagrangian can be obtained. Its density can be written as

$$\mathcal{L}_1[\boldsymbol{\xi}] = q^2 \rho |\boldsymbol{\xi}|^2 + |\mathbf{Q}|^2 + \mathbf{J} \times \boldsymbol{\xi}^* \cdot \mathbf{Q} + (\boldsymbol{\xi} \cdot \nabla P)(\nabla \cdot \boldsymbol{\xi}^*) + \gamma_s P |\nabla \cdot \boldsymbol{\xi}|^2, \quad (4)$$

with $\mathbf{Q} \equiv \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$, $|\boldsymbol{\xi}|^2 \equiv \boldsymbol{\xi}^* \cdot \boldsymbol{\xi}$, and $*$ denotes complex conjugate. The first term in eq.(4) corresponds to the kinetic energy, and the other terms to the potential energy.[4] The eigenvalue q is real (for unstable modes) or pure imaginary (for stable modes).

When the resistivity is introduced, the self-adjointness of the linearized operator is lost, and the eigenvalue q becomes to a complex number. We shall rewrite eqs.(2) and (3) in terms of perturbed electric displacement $\mathbf{a} = \nabla \times \mathbf{b}/q$, instead of \mathbf{b} .

$$q^2 \rho \boldsymbol{\xi} + \mathbf{Q} \times \mathbf{J} + \mathbf{B} \times \nabla \times \mathbf{Q} - \nabla [\boldsymbol{\xi} \cdot \nabla P + \gamma_s P \nabla \cdot \boldsymbol{\xi}] - \mathbf{B} \times \nabla \times \nabla \times (\eta \mathbf{a}) + \mathbf{J} \times \nabla \times (\eta \mathbf{a}) = 0, \quad (5)$$

$$q \mathbf{a} - \nabla \times \mathbf{Q} + \nabla \times \nabla \times (\eta \mathbf{a}) = 0. \quad (6)$$

The adjoint equations are written as

$$q^2\rho\bar{\xi} + \bar{\mathbf{Q}} \times \mathbf{J} + \mathbf{B} \times \nabla \times \bar{\mathbf{Q}} - \nabla[\bar{\xi} \cdot \nabla p + \gamma_s p \nabla \cdot \bar{\xi}] - \mathbf{B} \times \nabla \times \nabla \times (\eta \bar{\mathbf{a}}) = 0, \quad (7)$$

$$q\bar{\mathbf{a}} - \nabla \times \bar{\mathbf{Q}} + \nabla \times \nabla \times (\eta \bar{\mathbf{a}}) = \nabla \times (\mathbf{J} \times \bar{\xi}), \quad (8)$$

with $\bar{\mathbf{Q}} \equiv \nabla \times (\bar{\xi} \times \mathbf{B})$. In the ideal case, the set of eqs.(7) and (8) are identical to that of eqs.(5) and (6), and the adjoint variables are either the complex conjugates of the original variables, or the original variables; however, such a relation does not hold in the resistive case. Hence, we shall introduce the new variables ξ_+ , ξ_- , \mathbf{a}_+ , and \mathbf{a}_- by the relations

$$\left. \begin{aligned} \xi &= \xi_+ + \xi_-, & \bar{\xi} &= \xi_+ - \xi_- \\ \mathbf{a} &= \mathbf{a}_+ + \mathbf{a}_-, & \bar{\mathbf{a}} &= \mathbf{a}_+ - \mathbf{a}_- \end{aligned} \right\}. \quad (9)$$

Equations for them can be written in the form

$$q^2\rho\xi_{\pm} + \mathbf{Q}_{\pm} \times \mathbf{J} + \mathbf{B} \times \nabla \times \mathbf{Q}_{\pm} - \nabla[\xi_{\pm} \cdot \nabla P + \gamma_s P \nabla \cdot \xi_{\pm}] - \mathbf{B} \times \nabla \times \nabla \times (\eta \mathbf{a}_{\pm}) + \frac{1}{2} \mathbf{J} \times \nabla \times (\eta[\mathbf{a}_+ + \mathbf{a}_-]) = 0, \quad (10)$$

$$q\mathbf{a}_{\pm} - \nabla \times \mathbf{Q}_{\pm} + \nabla \times \nabla \times (\eta \mathbf{a}_{\pm}) = \pm \frac{1}{2} \nabla \times (\mathbf{J} \times [\xi_+ - \xi_-]). \quad (11)$$

Then, it is a easy task to write down the Lagrangian deriving these equation.

