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Y. Kondoh and T. Sato

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# Thought Analysis on Self-Organization Theories of MHD Plasma

Yoshiomi KONDOH and Tetsuya SATO[†]

*Department of Electronic Engineering, Gunma University,*

*Kiryu, Gunma 376, Japan*

[†]*National Institute for Fusion Science, Nagoya 464-01, Japan*

**Abstract**

A thought analysis on the self-organization theories of dissipative MHD plasmas is presented to lead to three groups of theories that lead to the same relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$, in order to find an essential physical picture embedded in the self-organization phenomena due to nonlinear and dissipative processes. The self-organized relaxed state due to the dissipation by the Ohm loss is shown to be formulated generally as the state such that yields the minimum dissipation rate of global auto- and/or cross-correlations between two quantities in $\mathbf{j}$, $\mathbf{B}$, and $\mathbf{A}$ for their own instantaneous values of the global correlations.

**Keywords** : thought analysis, self-organization, magnetic energy relaxation, minimum dissipation state, self-similar decay phase, resistive MHD plasma, variational calculus, RFP

§ 1. Inroduction

The energy-relaxation theory of the resistive magnetohydrodynamic (MHD) plasma by J. B. Taylor [1,2] has been applied successfully to the relaxation phenomena of magnetically confined plasmas in toroidal devices such as for the reversed field pinch (RFP) experiment [3-8] and for the spheromak experiment [9-11]. J. B. Taylor has clarified from his idealized theory that the equation of the force-free field, $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with a constant profile of $\lambda$, represents "the minimum-energy state" which is called "the fully relaxed state", by introducing the conjecture on "the time invariant" of "the total helicity". For a cylindrical plasma he derived the well-known $\beta = 0$ Bessel function model (BFM) configuration from the equation [1,2]. The gross features of the relaxed plasmas in the experiments of the RFP and the spheromak are well described by the force-free field equation, $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with the constant profile of $\lambda$ [1-11]. The detailed experimental measurements show, however, that the relaxed states of plasmas deviate somewhat from the fully relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$, and have finite pressure gradient and nonuniform profile of $\lambda$ [3-11]. This deviation is considered to result from the high resistive boundary plasmas [2]. Taking account of the experimental RFP plasma which has the finite pressure gradient and satisfies the boundary condition that the current density $\mathbf{j} = \mathbf{0}$ at the wall, one of the authors (Y.K.) had introduced the partially relaxed state model (PRSM) [12-14] and developed numerical codes for the RFP equilibria and for the mode transition point of the relaxed states by introducing the energy principle with partial loss of helicity in the boundary region [15-20]. It has been shown that the experimental data of the RFP plasma in the TPE-1RM15 device [7,8] are well fitted by the numerical results of the PRSM [14,20].

On the other hand, the self-organization process in the resistive MHD plasma has been investigated in details with use of the three-dimensional MHD simulations by one of the authors (T.S.) and his co-workers [21-26]. R. Horiuchi and T. Sato have demonstrated by the three-dimensional simulation that there exists some energy relaxation process where the total helicity is approximately conserved during the rapid dissipation of the magnetic energy [22,23]. They also have demonstrated by the three-dimensional simulation that fairly high dissipation of the total helicity, up to about 20 percent in one example, accompanies the magnetic energy relaxation which leads to the force-free field of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ ( see Fig.3 in ref.[24] ). We notice clearly from this result of their three-dimensional simulation shown in Fig.3 in ref.[24] that the total helicity is no longer the time invariant during the magnetic energy ralaxation and the system still relaxes to the force-free field of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$. In other words, the results of the three-dimensional MHD simulation mentioned above make it clear that the conjecture of the total helicity invariant is not the essential physical condition necessary for the realization of the relaxed state $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ in the energy relaxation process. Using a variational method, they also have shown in ref.[24] that the relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ is obtained as the maximum entropy state under the constraints of the total helicity invariant, the total energy- and the total mass conservations. Using the so-called reciprocity of the variational calculus, one of the author (Y.K.) has shown in ref.[16] that the maximum entropy state with the global constraints on the helicity, the energy and the mass is equivalent to the minimum energy state with the global constraints on the helicity, the entropy, and the mass, and both of them lead to the same equilibrium equation ( if there is no partial loss, the equilibrium equation becomes the force-free field of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ ).

Some other energy relaxation theories or self-organization theories have been also reported, as the modifications of the theory by Taylor [1,2] for the explanation of experimental plasmas, for example, by using infinit set of global invariant concerning with helicity [27] or by using the minimun dissipation rate or the minimum entropy production rate under the constraint of the constant time-averaged rate of supply of helicity [28] or the assumption of the total helicity invariant [29]. All of these theories mention above are based essentially on the concept of "helicity", and lead to the same force-free field of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$.

On the other hand, recent experimental data have clarified that in the ZP-2 device [30], which is a simple toroidal $Z$ pinch without toroidal coiles for the toroidal flux and therefore has no initial total helicity, there still appears the relaxation of the field configuration to lead to the spontaneous generation of the toroidal field within a few tens of $\mu s$ in the produced toroidal plasma [30-32]. The relaxed state of the plasma becomes to have finite total helicity and to be close to the state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ that cannot be determined by the initial total helicity [30-32], contrary to the theory by Taylor [1,2]. The total helicity is not the invariant during the magnetic energy relaxation in the experiment of the ZP-2 device, just the same as the case of the three-dimensional simulation shown in Fig.3 in ref.[24] mentioned above. Another important point to consider is that in the MHD simulations reported in refs.[21]-[26], they do not solve any equations for hilicity but they do only solve equations of mass, momentum, and energy ( or equivalently the entropy equation ) together with Maxwell's equations and Ohm's law, where $\mu_o \, \mathbf{j} = \nabla \times \mathbf{B}$ is used by neglecting the displacement current. This fact indicates that the quantity of helicity does not dominate the process of relaxation but is used for a kind of classification or labeling to describe some part of the process. The both results by the experiments in the ZP-2 device and the MHD simulations mentioned above suggest that we need a new theory for obtaining the relaxed state without using "helicity invariant for time".

The set of general thoughts to find internal structures of the self-organized relaxed states without using any invariant for time has been reported by one of the authors (Y.K.), using a thought analysis on relaxation due to nonlinear processes with dissipation itself [33]. Here, the word "thought analysis" means that we investigate logical structures, ideas or thoughts used in the objects being studied, and try to find some key elements for improvement and/or some other new thoughts which involve generality, by using such as a kind of thought experiments and mathematically reversible processes [33-35]. The applications of the set of general thoughts to the energy relaxation of the MHD plasma, the incompressible viscous fluids, and the incompressible viscous MHD fluids have been shown in refs.[33] and [36]. A detailed description of the thought analysis on the self-organization due to nonlinear processes with dissipation is presented in ref.[37] to clarify that the internal structures of the self-organized relaxed states are such structures that are hardest to change themselves in their time evolutions and therefore followed by the self-similar decay phase without significant change of their internal distributions. Detailed descriptions of the three applications of the set of general thoughts to the resistive MHD plasmas, the incompressible viscous fluids, and the incompressible MHD fluids are also presented in ref.[37] to lead to the internal spatial structures of the self-organized relaxed states and their self-similar decay phases, together with some examples of axisymmetric plasmas such as the diffused $Z$ pinch plasma, the screw pinch plasma, the RFP plasma in the cylindrical approximation, and the field reversal configuration (FRC) plasma. Remarkable points of the applied theory of the set of general thoughts, summarized in ref.[37], are the followings:

(a) The relaxed state of the force-free field of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ and the mode transition condition are derived generally as the low $\beta$ plasma limit of the self-organized relaxed state from the set of general thoughts without using "helicity" and "invariant", whose concepts are essential in the theory by Taylor [1,2].

(b) The applied theory permits the quasi-steady energy flow through the boundary surface, as is indeed the case in most experiments, and leads to a more general relaxed state of $2\eta\mathbf{j} = \alpha\mathbf{A}$ for plasmas having spatially dependent resistivity $\eta$. This result leads directly to the experimental fact of $\mathbf{j} = 0$ near the wall, as is indeed the case in all experiments where $\eta$ goes up to infinity near the boundary wall.

(c) The self-organized relaxed states are proved directly to be followed by the self-similar decay phase without significant change of their own spatial distributions.

(d) The self-organized relaxed states of flow and/or magnetic field after turbulent phases with dissipation in the incompressible viscous fluid and/or in the incompressible viscous MHD fluid are also derived and proved to be followed by the self-similar decay phase.

We now confront the fact that there exist several different theories, all of which lead to the same relaxed state of $\nabla \times \mathbf{B} = \lambda\mathbf{B}$ as a branch, as mentioned above. This fact itself suggests that there may exist some other theories that lead to the same relaxed state of $\nabla \times \mathbf{B} = \lambda\mathbf{B}$. It is interesting to investigate these theories themselves to find other possible theories leading to the same relaxed state. Overall investigation on those possible theories on the self-organization would give us some essential physical picture on the self-organization phenomena.

In this paper, a thought analysis on the self-organization theories ( or the energy-relaxation theories ) for the dissipative MHD plasma is presented to lead to three groups of the self-organization theories that lead to the same self-organized relaxed state of $\nabla \times \mathbf{B} = \lambda\mathbf{B}$. We also compare and investigate the obtained groups of the self-organization theories to find their common origin and an essential physical picture on the self-organization phenomena. In Section 2, a thought analysis on the self-organization theories ( or the energy-relaxation theories ) is presented to find other possible theories that lead to the same relaxed state. The first group of the self-organization theories connected to the theory by Taylor [1,2] is shown in the subsection 2.1. The second group connected to the theory by Kondoh [33,36,37] is presented in the subsection 2.2, and the third group connected to the theory by T. Kato and T. Furusawa [29] is in the sebsection 2.3. The comparison among the obtained groups of theories and some discussion are presented in Section 3.

## § 2. Thought Analysis on Self – Organization Theory

We try to analyze here the logical and the mathematical structures of the self-organization theory ( or the energy-relaxation theory ) in order to find groups of thoughts for self-organization theories that leads to the same self-organized relaxed state. For simplicity, it is assumed here that the plasma internal energy is negligible compared to the magnetic energy, as is indeed the case in most experiments. In other word, we are dealing with a self-organization theory due to the magnetic energy-relaxation. The set of the physical quantities to be used in the self-organization theory due to the magnetic energy-relaxation is

$$\{\, \mathbf{A},\ \mathbf{B},\ \mathbf{j},\ \mathbf{E},\ \mathbf{u},\ w_m,\ \eta \,\}, \tag{1}$$

where $\mathbf{A}(t,\mathbf{x})$, $\mathbf{B}(t,\mathbf{x})$, $\mathbf{j}(t,\mathbf{x})$, $\mathbf{E}(t,\mathbf{x})$, $\mathbf{u}(t,\mathbf{x})$, $w_m(t,\mathbf{x})$, and $\eta(t,\mathbf{x})$ are the vector potential, the magnetic field, the current density, the electric field, the fluid velocity of plasma, the magnetic energy density, and the resistivity, respectively, and $t$ and $\mathbf{x}$ denote the time and the spatial coordinates, respectively. The resistivity $\eta$ is assumed to be constant and spatially uniform, for simplicity for a while. The relations among the physical quantities to be used are the followings:

$$\mathbf{B} = \nabla \times \mathbf{A}\,. \tag{2}$$

$$\mu_o\, \mathbf{j} = \nabla \times \mathbf{B}\,. \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{4}$$

$$\eta \mathbf{j} = \mathbf{E} + \mathbf{u} \times \mathbf{B} \quad \text{Ohm's law} . \tag{5}$$

$$w_m = \frac{B^2}{2\mu_o} . \tag{6}$$

Integrating $w_m$ over the volume of the system, we obtain the total magnetic energy, $W_m$, as the global quantities, as follows:

$$W_m = \int \frac{B^2}{2\mu_o} \mathrm{d}v . \tag{7}$$

The total helicity, $K$, is defined by

$$K = \int \frac{\mathbf{A} \cdot \mathbf{B}}{2\mu_o} \mathrm{d}v . \tag{8}$$

Using eqs.(2) and (3) ( Maxwell's equations ), the vector formula of $\nabla \cdot (\mathrm{a} \times \mathrm{b}) = \mathrm{b} \cdot \nabla \times \mathrm{a}$ - $\mathrm{a} \cdot \nabla \times \mathrm{b}$, Ohm's law of eq.(5), and the Gauss theorem, we obtain the time derivatives of $W_m$ and $K$ as follows,

$$\frac{\mathrm{d}W_m}{\mathrm{d}t} = -\int \{ \eta \mathbf{j} \cdot \mathbf{j} + (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{u} \} \mathrm{d}v - \oint ( \mathbf{E} \times \mathbf{H} ) \cdot \mathrm{ds}, \tag{9}$$

$$\frac{\mathrm{d}K}{\mathrm{d}t} = -\frac{1}{\mu_o} \int \eta \mathbf{j} \cdot \mathbf{B} dv - \frac{1}{2\mu_o} \oint (\mathbf{E} \times \mathbf{A} + \phi \mathbf{B}) \cdot \mathrm{ds}, \tag{10}$$

where $\oint$ denotes the surface integral over the boundary, and $\phi$ is the scalar potential.

In Ref.[38], H. Ito developed a theory of helicities in MHD by using the terminologies of "autohelicity" and "crosshelicity" for $K$ and $W_m$. The quantities of $W_m$ and $K$ are in other words, however, the "global autocorrelation" and the "global crosscorrelation", respectively, with respect to **B** and **A** [25]. The first term of the right-hand side of eq.(9) is the dissipation term of the magnetic energy and also, in other words, the dissipation term of the global autocorrelation with respect to **B**.

In the same way, the first term of the right-hand side of eq.(10) is the dissipation term of the total helicity and also, in other words, the dissipation term of the global crosscorrelation with respect to **B** and **A**.

### 2.1 *First Group of Self – Organization Theory*

The logical structure of Taylor's theory on the energy-relaxation of the MHD plasma consists of the following main set of three thoughts, { [A-1], [A-2], [A-3] } [1,2]: Here, the boundary is assumed to be ideally conducting wall, for simplicity.

[A-1] When the resistivity $\eta$ of plasma is negligibly small or zero, then $\mathrm{d}W_m/\mathrm{d}t = 0$ and $\mathrm{d}K/\mathrm{d}t = 0$, as is seen from eqs.(9) and (10), and therefore both the total mangetic energy $W_m$ and the total helicity $K$ are the global invariants for motions of plasmas in the system of the ideal MHD plasma.

[A-2] When we introduce very small but finite resistivity into the plasma, then magnetic energy dissipation and reconnection of magnetic field lines would take place, and therefore $W_m$ is no longer the global invariant. It is, however, considered that the total helicity $K$ would be conserved during the field reconnection. ( This is known as "Taylor's conjecture" on the total helicity for plasmas with small but finite resistivity.)

[A-3] The MHD plasmas with the small but finite resistivity would relax to the state with the minimum value of $W_m$ under this global invariant of $K$, which is expressed by the following form;

$$\textit{the minimum } W_m \textit{ state} \text{ with } K \;=\; K_B\;, \tag{11}$$

where $K_B$ is the value of $K$ measured at the time just "before the relaxation phase". The set of three thoughts, { [A-1], [A-2], [A-3] } is understood usually as a theory by "the energy principle" or "the variational principle". Since $K$ is assumed to be the global invariant during the relaxation phase, the value of $K$ measured "at the time of the relaxed state", $K_R$, just "after the relaxation phase" is assumed to be $K_B$, i.e. $K_R = K_B$. Here, the word "relaxation phase" means that in this phase some nonlinear processes take place to change the internal spatial structure so drastically that the value of $W_m$ decreases very rapidly. This relaxation phase is assumed to lead the system finaly to the relaxed state with a peculiar internal structure that yields the minimum value of $W_m$. The value of $W_m$ takes the minimum value under the condition of $K = K_B$ at the time $t = t_R$ when the relaxed state is realized.

Using the variational technique with respect to the spatial variable $\mathbf{x}$, the mathematical expression for eq.(11) of the thought [A-3] is written in the following forms,

$$\delta F = 0, \tag{12}$$

$$\delta^2 F > 0, \tag{13}$$

where $F$ is the functional defined by $F = W_m$ - $\lambda K$; $\delta F$ and $\delta^2 F$ are the first and second variations of $F$; and $\lambda$ is the Lagrange multiplier. It should be emphasized here that the variations of $\mathbf{A}(t, \mathbf{x})$, $\mathbf{B}(t, \mathbf{x})$ and $\mathbf{j}(t, \mathbf{x})$ are taken only with respect to the spatial vairables $\mathbf{x}$ and the time is fixed at $t = t_R$, like as $\delta\mathbf{A}(\mathbf{x})$, $\delta\mathbf{B}(\mathbf{x})$ and $\delta\mathbf{j}(\mathbf{x})$, in the following calculations of the variational technique.

Substituting $W_m$ and $K$ in eqs.(7) and (8) into eqs.(12) and (13), and using eq.(2), $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem again, we obtain the followings,

$$\delta F = \frac{1}{\mu_o} \int \delta\mathbf{A} \cdot (\nabla \times \mathbf{B} - \lambda \mathbf{B}) \mathrm{d}v = 0 , \tag{14}$$

$$\delta^2 F = \frac{1}{\mu_o} \int \delta\mathbf{A} \cdot (\nabla \times \delta\mathbf{B} - \lambda \delta\mathbf{B}) \mathrm{d}v > 0, \tag{15}$$

where the boundary condition of $\delta\mathbf{A} \times \mathrm{d}\mathbf{s} = 0$ for the ideally conducting wall is used and no singular surface of $\delta\mathbf{B}$ in the volume of the system is assumed [17,18]. We then obtain the so-called Taylor state from eq.(14) as the Euler-Lagrange equation for arbitrary variations of $\delta\mathbf{A}$ as follows,

$$\nabla\times\mathbf{B} = \lambda\mathbf{B}\,. \tag{16}$$

Using the associated eigenvalue problem for the critical perturbation $\delta\mathbf{B}$ that makes $\delta^2 F_T$ in eq.(15) become zero, we can obtain the mode transition condition of the relaxed state, for example from the cylindrical mode to the mixed helical one in the cylindrical plasma [1,2,18,20,37].

On the other hand, the three-dimensional MHD simulations shows clearly that, while there exists some magnetic energy relaxation process which keeps the total helicity nearly constant, fairly high dissipation of the total helicity, up to about 20 percent in one example of Fig.3 in ref.[24], accompanies the magnetic energy relaxation process which still leads to the force-free field of $\nabla\times\mathbf{B} = \lambda\mathbf{B}$. We notice clearly from the results of the three-dimensional simulation shown in Fig.3 in ref.[24] that there exists actually the magnetic energy relaxation process where the total helicity is no longer the time invariant and the system still relaxes to the force-free field of $\nabla\times\mathbf{B} = \lambda\mathbf{B}$. In other words, the results of the three-dimensional MHD simulation mentioned above make it clear that the conjecture of the total helicity invariant is not the essential physical condition necessary for the realization of the relaxed state $\nabla\times\mathbf{B} = \lambda\mathbf{B}$ in the energy relaxation processes. Taking account of this important fact, we have to reconstruct the thoughts, { [A-1], [A-2], [A-3] }, of the relaxation theory by Taylor. When we investigate eqs.(9) and (10) more closely, we notice that the two equations of (9) and (10) indicate only that the dissipation rate of $W_m$ is always greater than that of $K$ which depends strongly on relative direction between $\mathbf{B}$ and $\mathbf{j}$. This fact means that the system of interest will tend to relax until the time when the magnetic energy $W_m$ becomes the minimum for the instantaneously containing value of the total helicity $K$ at each time, even if the total helicity dissipates finitely during the relaxation process. In other words, the system will relax to the state with the minimum value of { $W_m$ normalized by instantaneous $K$ }, as is demonstrated by the curve of the energy-to-helicity ratio in Fig.3 of ref.[24]. We therefore come to the following thought [B] to find the relaxed state with the minimum value of $W_m$, which is available for both cases with and without the total helicity conservation during the magnetic energy relaxation processes:

[B] The relaxation phase continues itself until and terminates itself at the time when the field distributions of $\mathbf{A}(t, \mathrm{x})$, $\mathbf{B}(t, \mathrm{x})$ and $\mathbf{j}(t, \mathrm{x})$ have reached the peculiar spatial structures such that yield the minimum value of $W_m$ for the instantaneous amount of the containing total helicity $K$ at that instant. The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the helicity $K = K_R$ and the minimum value of $W_m$, that is expressed by the following form;

$$\textit{the minimum } W_m \textit{ state} \text{ with } K = K_R \,, \tag{17}$$

where $K_R$ is the value of $K$ measured "at the time of the relaxed state" just after the relaxation phase. Here, "$K = K_R$" is the necessary condition which must be satisfied by the internal structure of the relaxed state because of the measured value, and becomes "the global constraint" for "finding out the objective internal structure of the relaxed state from the set of various distributions". The mathematical expressions of eq.(17) in the thought [B] by the variational technique are eqs.(12) and (13) themselves, and they lead us to the same processes from eq.(14) to eq.(16) and also to the same condition for the mode transition point of the relaxed state. It is clear that $K$ need not be the invariant for time in eq.(17) in the thought [B]. This is because, as mentioned above, that the two eqs.(9) and (10) do indicate only that the resistive dissipation rate of $W_m$ is greater than that of $K$ and both of them are equivalent at the relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ . We should notice here that the theory by the thought [B] is no longer "the energy principle" or "the variational principle" based on "the invariant" for time, and it still leads to the same relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ . Essential difference of the relaxation theory by the thought [B] from Taylor's theory by the thoughs { [A-1], [A-2], [A-3] } is that the relaxation process in the thought [B] is recognized clearly to have no concern with the total helicity conservation which Taylor's theory bases on. Even if the relaxation phase of interest contains a finite dissipation of $K$ itself, like as $K_R < K_B$, as is indeed the case in most of all experiments and also in the three-dimensional MHD simulations like as shown in Fig.3 of Ref.[24] and therefore $K$ is no longer the invariant even in the approximate meaning, the internal structure derived by the thought [B] yields the correct relaxed profile of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ for the relaxed state which has the value of $K_R$.