The Lagrangian density can be written in the form

$$\mathcal{L}_R = \mathcal{L}_0[\xi_+, \mathbf{a}_+] - \mathcal{L}_0[\xi_-, \mathbf{a}_-] + \mathcal{M}, \quad (12)$$

where

$$\begin{aligned} \mathcal{L}_0[\xi, \mathbf{a}] &= q^2\rho|\xi|^2 + q\eta|\mathbf{a}|^2 + \frac{1}{2}[(\xi^* \cdot \nabla P)\nabla \cdot \xi + (\xi \cdot \nabla P)\nabla \cdot \xi^*] \\ &\quad + \left| \mathbf{Q} + \frac{1}{2} \mathbf{J} \times \xi - \nabla \times (\eta \mathbf{a}) \right|^2 - \frac{1}{4} |\mathbf{J} \times \xi|^2 + \gamma_s P |\nabla \cdot \xi|^2, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{M} &= \frac{1}{2} \{ \mathbf{J} \times \xi_- \cdot \nabla \times (\eta \mathbf{a}_+) + \mathbf{J} \times \xi_+ \cdot \nabla \times (\eta \mathbf{a}_-) \\ &\quad - \mathbf{J} \times \xi_+ \cdot \nabla \times (\eta \mathbf{a}_-) - \mathbf{J} \times \xi_- \cdot \nabla \times (\eta \mathbf{a}_+) \}. \end{aligned} \quad (14)$$

In the next step, the perturbation of the total pressure

$$\varpi \equiv -\xi \cdot \nabla P - \gamma_s P (\nabla \cdot \xi) + \mathbf{B} \cdot \mathbf{b} \quad (15)$$

is introduced as the dependent variable, in order to recover the NOVA[1,5] and NOVA-R[6] equations. For this purpose, we introduce the variable $\mathcal{D} \equiv \nabla \cdot \xi$, and write the Lagrangian (4) in the form

$$\begin{aligned} \mathcal{L}_1[\xi, \mathcal{D}, \varpi] &= q^2\rho|\xi|^2 + |\hat{\mathbf{Q}} - \mathbf{B}\mathcal{D}|^2 + \mathbf{J} \times \xi^* \cdot (\hat{\mathbf{Q}} - \mathbf{B}\mathcal{D}) + (\xi \cdot \nabla P)\mathcal{D}^* \\ &\quad + \gamma_s P |\mathcal{D}|^2 + \varpi^* (\mathcal{D} - \nabla \cdot \xi) + \varpi (\mathcal{D}^* - \nabla \cdot \xi^*), \end{aligned} \quad (16)$$

where $\hat{\mathbf{Q}} \equiv \mathbf{B} \cdot \nabla \xi - \xi \cdot \nabla \mathbf{B}$, and ϖ is introduced as the Lagrange multiplier. Since \mathcal{D} appears only in algebraic form without derivatives in eq.(16), by

carrying out the minimization with respect to it, we can eliminate \mathcal{D} from the Lagrangian. The result is

$$\begin{aligned} \mathcal{L}_1[\xi, \varpi] = & q^2 \rho |\xi|^2 + |\hat{Q}|^2 + \mathbf{J} \times \xi^* \cdot \hat{Q} - \varpi^* (\nabla \cdot \xi) \\ & - \varpi (\nabla \cdot \xi^*) - \frac{1}{B^2 + \gamma_s P} |\varpi - \mathbf{B} \cdot \hat{Q} + \xi \cdot \nabla P|^2. \end{aligned} \quad (17)$$

It is easy to see that ϖ satisfies eq.(15).

If we put

$$\xi = \frac{\xi_\psi \nabla \psi}{|\nabla \psi|^2} + \frac{\xi_s \mathbf{B} \times \nabla \psi}{B^2} + \xi_b \mathbf{B}, \quad (18)$$

and define the quantities

$$\sigma \equiv \frac{\mathbf{J} \cdot \mathbf{B}}{B^2}, \quad (19)$$

$$\hat{S} \equiv \frac{\nabla \psi \times \mathbf{B}}{|\nabla \psi|^2} \cdot \nabla \times \left(\frac{\nabla \psi \times \mathbf{B}}{|\nabla \psi|^2} \right), \quad (20)$$

$$K_\psi \equiv \frac{2\kappa \cdot \nabla \psi}{|\nabla \psi|^2}, \quad K_s \equiv \frac{2\kappa \cdot \mathbf{B} \times \nabla \psi}{B^2}, \quad (21)$$