Using the so-called reciprocity of the variational calculus, we obtain the following thought which is equivalent to eq.(17) of the thought [B],

[C] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the magnetic energy $W_m = W_R$ and the maximum value of $K$, that is expressed by the following form;

$$\textit{the maximum } K \textit{ state} \text{ with } W_m = W_R \,, \tag{18}$$

where $W_R$ is the value of $W_m$ measured at the time of the relaxed state just after the relaxation phase, and $W_m$ is, of course, not an invariant for time. The mathematical expression for eq.(18) by the variational technique is written in the following forms, which are equivalent to eqs.(12) and (13),

$$\delta F = 0, \tag{19}$$

$$\delta^2 F < 0, \tag{20}$$

where $F$ is now the functional defined by $F = K - (1/\lambda)W_m$; $\delta F$ and $\delta^2 F$ are the first and second variations of $F$; and $1/\lambda$ is the positive Lagrange multiplier. The mathematical expressions of eq.(18) in the thought [C] lead us to the same processes from eq.(14) to eq.(16) and also to the same condition for the mode transition point of the relaxed state.

Since we have used reversible mathematical processes from eq.(11) to eq.(16), we can follow back from eq.(16) to eq.(11) in the theory by the thought [B]. Using this property of the logical and the mathematical structures used for obtaining the relaxed state of eq.(16), and reconstructing eqs.(14) and (15) themselves, we may find a group of thoughts for relaxation theories which lead to the same relaxed state of eq.(16) and the same mode transition condition of the relaxed state in the following way: Changing the variation of $\delta\mathbf{A}$ in eqs.(14) and (15) to a variation of a more general quantity $\mathbf{q}$, as $\delta\mathbf{q}$, we may have the following expressions for the group of relaxation theories that lead to eq.(16) as the Euler-Lagrange equation from the volume integral term for arbitrary variations of $\delta\mathbf{q}$ and also lead to the mode transition condition of the relaxed state,

$$\delta F[\delta\mathbf{q}] = \frac{1}{\mu_o}\int \delta\mathbf{q}\cdot(\nabla\times\mathbf{B} - \lambda\mathbf{B})\mathrm{d}v = 0\,, \tag{21}$$

$$\delta^2 F[\delta\mathbf{q}] = \frac{1}{\mu_o}\int \delta\mathbf{q}\cdot(\nabla\times\delta\mathbf{B} - \lambda\delta\mathbf{B})\mathrm{d}v > 0\,. \tag{22}$$

At first, we adopt $\delta\mathbf{B}$ for $\delta\mathbf{q}$ in eqs.(21) and (22), and then we obtain the followings,

$$\delta F[\delta\mathbf{B}] = \frac{1}{\mu_o}\int \delta\mathbf{B}\cdot(\nabla\times\mathbf{B} - \lambda\mathbf{B})\mathrm{d}v = 0\,. \tag{23}$$

$$\delta^2 F[\delta\mathbf{B}] = \frac{1}{\mu_o}\int \delta\mathbf{B}\cdot(\nabla\times\delta\mathbf{B} - \lambda\delta\mathbf{B})\mathrm{d}v > 0\,. \tag{24}$$

Using eq.(3), $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the followings from eq.(23),

$$\delta F[\delta \mathbf{B}] = \int \{ \frac{1}{2}(\delta \mathbf{j} \cdot \mathbf{B} + \mathbf{j} \cdot \delta \mathbf{B}) - \frac{\lambda}{\mu_o} \delta \mathbf{B} \cdot \mathbf{B} \} \mathrm{d}v + \frac{1}{\mu_o} \oint (\mathbf{B} \times \delta \mathbf{B}) \cdot \mathrm{d}\mathbf{s} = 0 . \quad (25)$$

We see from comparison between the volume integral term of eq.(25) and eqs.(10) and (7) that $F$ in eq.(25) is the fuctional defined by $F$ = - $\mu_o(\mathrm{d}K/\mathrm{d}t)/2\eta$ - $\lambda W_m$ for the case of the ideal conducting wall. Since the first term of $\mathrm{d}K/\mathrm{d}t$ in eq.(10) is the dissipation rate of the total helicity $K$ by the resistivity $\eta$ and has the negative value, we therefore come to the following another thought [D] for the relaxed state that leads to eq.(16):

[D] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structure yields the minimum dissipation rate of $K$ with $W_m = W_R$, which is expressed by

$$\textit{the maximum } \frac{\mathrm{d}K}{\mathrm{d}t} \textit{ state} \text{ with } W_m = W_R , \quad (26)$$

where the functional $F$ is defined by $F$ = - $\mathrm{d}K/\mathrm{d}t$ - $\alpha W_m$, and $\lambda$ in eq.(16) is given as $\lambda = 2\eta\alpha/\mu_o$ with use of another Lagrange multiplier $\alpha$ to lead to eq.(23). Here, $W_m$ is, of course, not the invariant for time. The mathematical expressions of eq.(26) in the thought [D] lead us to eqs.(23) and (24), and therefore we obtain eq.(16) and also the same condition for the mode transition point of the relaxed state. The reciprocity of the variational calculus gives us the following thought [E] equivalent to the thought [D] given by eq.(26):

[E] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the minimum value of $W_m$ with $\mathrm{d}K/\mathrm{d}t = \mathrm{d}_t K_R$, which is expressed by

$$\textit{the minimum } W_m \textit{ state} \text{ with } \frac{\mathrm{d}K}{\mathrm{d}t} = \mathrm{d}_t K_R\ , \tag{27}$$

where $\mathrm{d}_t K_R$ is the value of $\mathrm{d}K/\mathrm{d}t$ measured at the time of the relaxed state just after the relaxation phase, and $\mathrm{d}K/\mathrm{d}t$ is, of course, not the invariant for time. The mathematical expressions of eq.(27) in the thought [E] lead us to eqs.(23) and (24), and therefore we obtain eq.(16) and also the same condition for the mode transition point of the relaxed state.

Next, we adopt $2\mu_o\eta\delta\mathbf{j}$ for $\delta\mathbf{q}$ in eqs.(21) and (22), and we obtain the followings in this case,

$$\delta F[\delta\mathbf{j}] = 2\eta \int \delta\mathbf{j}\cdot(\nabla\times\mathbf{B}-\lambda\mathbf{B})\mathrm{d}v = 0\,. \tag{28}$$

$$\delta^2 F[\delta\mathbf{j}] = 2\eta \int \delta\mathbf{j}\cdot(\nabla\times\delta\mathbf{B}-\lambda\delta\mathbf{B})\mathrm{d}v > 0\,. \tag{29}$$

Using the same procedure used from eq.(23) to eq.(26), and refering to eqs.(9) and (10), we come to the following another thought [F] that leads to eq.(16):

[F] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structures yield the minimum value of $|\ \mathrm{d}W_m/\mathrm{d}t\ |$ with $\mathrm{d}K/\mathrm{d}t = \mathrm{d}_t K_R$, which is expressed by

$$\textit{the minimum } \left|\frac{\mathrm{d}W_m}{\mathrm{d}t}\right| \textit{ state} \text{ with } \frac{\mathrm{d}K}{\mathrm{d}t} = \mathrm{d}_t K_R\,. \tag{30}$$

Here, the functional is defined by $F = |\ \mathrm{d}W_m/\mathrm{d}t\ | + \alpha\mathrm{d}K/\mathrm{d}t$, and $\lambda$ in eq.(16) is given as $\lambda = \alpha/\mu_o$ with use of another Lagrange multiplier $\alpha$ to lead to eq.(28). The mathematical expressions of eq.(30) in the thought [F] lead us to eqs.(28) and (29), and therefore we obtain eq.(16) and also the same condition for the mode transition point of the relaxed state. It is interesting to note here that the theory by the thought [F] is equivalent to the theory by Montgomery and Phillips [28], where they deal with the state with the minimum dissipation rates under the constraint of the constant time-averaged rate of supply of helicity. The reciprocity of the variational calculus gives us the following thought [G] equivalent to the thought [F] given by eq.(30):

[G] The relaxed state of the MHD plasma is the state whose internal structures yield the maximum value of $|\, \mathrm{d}K/\mathrm{d}t \,|$ with $|\, \mathrm{d}W_m/\mathrm{d}t \,| = \mathrm{d}_t W_{mR}$, which is expressed by

$$\textit{the maximum } \frac{\mathrm{d}K}{\mathrm{d}t} \textit{ state } \text{ with } \left| \frac{\mathrm{d}W_m}{\mathrm{d}t} \right| = |\, \mathrm{d}_t W_{mR} \,| , \tag{31}$$

where $|\, \mathrm{d}_t W_{mR} \,|$ is the value of $|\, \mathrm{d}W_m/\mathrm{d}t \,|$ measured at the time of the relaxed state just after the relaxation phase, and $|\, \mathrm{d}W_m/\mathrm{d}t \,|$ is, of course, not the invariant for time. The mathematical expressions of eq.(31) in the thought [G] lead us to eqs.(28) and (29), and therefore we obtain eq.(16) and also the same condition for the mode transition point of the relaxed state.

We have obtained a group of thoughts, { from [B] to [G] }, that lead to the same relaxed state of eq.(16) and the mode transition condition of the relaxed state, starting from Taylor's theory. The group of thoughts, { from [B] to [G] }, has no concern with the conjecture of the total helicity conservation in their physical picture for the relaxation process. The three thoughts of [B], [E] and [F] may be acceptable as the theories that can be derived from the relation between eq.(9) and eq.(10) in order to find the internal spatial structure of the self-organized relaxed state. The thought of [F] is equivalent to the thought used in the theory by Montgomery and Phillips [28] dealing with the state with the minimum dissipation rate under the constraint of the constant time-averaged rate of supply of helicity. The common physical laws used for the derivation of eq.(16) from the group of thoughts { from [A-3] to [G] } are eqs.(2) and (3), i. e. $\{\, \mathbf{B} = \nabla \times \mathbf{A},\ \ \mu_o \mathbf{j} = \nabla \times \mathbf{B} \,\}$. It is interesting to note here that the two physical laws of eqs.(4) and (5), i. e. $\{\, \nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t,\ \ \eta \mathbf{j} = \mathbf{E} + \mathbf{u} \times \mathbf{B} \,\}$, are not used directly for the derivation of eq.(16) from the group of thoughts { from [A-3] to [G] }.

2.2 *Second Group of Self – Organization Theory*

We now proceed to the theory by Kondoh [33,36,37] that has quite different logical structure from Taylor's theory, and still leads to the same relaxed state of eq.(16) as the low $\beta$ plasma limit, without using the concepts "helicity" and "invariant".

In the thought analysis on "relaxation due to nonlinear processes with dissipation" [37], we consider a general nonlinear dynamical system with dissipation that consists of quantities $\mathrm{q}(t, \mathrm{x})$, and investigate the internal spatial structure of the self-organized quasi-steady relaxed state by using a kind of thought experiments. Here, $t$ is time, $\mathrm{x}$ denotes m-dimensional space variables, and $\mathrm{q}$ represents a set of physical quantities having $n$ elements, some of which are vectors such as $\mathbf{B}$ and $\mathbf{j}$, and others are scalars such as the mass density, the energy density, the specific entropy and so on. Time evolutions of $\mathrm{q}$ are given by definite equations such as the equations of mass, momentum, and energy, and the Maxwell equations or the laws ruling $\mathrm{q}(t, \mathrm{x})$ of the nonlinear dynamical system in general sense. Integrating one element, $w$, such as the energy density in $\mathrm{q}$ over the space volume, we can define a global quantity of $W(t)$, such as the energy of the system, as $W(t) = \int w(\mathrm{q})\, \mathrm{dx}$ . We can recognize from this definition of $W(t)$ that the values of both $W(t)$ and its time derivative $\mathrm{d}W/\mathrm{d}t$ depend essentially on the internal structure [ i.e. the internal distributions of $\mathrm{q}(t, \mathrm{x})$ ] of the dynamical system. The value of $\mathrm{d}W/\mathrm{d}t$ represents the loss rate or the dissipation rate with respect to $W$ of the system. We can also understand that the relation between $\mathrm{d}W/\mathrm{d}t$ and $W(t)$ is determined essentially by the laws ruling $\mathrm{q}(t, \mathrm{x})$ of the nonlinear dynamical system with dissipation.

The fact that the nonlinear dynamical system of interest is dissipative with respect to $W$ means that the system is an open system with respect to $W$. If the internal spatial distribution of the system is unstable against keeping or sustaining the instantaneous amount of containing quantity of $W$, drastic change of the internal spatial distribution will be induced and develop nonlinearly to release and dissipate $W$ rapidly, through driving elements of the system. This rapid decay phase of $W$ with the nonlinear drastic change of internal spatial structure is recognized and called as "the relaxation phase". The relaxation phase will continue itself until and terminate itself at the time when the internal spatial distribution has come to have a peculiar internal spatial structure such that yields the minimum dissipation rate of $W$ and therefore is hardest to change its own spatial distribution for the instantaneous amount of the containing $W$ at that instant. The state with this peculiar internal spatial structure yielding the minimum dissipation rate of $W$ for the instantanious amount of $W$ at the instant is recognized and called as "the self-organized relaxed state". These thoughts are summarized to the following set of general thoughts, { [I] and [II] }, to find internal structures of the self-organized quasi-steady relaxed state with respect to $W$ in the system, where the thought [I] is on the relaxation phase and the thought [II] is on the internal structure of the relaxed state [33,37]:

[I] In the relaxation phases, some nonlinear processes with dissipation take place to change the internal spatial structure of $\mathrm{q}(t, \mathrm{x})$ so drastically that the value of $W(t)$ decreases ( or increases ) very rapidly. The relaxation phase continues itself until and terminates itself at the time when the internal spatial structure has reached the peculiar spatial structure such that yields the minimum dissipation rate of $W$ and therefore is hardest to change its own internal spatial distribution for its own instantaneous amount of the containing quantity $W$ at that instant.

[II] The self-organized quasi-steady relaxed state just after each relaxation phase is the state with the minimum dissipation rate of $W$, whose peculiar internal spatial structure of $q(t_R, x)$ has

$$\textit{the minimum value of } \ |\, dW/dt \,| \ \text{ with } W \ = \ W_R \, , \tag{32}$$

where $t_R$ denotes the time when the self-organized relaxed state has realized after the relaxation phase of interest, and $W_R$ is the value of $W$ measured at the time of $t_R$. Here, "$W = W_R$" is the necessary condition that must be satisfied by the internal strauctures of the relaxed state because of the measured value and becomes "the global constraint" for "finding out the objective internal spatial structure of the relaxed state from the set of various distributions", and $W$ is, of course, not the invariant for time.

Using the variational technique with respect to the spatial variables x for $q(t_R, x)$, i.e. using the variations of $\delta q(x)$, in order to find out the objective internal spatial structure and the minimum value of $|dW/dt|$ at the time of the quasi-steady relaxed state, which is given by the two thoughts { [I], [II] } with eq.(32), we obtain the following mathematical expressions [33,37]:

$$\delta F \ = \ 0, \tag{33}$$

$$\delta^2 F \ > \ 0, \tag{34}$$

where $F$ is the functional defined by $F = |dW/dt| - \alpha W$; $\delta F$ and $\delta^2 F$ are the first and second variations of $F$; and $\alpha$ is the Lagrange multiplier. When the boundary values of $x_k$ component of some elements $q_j$ in $\mathbf{q}$ are given such as by the property of the given boundary materials and/or measurements at the relaxed state, then the boundary conditions of the variations $\delta q(x)$ are written as

$$\delta q_{jk} \ = \ 0 \quad \text{at the boundary} \, . \tag{35}$$

We should notice here that the present theory shown above is neither "the energy principle" nor "the variational principle" based on some "invariant for time".

Since the self-organized relaxed state has the peculiar internal spatial structure such that is hardest to change its own spatial distribution, the relaxed state should be followed by the self-similar decay phase without significant change of the spatial structure. We should bear in mind, however, that the dissipation and being open of the system with respect to $W$ will still lead to some gradual deviation from the self-similar decay. When we observe the time evolution of the system of interest during long time interval, we come to find that the system behaves as if it is repeatedly attracted to and trapped in the self-organized relaxed state where the system stays longest time during one cycle of the time evolution. In this meaning, the internal spatial distribution $\mathrm{q}(t_R, \mathrm{x})$ of the self-organized relaxed state is "the attractor of the dissipative structure" introduced by Prigogine [39,40]. The set of general thoughts { [I], [II] } with eqs.(32) - (35) is useful to find this attractor of the dissipative structure. In order to find the attractor $\mathrm{q}(t_R, \mathrm{x})$, we can use, for examples of the quantity $W$, the global autocorrelation $W_{jj}$ or the global crosscorrelation $W_{jk}$ among the elements $q_j$ in the set of quantities $\mathrm{q}(t, \mathrm{x})$, where $W_{ji}$ ( $i = j,\ k$ ) is defined as $W_{ji}(t) \ = \ \int q_j(t, \mathrm{x}) q_i(t, \mathrm{x})\ \mathrm{dx}$ . It is because that the dissipation rate $\mathrm{d}W_{ji}/\mathrm{d}t$ of the global correlations $W_{ji}$ has the minimum value at the self-organized relaxed state whose internal structure $\mathrm{q}(t_R, \mathrm{x})$ is hardest to change its own spatial distribution against the dissipation process.

When we use instantaneous value of $W$ at each time $t$ in the relaxation phase instead of $W_R$ in eq.(32), we can obtain a "calculated spatial distribution" such that yields the minimun dissipation rate of $W$ for the instantaneously containing $W$, by using the same variational technique with respect to the spatial variables $\mathrm{x}$ from eq.(33) to eq.(35). We denote here the "calculated spatial distribution" with the minimum dissipation rate as $\mathbf{q}^*(W, \mathbf{x})$, which is the function of $W$. We further define a kind of distance, $D$, of the temporal distribution $\mathbf{q}(t, \mathbf{x})$ of the system from $\mathbf{q}^*(W, \mathbf{x})$ as $D^2(t) \ = \ \int[\ q_j^*(W, \mathbf{x})\ \text{-}\ q_j(t, \mathbf{x})\ ]^2\ \mathrm{d}\mathbf{x}$ . Since the value of $W$ at the time of $t_R$ is $W_R$ and $\mathbf{q}(t_R, \mathbf{x}) = \mathbf{q}^*(W_R, \mathbf{x})$, therefore the distance $D$ at $t = t_R$ becomes zero, like as $D^2(t_R) \ = \ \int[\ q_j^*(W_R, \mathbf{x})\ \text{-}\ q_j(t_R, \mathbf{x})\ ]^2\ \mathrm{d}\mathbf{x} \ = 0$. Since the Euler-Lagrange equation obtained for $\mathbf{q}^*(W, \mathbf{x})$ is the same as that for $\mathbf{q}(t_R, \mathbf{x})$ by using the same variational calculus of eqs.(33)-(35), we may say as follows: $\mathbf{q}^*(W, \mathbf{x})$ is the "attractor" of the present system. The system of $\mathbf{q}(t, \mathbf{x})$ relaxes toward the attractor $\mathbf{q}^*(W, \mathbf{x})$ during the relaxation phase, and the self-organized relaxed state is the state whose internal structure has become to coincide with the attractor $\mathbf{q}^*(W_R, \mathbf{x})$ which is hardest to change its own spatial structure during the nonlinear dissipation process.

It is interesting to note the following possibility which can be deduced from the thought analysis shown above. If the system of interest has plural different laws ruling the elements $\mathbf{q}(t, \mathbf{x})$ to be dissipated, and if there exist plural different functional forms due to different dissipations in the relations between the dissipation rate $\mathrm{d}W_{ji}/\mathrm{d}t$ and the global correlations $W_{ji}$, then the system may possibly have plural different attractors $\mathbf{q}^*(W_{ji}, \mathbf{x})$ for different global correlations $W_{ji}$. This kind of system may be attracted to those plural different attractors $\mathbf{q}^*(W_{ji}, \mathbf{x})$ at different time during the time evolutions of the system.

All of thoughts shown above, including the set of general thoughts { [I], [II] } with eqs.(32)-(35) to find internal structures of the self-organized relaxed states, would be applicable to all dynamical systems including physical systems, chemical systems, biological systems, and/or economical systems in general. The realization of the internal spatial structure of the self-organized relaxed state comes essentially from the fact that the dissipative nonlinear dynamical system of interest is the open system with respect to the global quantity $W$ subject to the dissipation. The realization of the self-organization is a global property that is embedded in the laws ruling the elements $\mathbf{q}(t, \mathbf{x})$ in the dissipative, open, and nonlinear dynamical system of interest.

This thought is connected to the well known thought of "the structure due to the dissipation" by Prigogine [39,40].

We now apply the set of general thoughts { [I], [II] } with eqs.(32) - (35) to the resistive MHD plasma which is described by the following simplified equations with Ohm's law of $\eta \mathbf{j} = \mathbf{E} + \mathbf{u} \times \mathbf{B}$,

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathbf{j} \times \mathbf{B} - \nabla p, \tag{36}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times ( \mathbf{u} \times \mathbf{B} ) - \nabla \times (\eta \mathbf{j}), \tag{37}$$

where the viscosity is assumed to be negligibly small. For simplicity, it is assumed here, as is indeed the case in most experiments, the plasma internal energy and the mass flow energy are negligible compared to the magnetic energy $W_m$. We pick up the magnetic energy $W_m$ of the system and look for the self-organized quasi-steady relaxed state with respect to $W_m$. Substituting $W_m$ and $|\mathrm{d}W_m/\mathrm{d}t|$ with $\mathbf{u} = 0$ respectively to $W$ and $|\mathrm{d}W/\mathrm{d}t|$ in the set of general thoughts { [I], [II] }, we obtain the following thought [II-A] to find the self-organized relaxed state:

[II-A] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structures yield the minimum value of $|\ \mathrm{d}W_m/\mathrm{d}t\ |$ ( i.e. the minimum dissipation rate of $W_m$ ) with $W_m = W_{mR}$, which is expressed by

$$\textit{the minimum} \ \left| \frac{\mathrm{d}W_m}{\mathrm{d}t} \right| \ \textit{state} \ \text{with} \ W_m = W_{mR}\,, \tag{38}$$

where $W_{mR}$ is the value of $W_m$ measured at the time of the relaxed state just after the relaxation phase, and $W_m$ is, of course, not the invariant for time.