κ being the curvature of the magnetic lines of force, the Lagrangian density can be written in the form

$$\begin{aligned} \mathcal{L}_1[\xi, \varpi] = & \frac{q^2 \rho}{|\nabla \psi|^2} |\xi_\psi|^2 + \frac{q^2 \rho |\nabla \psi|^2}{B^2} |\xi_s|^2 + q^2 \rho B^2 |\xi_b|^2 + \frac{1}{|\nabla \psi|^2} |\mathbf{B} \cdot \nabla \xi_\psi|^2 \\ & + \frac{|\nabla \psi|^2}{B^2} |\mathbf{B} \cdot \nabla \xi_s - \hat{S} \xi_\psi|^2 + \sigma \xi_\psi^* (\mathbf{B} \cdot \nabla \xi_s) + \sigma \xi_\psi (\mathbf{B} \cdot \nabla \xi_s^*) \\ & - (P' K_\psi + \sigma \hat{S}) |\xi_\psi|^2 + \frac{B^2 \gamma_s P}{B^2 + \gamma_s P} |\mathbf{K} \cdot \xi_\perp - \mathbf{B} \cdot \nabla \xi_b|^2 \\ & - \frac{|\varpi|^2}{B^2 + \gamma_s P} - \frac{B^2 \varpi^*}{B^2 + \gamma_s P} \left[\mathbf{K} \cdot \xi_\perp + \frac{\gamma_s P}{B^2} \mathbf{B} \cdot \nabla \xi_b \right] - \varpi^* \nabla \cdot \xi_\perp \\ & - \frac{B^2 \varpi}{B^2 + \gamma_s P} \left[\mathbf{K} \cdot \xi_\perp + \frac{\gamma_s P}{B^2} \mathbf{B} \cdot \nabla \xi_b \right]^* - \varpi \nabla \cdot \xi_\perp^*, \end{aligned} \quad (22)$$

where $\mathbf{K} \cdot \xi_\perp = K_\psi \xi_\psi + K_s \xi_s$, and

$$\nabla \cdot \xi_\perp \equiv \nabla \cdot \left(\frac{\xi_\psi \nabla \psi}{|\nabla \psi|^2} \right) + \nabla \cdot \left(\frac{\xi_s \mathbf{B} \times \nabla \psi}{B^2} \right). \quad (23)$$

In writing eq.(22) the following relation is used.

$$\mathbf{B} \cdot \nabla \sigma = p' K_s. \quad (24)$$

The part of (22) containing ξ_s , and ξ_b

$$\begin{aligned} \mathcal{L}_{\text{con}}[\xi_s, \xi_b] = & \frac{|\nabla \psi|^2}{B^2} \left\{ q^2 \rho |\xi_s|^2 + |\mathbf{B} \cdot \nabla \xi_s|^2 \right\} \\ & + q^2 \rho B^2 |\xi_b|^2 + \frac{B^2 \gamma_s P}{B^2 + \gamma_s P} |\mathbf{B} \cdot \nabla \xi_b - K_s \xi_s|^2, \end{aligned} \quad (25)$$

is the Lagrangian[1] describing the continuous spectrum. The essentially same equations given in Ref.[1,5] can be derived from this Lagrangian; the difference is that the variable \mathcal{D} is used in Ref.[1,5] instead of ξ_b ,

$$\xi_b = \frac{\gamma_s P}{q^2 \rho} \mathbf{B} \cdot \nabla \mathcal{D}. \quad (26)$$

In addition to that, equations in [1,5] do not have Hermitian nature, while those derived from eq.(22) are naturally Hermitian.

Similarly, we obtain for the resistive case

$$\mathcal{L}_R = \mathcal{L}_0[\xi_+, \varpi_+, \mathbf{a}_+] - \mathcal{L}_0[\xi_-, \varpi_-, \mathbf{a}_-] + \mathcal{M}, \quad (27)$$

$$\begin{aligned} \mathcal{L}_0[\xi, \varpi, \mathbf{a}] = & q^2 \rho |\xi|^2 + \left| \hat{Q} + \frac{1}{2} \mathbf{J} \times \xi - \nabla \times (\eta \mathbf{a}) \right| - \frac{1}{4} |\mathbf{J} \times \xi|^2 \\ & - \varpi^* (\nabla \cdot \xi) - \varpi (\nabla \cdot \xi^*) + q \eta |\mathbf{a}|^2 \\ & - \frac{1}{B^2 + \gamma_s P} \left| \mathbf{B} \cdot \hat{Q} - \varpi - \xi \cdot \nabla P - \mathbf{B} \cdot \nabla \times (\eta \mathbf{a}) \right|^2. \end{aligned} \quad (28)$$