In the quasi-steady relaxed state, we may assume $\mathbf{u} \cong 0$, and obtain the following equilibrium equation from eq.(36),

$$\nabla p = \mathbf{j}_R \times \mathbf{B}_R \,, \tag{39}$$

where the subscript R denotes the quantities at the quasi-steady relaxed state. We assume here, for simplicity, that the resistivity $\eta$ at the quasi-steady relaxed state has a fixed spatial dependence like as $\eta(\mathbf{x})$, as is indeed the case in all experiments where $\eta$ goes up to infinity near the boundary wall. Substituting $W_m$ of eq.(7) and $|\mathrm{d}W_m/\mathrm{d}t|$ of eq.(9) with $\mathbf{u} = 0$ respectively into $W$ and $|\mathrm{d}W/\mathrm{d}t|$ in the set of general thoughts { [I], [II] } with eqs.(32)-(35) to find the internal structures of the self-organized quasi-steady relaxed state, we obtain the followings [33,37],

$$\delta F = \int (2\eta \, \delta\mathbf{j} \cdot \mathbf{j} - \frac{\alpha}{\mu_o}\delta\mathbf{B} \cdot \mathbf{B})\mathrm{d}v = 0, \tag{40}$$

$$\delta^2 F = \int (2\eta \, \delta\mathbf{j} \cdot \delta\mathbf{j} - \frac{\alpha}{\mu_o}\delta\mathbf{B} \cdot \delta\mathbf{B})\mathrm{d}v > 0, \tag{41}$$

where the values of the Poynting vector $\mathbf{E} \times \mathbf{H}$ on the boundary surface in $\mathrm{d}W_m/\mathrm{d}t$ are assumed to be given so that the surface integral terms vanish in both $\delta F$ and $\delta^2 F$ by the boundary conditions of eq.(35), for simplicity. Using $\mu_o \delta\mathbf{j} = \nabla \times \delta\mathbf{B}$, the vector formula of $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the followings from eqs.(40) and (41),

$$\delta F = \frac{2}{\mu_o} \int \delta\mathbf{B} \cdot \{ \nabla \times (\eta\mathbf{j}) - \frac{\alpha}{2}\mathbf{B}\}\mathrm{d}v - \frac{2}{\mu_o} \oint ( \eta\mathbf{j} \times \delta\mathbf{B} ) \cdot \mathrm{d}\mathbf{s} = 0, \tag{42}$$

$$\delta^2 F = \frac{2}{\mu_o} \int \delta\mathbf{B} \cdot \{\nabla \times (\eta\delta\mathbf{j}) - \frac{\alpha}{2}\delta\mathbf{B}\}\mathrm{d}v - \frac{2}{\mu_o} \oint (\eta\delta\mathbf{j} \times \delta\mathbf{B}) \cdot \mathrm{d}\mathbf{s} > 0. \tag{43}$$

We then obtain the Euler-Lagrange equation from the volume integral term in eq.(42) for arbitrary variations of $\delta\mathbf{B}$ as follows,

$$\nabla \times (\eta \mathbf{j}) = \frac{\alpha}{2}\mathbf{B}\,. \tag{44}$$

When we use $\mu_o \mathbf{j} = \nabla \times \mathbf{B}$ instead of $\mu_o \delta\mathbf{j} = \nabla \times \delta\mathbf{B}$, we obtain the followings from eqs.(40) and (41), corresponding to eqs.(42) and (43),

$$\delta F = \int \delta\mathbf{j} \cdot (2\eta\mathbf{j} - \alpha\mathbf{A})\mathrm{d}v - \frac{\alpha}{\mu_o}\oint(\mathbf{A} \times \delta\mathbf{B}) \cdot \mathrm{d}\mathbf{s} = 0, \tag{45}$$

$$\delta^2 F = \int \delta\mathbf{j} \cdot (2\eta\delta\mathbf{j} - \alpha\delta\mathbf{A})\mathrm{d}v - \frac{\alpha}{\mu_o}\oint(\delta\mathbf{A} \times \delta\mathbf{B}) \cdot \mathrm{d}\mathbf{s} > 0\,. \tag{46}$$

We then obtain the Euler-Lagrange equation from the volume integral term in eq.(45) for arbitrary variations of $\delta\mathbf{j}$ as follows [33,37],

$$\eta\mathbf{j} = \frac{\alpha}{2}\mathbf{A}. \tag{47}$$

Taking the rotation of eq.(47), we obtain eq.(44) again. Since $\mathbf{A}$ is finite near the boundary wall, the present result of eq.(47) leads directly to the experimental fact that the current density $\mathbf{j}$ goes to zero near the wall where $\eta$ goes to infinity, as is indeed the case in all experiments.

We now have found that the self-organized quasi-steady relaxed state has the peculiar internal structure which satisfies eq.(44). Taking account of the assumption $\mathbf{u} \cong 0$ for the self-organized quasi-steady relaxed state, and substituting eq.(44) into eq.(37), we obtain the following,

$$\frac{\partial \mathbf{B}}{\partial t} \cong -\frac{\alpha}{2}\mathbf{B}. \tag{48}$$

Equation (48) gives us the following solution

$$\mathbf{B}(\mathbf{x}, t) \cong \mathbf{B}_R(\mathbf{x})\mathrm{e}^{-\frac{\alpha}{2}t}, \tag{49}$$

where $\mathbf{B}_R(\mathbf{x})$ is the solution of eq.(44) for the self-organized quasi-steady relaxed state. We see from eq.(49) that the field profile of $\mathbf{B}$ just after the realization of the self-organized relaxed state has the self-similar decay phase without significant change of the spatial structure. The first term of right-hand side of eq.(37) and boundary conditions such as the ideally conducting wall would, however, lead gradually to finite deviation from the self-similar decay shown by eq.(49). We may recognize from eq.(44) for the self-organized quasi-steady relaxed state and eq.(49) for the time evolution of the relaxed **B** field that the present nonlinear dynamical system with dissipation relaxes to the state that has attained such a peculiar internal spatial structure that yields the minimum dissipation rate of $W_m$ and thereafter leads to the self-similar decay phase without significant change of the spatial structure.

Chandrasekhar and Woltjer derived already the same equation with eq.(44) more than thirty years ago as the equation of the state of minimum dissipation for a given magnetic energy, in order to explain and lead to the constancy of $\lambda$ of the force-free fields $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ in the cosmic magnetic fields [41]. The current density j of experimental relaxed state plasmas, however, goes to zero near the wall where the resistivity $\eta$ of the plasma goes to infinity, as is indeed the case in all experiments. Therefore, the value of $\lambda$ for the experimental relaxed state plasma must be zero near the wall when $\mathbf{B} \neq 0$ at the wall, even if the relaxed state becomes to be expressed approximately by $\nabla \times \mathbf{B} = \lambda \mathbf{B}$. The present theory by the set of general thoughts { [I], [II] } with eqs.(32) - (35) has been motivated rather by this experimental fact of j = 0 near the wall and includes the key concept of the self-similar decay phase without significant change of the spatial structure that must be satisfied by the self-organized relaxed state [33,36,37]. { The set of general thoughts { [I], [II] } with eqs.(32) - (35) can be applicable to other nonlinear dynamical systems with dissipation like as the incompressible vicous fluids [36,37] }. The state described by eq.(44) and/or eq.(47) does represent more general self-organized relaxed state that is proved directly to be followed by the self-similar decay phase, as was shown at eq.(49), and also satisfies the experimental fact of $\mathbf{j} = 0$ near the wall, as was mentioned after eq.(47). The relaxed state of the force-free fields $\nabla \times \mathbf{B} = \lambda\mathbf{B}$ with a constant $\lambda$ is rather one example of the general self-organized relaxed states of eq.(44) that takes place in a peculiar situation where spatially uniform resistivity profile and low $\beta$ plasma limit have become to be assumed as a result, as will be shown in the following analytical examples [37].

In order to show some analytical examples, we now assume the resistivity $\eta$ to be spatially constant, for simplicity. We then obtain the following from eq.(44) [33,37],

$$\nabla \times \nabla \times \mathbf{B} = \lambda^2 \mathbf{B} , \tag{50}$$

$$| \lambda | = \sqrt{\frac{\alpha \mu_o}{2\eta}} , \tag{51}$$

where the Lagrange multiplier $\alpha$ is assumed to be positive. Equation (50) is the same with the equation used for the classical spheromak [42,43]. According to ref.[42], three independent solutions of eq.(50) with $\nabla \cdot \mathbf{B} = 0$ are given by

$$\mathbf{L}_m = \mathrm{grad}\psi_m, \;\; \mathbf{T}_m = \nabla \times (\mathbf{e}\psi_m), \;\; \text{and} \;\; \mathbf{S}_m = \frac{1}{\lambda} \nabla \times \mathbf{T}_m , \tag{52}$$

where $\mathbf{e}$ is a fixed unit vector, and $\psi_m$ is a scalar function such that

$$\nabla^2 \psi_m + \lambda^2 \psi_m = 0 . \tag{53}$$

Here, the solution of $\mathbf{L}_m$ may be excluded from the solutions for eq.(50), because $\nabla \times \mathrm{grad}\ \psi_m \equiv 0$. The general solution of eq.(50), $\mathbf{B}_R(\mathbf{x})$, for the self-organized quasi-steady relaxed state is then written as

$$\mathbf{B}_R(\mathbf{x}) = c_{m1} \mathbf{T}_m + c_{m2} \mathbf{S}_m . \tag{54}$$

Using eq.(54) and $\mu_o \mathbf{j} = \nabla \times \mathbf{B}$, we obtain the current density of the relaxed state, $\mathbf{j}_R(\mathbf{x})$, as follows,

$$\mathbf{j}_R(\mathbf{x}) = \frac{\lambda}{\mu_o}(c_{m1}\mathbf{S}_m + c_{m2}\mathbf{T}_m), \tag{55}$$

where eq.(52) and $\nabla \times \nabla \times \mathbf{T}_m = \lambda^2 \mathbf{T}_m$ are used. There are three unknown factors of $\{\lambda, c_{m1}, c_{m2}\}$ in eqs.(52)-(55). In order to determine the values of the three unknown factors $\{\lambda, c_{m1}, c_{m2}\}$, it is enough to use three measured values of the magnetic energy $W_m$, the toroidal magnetic flux $\Phi$ and the toroidal plasma current $I$ inside the boundary at the time of the relaxed state, which are denoted here respectively by $W_{mR}$, $\Phi_R$, and $I_R$. It is because that we obtain $\Phi_R$ and $I_R$ by integrating eqs.(54) and (55) respectively across the poloidal cross-section of the toroidal plasma.

Using eqs.(39),(54) and (55), we obtain the followings,

$$\nabla p = \mathbf{j}_R \times \mathbf{B}_R = \frac{\lambda}{\mu_o}({c_{m2}}^2 - {c_{m1}}^2)\mathbf{T}_m \times \mathbf{S}_m, \tag{56}$$

$$\frac{\lambda}{\mu_o}\Phi_R - I_R = \frac{\lambda}{\mu_o}(c_{m2} - c_{m1}) \int_{S_p} (\mathbf{S}_m - \mathbf{T}_m) \cdot \mathrm{ds}, \tag{57}$$

where $\int_{S_p}$ denotes the integral across the poloidal cross-section of the toroidal plasma. It is seen from eqs.(56) and (57) that the difference between $c_{m1}$ and $c_{m2}$ yields the non-force-free component which is balanced with the pressure gradient.

In the limit of the low $\beta$ plasma, we come to have the profiles with $c_{m1} = c_{m2}$ from eq.(56) because of two independent vector solutions of $\mathbf{T}_m$ and $\mathbf{S}_m$, and obtain the followings from eq.(54),

$$\mathbf{B}_R(\mathbf{x}) = c_{m1}(\mathbf{T}_m + \mathbf{S}_m), \tag{58}$$

which satisfies the following as was shown in ref.[42],

$$\nabla \times \mathbf{B} = \pm\lambda \mathbf{B}\,. \tag{59}$$

We see from eqs.(58) and (59) that the force-free fields of $\nabla \times \mathbf{B} = \pm\lambda\mathbf{B}$, derived by Taylor based on "the minimum energy state under the time invariant of the total helicity" [1,2], can be derived generally as the low $\beta$ plasma limit of the self-organized relaxed state which has the minimum dissipation rate profile and therefore is hardest to change its own profile for its own instantaneous amount of the containing magnetic energy, in the nonlinear and dissipative MHD system, without using the "helicity" and the "invariant" for time.

We show here some examples of the self-organized relaxed state of axisymmetric plasmas in the cylindrical coordinates ( $r$, $\theta$, $z$ ). We consider simple cases of the straight axisymmetric plasmas such as the diffused $Z$ pinch, the screw pinch, and the reversed field pinch (RFP) in the cylindrical approximation. The $z$ direction is now the toroidal direction and we use the unit vector along the $z$ direction, $\mathbf{e}_z$, for the fixed unit vector $\mathbf{e}$ in eq.(52). In this case, eq.(53) becomes one dimensional problem, and the solution of $\psi_m$ is known to be the 0th order Bessel function written as $\psi_m = J_o(\lambda r)$, by solving eq.(53). Then the vector solutions of $\mathbf{T}_m$ and $\mathbf{S}_m$ in eq.(52) are obtained respectively as

$$\mathbf{T}_m \;=\; \lambda J_1(\lambda r)\mathbf{e}_\theta \,, \tag{60}$$

$$\mathbf{S}_m \;=\; \lambda J_o(\lambda r)\mathbf{e}_z \,, \tag{61}$$

where $J_1(\lambda r)$ is the 1st order Bessel function, and $\mathbf{e}_\theta$ is the unit vector of the $\theta$ direction. For the first example, we consider the self-organized relaxed state of the diffused $Z$ pinch. Since the measured velue $\Phi_R$ of the toroidal flux for this $Z$ pinch is zero, we obtain the followings from eqs.(54), (60) and (61),

$$\Phi_R = \int_{S_p} \mathbf{B}_R \cdot \mathrm{d}\mathbf{s} \;=\; 2\pi c_{m2}\lambda \int_0^{r_w} J_o(\lambda r) r \mathrm{d}r \;=\; 0 \,, \tag{62}$$

where $r_w$ is the wall ( boundary ) radius. We therefore obtain $c_{m2} = 0$ from eq.(62) and find from eqs.(54), (55), (60) and (61) that the configurations of the relaxed state of the diffuse $Z$ pinch are given by

$$\mathbf{B}_R = c_{m1}\lambda J_1(\lambda r)\mathbf{e}_\theta , \tag{63}$$

$$\mathbf{j}_R = \frac{c_{m1}\lambda^2}{\mu_o} J_o(\lambda r)\mathbf{e}_z . \tag{64}$$

The two factors of $c_{m1}$ and $\lambda$ are determined by using the other two measured values of $W_{mR}$ and $I_R$. Substituting eqs.(63) and (64) into the equilibrium equation of eq.(56), we obtain the pressure gradient that leads to the pressure profile at the relaxed state as follows,

$$\nabla p = - \frac{{c_{m1}}^2\lambda^3}{\mu_o} J_o(\lambda r) J_1(\lambda r)\mathbf{e}_r , \tag{65}$$

where $\mathbf{e}_r$ is the unit vector of the $r$ direction. We can expect from eq.(49) and eqs.(63)-(65) that the obtained profiles of $\mathbf{B}_R$, $\mathbf{j}_R$ and $p$ at the relaxed state for the straight diffused $Z$ pinch are followed by the self-similar decay phase. We should bear in mind, however, that the change of the spatial distribution of resistivity $\eta$ caused such as by ohmic heating and also the first term of right-hand side of eq.(37) would result in some gradual deviation from the self-similar decay.

For the second example, we consider the self-organized relaxed state of the straight screw pinch. We now express $c_{m2}$ as $c_{m2} = c_{m1} - \Delta c$. We then obtain the followings from eqs.(54)-(57) and eqs.(60) and (61),

$$\mathbf{B}_R = c_{m1}\lambda[\, J_1(\lambda r)\mathbf{e}_\theta + J_o(\lambda r)\mathbf{e}_z \,] - \Delta c\lambda J_o(\lambda r)\mathbf{e}_z , \tag{66}$$

$$\mathbf{j}_R = \frac{c_{m1}\lambda^2}{\mu_o}[\, J_1(\lambda r)\mathbf{e}_\theta + J_o(\lambda r)\mathbf{e}_z \,] - \frac{\Delta c\lambda^2}{\mu_o} J_1(\lambda r)\mathbf{e}_\theta , \tag{67}$$

$$\frac{\lambda}{\mu_o}\Phi_R - I_R = - \frac{2\pi\Delta c\lambda^2}{\mu_o}\int_0^{r_w} J_o(\lambda r) r \mathrm{d}r , \tag{68}$$

$$\nabla p = - \frac{\Delta c(2c_{m1} - \Delta c)\lambda^3}{\mu_o} J_o(\lambda r) J_1(\lambda r)\mathbf{e}_r . \tag{69}$$

The three factors of $\Delta c$, $c_{m1}$, and $\lambda$ are determined by using the three measured values of $W_{mR}$, $\Phi_R$, and $I_R$. The screw pinch is usually operated at the high toroidal field without the field reversal. We see from eqs.(68) and (69) that the value of $\Delta c$ depends on the $\beta$ value of the confined plasma. We find from eqs.(66) and (67) that the configurations of $\mathbf{B}_R$ and $\mathbf{j}_R$ at the relaxed state of the screw pinch containes the force-free field component of the Bessel function model, i.e. the first terms of right-hand sides of eqs.(66) and (67), which would be fairly high compared with the non-force-free field component that depends on the $\beta$ value of the confined plasma. We can also expect from eq.(49) and eqs.(66)-(69) that the obtained profiles of $\mathbf{B}_R$, $\mathbf{j}_R$ and $p$ at the relaxed state for the screw pinch are followed by the self-similar decay phase with some gradual deviation, just as the same as the diffused $Z$ pinch shown above. In the experimental screw pinch plasma, the spatial distribution of the resistivity $\eta$ would fairly modify the profiles of $\mathbf{B}_R$, $\mathbf{j}_R$ and $p$, especially in the boundary region.

For the third example, we consider the RFP plasma which has the toroidal field reversal. The profiles of $\mathbf{B}_R$, $\mathbf{j}_R$ and $p$ at the relaxed state for the RFP are also shown by eqs.(66)-(69), just the same as for the screw pinch. In the limit of the low $\beta$ plasma, $\Delta c$ becomes zero from eq.(69), and we obtain the followings for the $\beta = 0$ RFP plasma from eqs.(66)-(69),

$$\mathbf{B}_R = c_{m1}\lambda[\, J_1(\lambda r)\mathbf{e}_\theta + J_o(\lambda r)\mathbf{e}_z \,] \,, \tag{70}$$

$$\mathbf{j}_R = \frac{c_{m1}\lambda^2}{\mu_o}[\, J_1(\lambda r)\mathbf{e}_\theta + J_o(\lambda r)\mathbf{e}_z \,] \,, \tag{71}$$

$$\frac{\lambda}{\mu_o}\Phi_R = I_R \,. \tag{72}$$

We easily recognize that eqs.(70)-(72) are the well known Bessel function model for the RFP plasma derived and discussed by Taylor based on the time invariant of the total helicity [1,2].

When we consider the finite $\beta$ RFP plasma with $\Delta c > 0$, the pressure profile would be given basically by eq.(69). However, we notice from eq.(69) that the direction of $\nabla p$ reverses across the field reversal point of $J_o(\lambda r) = 0$. This result suggests that the RFP plasma outside the field reversal point at the relaxed state, based on the assumption of $\eta =$ const., is unstable or tends to have uniform pressure profile in the field reversal region through the interaction between the plasma and the boundary wall. In the experimental RFP plasma, the resultant spatial distribution of the resistivity $\eta$, affected by the plasma-wall interaction, would fairly modify the profiles of $\mathbf{B}_R$, $\mathbf{j}_R$ and $p$ consequently, especially in the boundary region [14,19].

Using eq.(43), we next discuss the mode transition point of the relaxed state, for example from the cylindrical mode to the mixed helical one in the cylindrical plasma [1,2,18,37]. We consider here the following associated eigenvalue problem for critical perturbations $\delta\mathbf{B}$ that make $\delta^2 F$ in eq.(43) vanish:

$$\nabla \times (\eta \nabla \times \delta\mathbf{B}_i) - \frac{\mu_o \alpha_i}{2}\delta\mathbf{B}_i = 0, \tag{73}$$

with the boundary conditions of $\delta\mathbf{B}\cdot \mathrm{d}\mathbf{s} = 0$, and $(\eta\delta\mathbf{j} \times \delta\mathbf{B})\cdot \mathrm{d}\mathbf{s} = 0$ at the boundary, where $\alpha_i$ and $\delta\mathbf{B}_i$ denote the eigenvalue and the eigensolution, respectively. Substituting the eigensolution $\delta\mathbf{B}_i$ into eq.(43) and using eq.(73), we obtain the following:

$$\delta^2 F = \frac{1}{\mu_o}(\alpha_i - \alpha)\int \delta\mathbf{B}_i \cdot \delta\mathbf{B}_i \,\mathrm{d}v > 0. \tag{74}$$

Since eq.(74) is required for all eigenvalues, we obtain the following condition for the self-organized relaxed state with the minimum $|\mathrm{d}W_m/\mathrm{d}t|$,

$$0 < \alpha < \alpha_1, \tag{75}$$

where $\alpha_1$ is the smallest of the positive eigenvalues, and $\alpha$ is assumed to be positive, as was assumed at eq.(51). When the value of $\alpha$ corresponding to $W_{mR}$ goes out of the condition of eq.(75), like as $\alpha_1 < \alpha$, then the mixed mode, which has the value of $W_{mR}$ and consists of the basic mode by the solution of eq.(44) with $\alpha = \alpha_1$ and the lowest eigenmode by eq.(73), becomes the self-organized relaxed state with the minimum value of $|dW_m/dt|$. By using definitions of $\eta(\mathbf{x}) = \eta_o g(\mathbf{x})$ and $|\lambda| = \sqrt{\alpha\mu_o/2\eta_o}$, the condition of eq.(75) can be rewritten to other form similar to the mode transition condition shown in refs.[18] and [20], where $\eta_o$ is the value of $\eta$ at the magnetic axis. The mode transition condition of eq.(75) is the generalization of the mode transition condition by Taylor [1,2,18,37].