The NOVA-R formulation[6] can be recovered by using the representation for the electric displacement

$$\mathbf{a} = A_\psi \nabla \psi + \frac{A_s \mathbf{B} \times \nabla \psi}{|\nabla \psi|^2} + A_b \mathbf{B}, \quad (29)$$

as well as eq.(18) for the plasma displacement. The boundary conditions in these variables are $\xi_\psi = A_s = A_b = 0$ on the boundary. The resulting expression is too cumbersome; therefore, we shall discuss only symbolically. We introduce the vectors

$$\mathbf{x} = (\xi_\psi, \varpi, \xi_s, \xi_b)^T, \quad \mathbf{y} = (A_\psi, A_s, A_b)^T, \quad (30)$$

where superscript T stands for the transposing, and write the Lagrangian in the form

$$\begin{aligned} \mathcal{L} = & (\mathbf{x}_+^T \cdot \mathbf{A} \cdot \mathbf{x}_+) + 2(\mathbf{x}_+^T \cdot \mathbf{B} \cdot \mathbf{y}_+) + (\mathbf{y}_+^T \cdot \mathbf{C} \cdot \mathbf{y}_+) + 2(\mathbf{x}_+^T \cdot \mathbf{D} \cdot \mathbf{y}_-) \\ & - (\mathbf{x}_-^T \cdot \mathbf{A} \cdot \mathbf{x}_-) - 2(\mathbf{x}_-^T \cdot \mathbf{B} \cdot \mathbf{y}_-) - (\mathbf{y}_-^T \cdot \mathbf{C} \cdot \mathbf{y}_-) - 2(\mathbf{x}_-^T \cdot \mathbf{D} \cdot \mathbf{y}_+), \end{aligned} \quad (31)$$

with operator matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} . Operator matrices \mathbf{A} and \mathbf{C} are symmetric

$$\mathbf{A}^T = \mathbf{A}, \quad \mathbf{C}^T = \mathbf{C}. \quad (32)$$

Especially, operator \mathbf{A} is the one appearing in the ideal MHD. Note that the symmetry relation (32) does not mean Hermitian, since the eigenvalue q is complex. The derivative in ψ direction applies only for ξ_ψ , A_s , and A_b , for which the boundary conditions are specified. Then equations are written in the form

$$\left. \begin{aligned} \mathbf{A} \cdot \mathbf{x}_+ + \mathbf{B} \cdot \mathbf{y}_+ + \mathbf{D} \cdot \mathbf{y}_- &= 0 \\ \mathbf{B}^T \cdot \mathbf{x}_+ + \mathbf{C} \cdot \mathbf{y}_+ - \mathbf{D}^T \cdot \mathbf{x}_- &= 0 \\ \mathbf{A} \cdot \mathbf{x}_- + \mathbf{B} \cdot \mathbf{y}_- + \mathbf{D} \cdot \mathbf{y}_+ &= 0 \\ \mathbf{B}^T \cdot \mathbf{x}_- + \mathbf{C} \cdot \mathbf{y}_- - \mathbf{D}^T \cdot \mathbf{x}_+ &= 0 \end{aligned} \right\} \quad (33)$$

For the physical variables, we have

$$\left. \begin{aligned} \mathbf{A} \cdot \mathbf{x} + (\mathbf{B} + \mathbf{D}) \cdot \mathbf{y} &= 0 \\ (\mathbf{B}^T - \mathbf{D}^T) \cdot \mathbf{x} + \mathbf{C} \cdot \mathbf{y} &= 0 \end{aligned} \right\} \quad (34)$$

Thus, the symmetric and anti-symmetric operators appear in the cross term between \mathbf{x} and \mathbf{y} .

In the equilibrium such that any scalar quantity satisfies the symmetry relation $u(\theta, \phi) = u(-\theta, -\phi)$, i.e. Fourier expanded in cosine series with respect to θ and ϕ , variables are divided into two classes according their parity: one (C) is Fourier expanded in cosine series and the other (S) is expanded into sine series. The structure of the Lagrangian shows that the variables $(\xi_\psi, \varpi, A_s, A_b)$ have the same parity and the variables (A_ψ, ξ_s, ξ_b) have the other parity. However, this fact does not mean the quantities with same parity have the same phase, because Fourier coefficients are not real. The variables ϖ, ξ_s, ξ_b , and A_ψ can be eliminated algebraically from the Fourier expanded equations and the 3 second order differential equations for ξ_ψ, A_s , and A_b are obtained.

In conclusion, the use of the Lagrangian simplifies the calculation of the coefficients in the linearized MHD equations.

References

- [1] C.Z.Chen and M.S.Chance: Phys. of Fluids 29 (1986) 3695.
- [2] K.Appert, T.Gruber and J.Vaclavik: Phys. of Fluids 17 (1974) 1471.
- [3] J.M.Greene, and J.L.Johnson: Plasma Phys. 10 (1968) 729.
- [4] I.B.Bernstein, E.A.Frieman, M.D.Kruskal, and R.M.Kulsrud: Proc. Roy. Soc.(London) A244 (1958) 17.
- [5] C.Z.Chen and M.S.Chance: J.Comp.Phys. 71 (1987) 124.
- [6] T.R.Harley, C.Z.Chen,S.C.Jardin: PPPL-2737 (1991), submitted to J.Comp.Phys.

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