It is easy to show from eqs.(50)-(59) that in the case of the low $\beta$ plasma limit with $\eta$ = const., the eigenvalue problem of eq.(73) includes the following eigenvalue problem as a force-free branch,

$$\nabla \times \delta\mathbf{B}_i = \pm\lambda_i\, \delta\mathbf{B}_i \tag{76}$$

with the boundary condition of $\delta\mathbf{B} \cdot d\mathrm{s} = 0$ at the boundary, where $\lambda_i$ is the eigenvalue, and this eigensolution $\delta\mathbf{B}_i$ makes the surface integral term of eq.(43) vanish automatically. Substituting the eigensolution $\delta\mathbf{B}_i$ into eq.(43) and using eq.(76), we obtain the following:

$$\delta^2 F = \frac{2\eta}{\mu_o^2}(\lambda_i^2 - \lambda^2)\int \delta\mathbf{B}_i \cdot \delta\mathbf{B}_i\, dv > 0, \tag{77}$$

where eq.(51) is used. Since eq.(77) is required for all eigenvalues, we obtain the following condition for the relaxed state with the minimum $|dW_m/dt|$,

$$\lambda_{-1} < \lambda < \lambda_1 , \tag{78}$$

where $\lambda_{-1}$ and $\lambda_1$ are the largest of the negative and the smallest of the positive eigenvalues, respectively. This mode transition condition is the same as that in Taylor's theory [1,2,18].

We now have shown that the relaxed state of eq.(16) is derived generally as the low $\beta$ plasma limit of the self-organized relaxed state from the thought [II-A] with eq.(38). Using the reciprocity of the variational calculus, we obtain the following thought [II-B] which is equivalent to the thought [II-A] with eq.(38),

[II-B] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the maximum value of $W_m$ and the dissipation rate of $\mathrm{d}W_m/\mathrm{d}t = \mathrm{d}_t W_{mR}$, which is expressed by

$$\textit{the maximum } W_m \textit{ state} \text{ with } \left| \frac{\mathrm{d}W_m}{\mathrm{d}t} \right| = \left| \mathrm{d}_t W_{mR} \right| , \tag{79}$$

where $\mathrm{d}_t W_{mR}$ is the value of $\mathrm{d}W_m/\mathrm{d}t$ measured at the time of the relaxed state after the relaxation phase, and $\mathrm{d}W_m/\mathrm{d}t$ is, of course, not the invariant for time. The mathematical expression for eq.(79) by the variational technique is written in the following forms, which are equivalent to eqs.(33) and (34),

$$\delta F = 0, \tag{80}$$

$$\delta^2 F < 0, \tag{81}$$

where $F$ is now the functional defined by $F = W_m$ - $(1/\alpha)|\ \mathrm{d}_t W_{mR}\ |$; $\delta F$ and $\delta^2 F$ are the first and second variations of $F$; and $1/\alpha$ is the positive Lagrange multiplier. The mathematical expressions of eq.(79) in the thought [II-B] lead us to the same processes from eq.(40) to eq.(72) and also to the same condition of eq.(75) or eq.(78) for the mode transition point of the relaxed state.

Since we have used reversible mathematical processes from eq.(40) to eq.(46), we investigate another group of thoughts connected to eqs.(45) and (46) in the thought [II-A] in the same way as was used at eqs.(21)-(31). We assume here, for simplicity, that the resistivity $\eta$ is spatially uniform. Changing the variation of $\delta\mathbf{A}$ in eqs.(45) and (46) to a variation of the more general quantity $\mathbf{q}$, as $\delta\mathbf{q}$, we may have the following expressions for the group of relaxation theories that lead to eq.(47) [ and eq.(59) in the low $\beta$ plasma limit ] as the Euler-Lagrange equation from the volume integral term for arbitrary variations of $\delta\mathbf{q}$ and also lead to the mode transition condition of the relaxed state:

$$\delta F[\delta\mathbf{q}] \;=\; \int \delta\mathbf{q}\cdot(2\eta\mathbf{j}-\alpha\mathbf{A})\mathrm{d}v \;=\; 0\,. \tag{82}$$

$$\delta^2 F[\delta\mathbf{q}] \;=\; \int \delta\mathbf{q}\cdot(2\eta\delta\mathbf{j}-\alpha\delta\mathbf{A})\mathrm{d}v \;>\; 0\,. \tag{83}$$

At first, we adopt $\delta\mathbf{B}$ for $\delta\mathbf{q}$ in eqs.(82) and (83), and then we obtain the followings,

$$\delta F[\delta\mathbf{B}] \;=\; \int \delta\mathbf{B}\cdot(2\eta\mathbf{j}-\alpha\mathbf{A})\mathrm{d}v \;=\; 0\,, \tag{84}$$

$$\delta^2 F[\delta\mathbf{B}] \;=\; \int \delta\mathbf{B}\cdot(2\eta\delta\mathbf{j}-\alpha\delta\mathbf{A})\mathrm{d}v \;>\; 0\,. \tag{85}$$

Using eqs.(2) and (3), $\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot\nabla\times\mathbf{a} - \mathbf{a}\cdot\nabla\times\mathbf{b}$, and the Gauss theorem, we obtain the followings from eq.(84),

$$\begin{aligned}\delta F[\delta\mathbf{B}] \;=\; &\int\{\,\eta(\delta\mathbf{j}\cdot\mathbf{B} \,+\, \mathbf{j}\cdot\delta\mathbf{B}\,) \,-\, \frac{\alpha}{2}(\delta\mathbf{A}\cdot\mathbf{B} \,+\, \mathbf{A}\cdot\delta\mathbf{B})\}\mathrm{d}v \\ &- \oint(\eta\delta\mathbf{B}\times\mathbf{B} \,+\, \frac{\alpha}{2}\mathbf{A}\times\delta\mathbf{A})\cdot\mathrm{d}\mathbf{s} \;=\; 0\,.\end{aligned} \tag{86}$$

We see from comparison between the volume integral term of eq.(86) and eqs.(10) and (8) that $F$ in eq.(86) is the fuctional defined by $F$ = - $\mu_o(\mathrm{d}K/\mathrm{d}t)$ - $\alpha\mu_o K$. Since the first term of $\mathrm{d}K/\mathrm{d}t$ in eq.(10) is the dissipation rate of the total helicity $K$ by the resistivity $\eta$ and has the negative value, we therefore come to the following another thought [II-C] for the relaxed state that leads to eq.(47):

[II-C] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structure yields the minimum dissipation rate of $K$ with $K = K_R$, which is expressed by

$$\textit{the minimum} \mid \frac{\mathrm{d}K}{\mathrm{d}t} \mid \textit{state} \text{ with } K = K_R \,, \tag{87}$$

where the functional $F$ is now defined by $F =$ - $\mathrm{d}K/\mathrm{d}t$ - $\alpha K$, $K_R$ is the value of $K$ measured at the time of the relaxed state just after the relaxation phase, and $K$ is, of course, not the invariant for time. The mathematical expressions of eq.(87) in the thought [II-C] lead us to the volume integral terms of eqs.(84) and (85), and therefore we obtain eq.(47) and also the same condition of eq.(75) for the mode transition point of the relaxed state. The reciprocity of the variational calculus gives us the following thought [II-D] equivalent to the thought [II-C] given by eq.(87):

[II-D] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the maximum value of $K$ with $\mathrm{d}K/\mathrm{d}t = \mathrm{d}_t K_R$, which is expressed by

$$\textit{the maximum } K \textit{ state} \text{ with } \frac{\mathrm{d}K}{\mathrm{d}t} = \mathrm{d}_t K_R \,, \tag{88}$$

where $\mathrm{d}_t K_R$ is the value of $\mathrm{d}K/\mathrm{d}t$ measured at the time of the relaxed state just after the relaxation phase, and $\mathrm{d}K/\mathrm{d}t$ is, of course, not the invariant for time. The mathematical expressions of eq.(88) in the thought [II-D] lead us to the volume integral terms of eqs.(84) and (85), and therefore we obtain eq.(47) and also the same condition of eq.(75) for the mode transition point of the relaxed state.

Next, we adopt $\delta\mathbf{A}$ for $\delta\mathbf{q}$ in eqs.(82) and (83), and we obtain the followings in this case,

$$\delta F[\delta\mathbf{A}] = \int \delta\mathbf{A} \cdot (2\eta\mathbf{j} - \alpha\mathbf{A})\mathrm{d}v = 0 \,, \tag{89}$$

$$\delta^2 F[\delta\mathbf{A}] = \int \delta\mathbf{A} \cdot (2\eta\delta\mathbf{j} - \alpha\delta\mathbf{A})\mathrm{d}v > 0 \,. \tag{90}$$

Using eqs.(2) and (3), $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the followings from eq.(89),

$$\delta F[\delta \mathbf{A}] = \int \{ \eta(\delta \mathbf{j} \cdot \mathbf{A} + \mathbf{j} \cdot \delta \mathbf{A}) - \frac{\alpha}{2}(\delta \mathbf{A} \cdot \mathbf{A} + \mathbf{A} \cdot \delta \mathbf{A}) \} \mathrm{d}v - \frac{1}{\mu_o} \oint (\delta \mathbf{B} \times \mathbf{A} + \delta \mathbf{A} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{s} = 0 . \qquad (91)$$

We consider here the autocorrelation with respect to the vector potential $\mathbf{A}$ in the resistive MHD plasma, and define the following global autocorrelation $Z$ of $\mathbf{A}$,

$$Z = \int \mathbf{A} \cdot \mathbf{A} \, \mathrm{d}v. \qquad (92)$$

Using Ohm's law and $\mathbf{E} = -\nabla \phi - \partial \mathbf{A}/\partial t$ deduced from eqs.(2) and (4), we obtain the time derivative of $Z$ as follows,

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -2 \int \{ \eta \mathbf{j} \cdot \mathbf{A} + (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{u} \} \mathrm{d}v - \oint \phi \mathbf{A} \cdot \mathrm{d}\mathbf{s}. \qquad (93)$$

We see from comparison between the volume integral term of eq.(91) and eqs.(92) and (93) that $F$ in eq.(91) is the fuctional defined by $F = -\,(\mathrm{d}Z/\mathrm{d}t)/2 - \alpha Z/2$ for the case of the quasi-steady relaxed state with $\mathbf{u} \cong 0$. Since the first term of $\mathrm{d}Z/\mathrm{d}t$ in eq.(93) is the dissipation rate of the autocorrelation $Z$ due to the resistivity $\eta$ and has the negative value, we therefore come to the following another thought [II-E] for the relaxed state that leads to eq.(47):

[II-E] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structure yields the minimum dissipation rate of the autocorrelation $Z$ with $Z = Z_R$, which is expressed by

$$\textit{the minimum} \ \left| \frac{\mathrm{d}Z}{\mathrm{d}t} \right| \ \textit{state} \text{ with } Z = Z_R , \qquad (94)$$

where the functional $F$ is now defined by $F = -\,\mathrm{d}Z/\mathrm{d}t - \alpha Z$, $Z_R$ is the value of $Z$ measured at the time of the relaxed state just after the relaxation phase, and $Z$ is, of course, not the invariant for time.

The mathematical expressions of eq.(94) in the thought [II-E] lead us to the volume integral terms of eqs.(89) and (90), and therefore we obtain eq.(47) and also the same condition of eq.(75) for the mode transition point of the relaxed state. The reciprocity of the variational calculus gives us the following thought [II-F] equivalent to the thought [II-E] given by eq.(94):

[II-F] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the maximum value of $Z$ with $\mathrm{d}Z/\mathrm{d}t = \mathrm{d}_t Z_R$, which is expressed by

$$\textit{the maximum Z state} \text{ with } \frac{\mathrm{d}Z}{\mathrm{d}t} = \mathrm{d}_t Z_R \,, \tag{95}$$

where $\mathrm{d}_t Z_R$ is the value of $\mathrm{d}Z/\mathrm{d}t$ measured at the time of the relaxed state just after the relaxation phase, and $\mathrm{d}Z/\mathrm{d}t$ is, of course, not the invariant for time. The mathematical expressions of eq.(95) in the thought [II-F] lead us to the volume integral terms of eqs.(89) and (90), and therefore we obtain eq.(47) and also the same condition of eq.(75) for the mode transition point of the relaxed state.

We now have obtained the second group of thoughts { from [II-A] to [II-F] } that lead to the same relaxed state of eq.(47) [ and eq.(59) in the low $\beta$ plasma limit ] which is followed by the self-similar decay phase of eq.(49), and the same mode transition condition eq.(75) of the relaxed state, starting from the set of general thoughts { [I], [II] } with eqs.(32)-(35). From comparison among the three thoughts of [II-A], [II-C], and [II-E], we find that in the thought [II-C] the total helicity $K$ is used instead of the total magnetic energy $W_m$ in the thought [II-A], and also in the thought [II-E] the global autocorrelation $Z$ is used instead of $W_m$ in the thought [II-A]. All of the three thoughts of [II-A], [II-C], and [II-E] are found to lead to the same self-organized relaxed state of eq.(47) [ and eq.(59) in the low $\beta$ plasma limit ] and the mode transition condition of the relaxed state, as were shown from eq.(82) to eq.(94). We easily understand from this fact that we can use any of the three quantities $W_m$, $K$, and $Z$ for $W$ in the set of general thoughts { [I], [II] } with eqs.(32)-(35), and we still obtain the same self-organized relaxed state and the mode transition condition of the relaxed state. Since we use all of eqs.(2)-(5) for the derivations of the time derivatives of $W_m$, $K$, and $Z$, we understand that all of the laws of eqs.(2)-(5) are embedded essentially in the relations between $\mathrm{d}W_m/\mathrm{d}t$ and $W_m$, between $\mathrm{d}K/\mathrm{d}t$ and $K$, and between $\mathrm{d}Z/\mathrm{d}t$ and $Z$. In the calculation of the variational technique with respect to the spatial variables $\mathbf{x}$ in order to find the self-organized relaxed state of eq.(46), we have used only eqs.(2) and (3). We see from these facts that the essential things, which lead to the same self-organized relaxed state in the three thoughts of [II-A], [II-C], and [II-E], are in the laws with dissipation { eqs.(2)-(5) } which are embedded in the relation between $\mathrm{d}W/\mathrm{d}t$ and $W$ ( $W = W_m$, $K$, and $Z$ ) and used in the derivation of the self-organized relaxed state by the variational calculus. The global quantity $W_m$ is, in other words, the autocorrelation with respect to $\mathbf{B}$ itself, and the global quantity $K$ is the crosscorrelation between $\mathbf{A}$ and $\mathbf{B}$, as mentioned at eqs.(7) and (8). The global quantity $Z$ is the autocorrelation with respect to $\mathbf{A}$, as mentioned at eq.(92). The three global quantities $W_m$, $K$, and $Z$ are the elements of the global correlations, $W_{ji}(t) = \int q_j(t,\mathbf{x}) q_i(t,\mathbf{x})\, \mathrm{d}\mathbf{x}$ , which were discussed generally after eqs.(32)-(35). Since the internal spatial structure of the self-organized relaxed state of interest is hardest to change its own spatial structure through the dissipation process with respect to Ohm's loss and therefore yields the minimum dissipation rate of the quantities of interst, it is reasonable to consider that the state with the minimum dissipation rate of all these global auto- and cross-correlations due to the common Ohm loss for their own instantaniously containing values become to be the same self-organized relaxed state of eq.(47) which is proved directly to be followed by the self-similar decay phase of eq.(49) without significant change of its own spatial structure. The internal structure given and expressed by eq.(47) [ or eq.(44) ] is the attractor $\mathbf{q}^*(W, \mathbf{x})$ discussed generally after eqs.(32)-(35), and the self-organized relaxed state is the state of $\mathbf{q}^*(W_R, \mathbf{x})$. Since the self-organized relaxed state is hardest to change its own spatial structure, when the nonlinear system of interest has come to this relaxed state, the system behaves like as to be trapped at this self-organized relaxed state during the time evolution of the system, just like as "the attractor of the dissipative structure" introduced by Prigogine [39,40]. These facts indicate that the set of general thoughts { [I], [II] } with eqs.(32)-(35) is useful to find the attractor of the dissipative structure which is the self-organized relaxed state in the nonlinear dynamical system with dissipation.

2.3 *Third Group of Self − Organization Theory*

We now proceed to the theory by T. Kato and T. Furusawa [29], where they deal with the state with the minimum entropy production rate of the irreversible thermodynamical system under the assumption of the total helicity invariant. Since the entropy production rate by the Joule heating is given by the term of $\eta \mathbf{j} \cdot \mathbf{j}$ in eq.(9), the relaxed state introduced by T. Kato and T. Furusawa [29] is expressed by the following thought [III-A],

[III-A] The MHD plasmas with small but finite resistivity would relax to the state with the minimum entropy production rate under the global invariant of the total helicity $K$, which is expressed by the following form;

$$\textit{the minimum} \mid \frac{\mathrm{d}W_m}{\mathrm{d}t} \mid \textit{state} \;\; \text{under}\; K \;=\; K_B \;, \tag{96}$$

where $K_B$ is the value of $K$ measured at the time just "before the relaxation phase" and $\mathbf{u} \;=\; 0$.

Since the total helicity $K$ is no longer the invariant for time during the relaxation process, as was discussed in detail in the derivation of the thought [B] with eq.(17) in the subsection 2.1, the thought [III-A] should be reconstructed to the following form in the same way as the thought [B]:

[III-B] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structure contains the helicity $K = K_R$ and the minimum value of the entropy production rate $\mid \mathrm{d}W_m/\mathrm{d}t \mid$, that is expressed by the following form;

$$\textit{the minimum} \ \left| \frac{\mathrm{d}W_m}{\mathrm{d}t} \right| \ \textit{state} \ \text{with} \, K \ = \ K_R \, , \tag{97}$$

where $K_R$ is the value of $K$ measured "at the time of the relaxed state" just after the relaxation phase and $\mathbf{u} \ = \ 0$. Here, "$K \ = \ K_R$" is the necessary condition which must be satisfied by the internal structure of the relaxed state because of the measured value, and becomes "the global constraint" for "finding out the internal structure of the relaxed state from the set of various distributions". The variational calculus for the thought [III-B] with respect to the spatial variable $\mathbf{x}$ is equivalent to that of the thought [III-A].

Using the similar procedure from eq.(40) to (43), $\mu_o \delta \mathbf{j} \ = \ \nabla \times \delta \mathbf{B}$, the vector formula of $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the following mathematical expression of the variational calculus for the thoughts [III-A] and [III-B],

$$\delta F \ = \ \frac{2}{\mu_o} \int \delta \mathbf{B} \cdot \{ \ \nabla \times (\eta \mathbf{j}) \ - \ \frac{\alpha}{2} \mathbf{A} \} \mathrm{d}v \ - \frac{2}{\mu_o} \oint ( \ \eta \mathbf{j} \times \delta \mathbf{B} \ + \ \frac{\alpha}{2} \mathbf{A} \times \delta \mathbf{A} \ ) \cdot \mathrm{d}\mathbf{s} = \ 0, \tag{98}$$

$$\delta^2 F \ = \ \frac{2}{\mu_o} \int \delta \mathbf{B} \cdot \{ \ \nabla \times (\eta \delta \mathbf{j}) \ - \ \frac{\alpha}{2} \delta \mathbf{A} \} \mathrm{d}v \ - \frac{2}{\mu_o} \oint ( \ \eta \delta \mathbf{j} \times \delta \mathbf{B} \ ) \cdot \mathrm{d}\mathbf{s} \ > \ 0 \, . \tag{99}$$

We then obtain the Euler-Lagrange equation from the volume integral term in eq.(98) for arbitrary variations of $\delta\mathbf{B}$ as follows,

$$\nabla \times (\eta \mathbf{j}) \;=\; \frac{\alpha}{2}\mathbf{A}\,. \tag{100}$$

Taking the rotation of eq.(100), and assuming $\eta$ to be spatially uniform, we obtain the following expression for the relaxed state which was derived by T. Kato and T. Furusawa [29],

$$\nabla \times \nabla \times \nabla \times \mathbf{B} \;=\; \gamma^3\mathbf{B}\,, \tag{101}$$

where $\gamma^3 = \alpha\mu_o/2\eta$, and $\alpha$ is the Lagrange multiplier. Solutions of eqs.(101) satisfy the following lower order differential equaion, as reported by T. Kato and T. Furusawa [29],

$$\nabla \times \mathbf{B}_i \;=\; \gamma\omega^i\mathbf{B}_i\,, \tag{102}$$

where $\omega$ and $\omega^2$ denote the cube root of the unity, and the solution with $i = 0$ expresses the force-free field of eq.(16) derived by Taylor. The relaxed state of eq.(101) has other solutions whose real and imaginary parts do not satisfy the equation of the force-free field [29]. The force-free field of eq.(16) is derived as the low $\beta$ plasma limit from the general solution of eq.(101), in the similar way from eq.(50) to eq.(59). However, the relaxed state expressed by eq.(100) does not lead directly to the self-similar decay phase by substituting eq.(100) and u $= 0$ into eq.(37), differently from the case of the second group of the relaxation theory in the previous subsection 2.2.

Using the reciprocity of the variational calculus, we obtain the following thought [III-C] which is equivalent to the thought [III-B] with eq.(97),

[III-C] The relaxed state of the MHD plasma after the nonlinear relaxation phase is the state whose internal structure contains the maximum value of $K$ and the entropy production rate of $\mid \mathrm{d}W_m/\mathrm{d}t \mid \, = \, \mid \mathrm{d}_t W_{mR} \mid$, that is expressed by the following form;

$$\textit{the maximum K state} \text{ with } \left| \frac{\mathrm{d}W_m}{\mathrm{d}t} \right| = \left| \mathrm{d}_t W_{mR} \right| \tag{103}$$

where $| \mathrm{d}_t W_{mR} |$ is the value of $| \mathrm{d}W_m/\mathrm{d}t |$ measured "at the time of the relaxed state" just after the relaxation phase and $\mathbf{u} = 0$, and $| \mathrm{d}W_m/\mathrm{d}t |$ is, of course, not the invariant for time. The mathematical expressions of eq.(103) in the thought [III-C] lead us to the volume integral terms of eqs.(98) and (99), and therefore we obtain eqs.(100) and (101).

Since we have used reversible mathematical processes from eq.(97) to eq.(102), we investigate another group of thoughts connected to eqs.(98) and (99) in the thought [III-B] in the same way as was used at eqs.(21)-(31) and at eqs.(82)-(95). We assume here, for simplicity, that the resistivity $\eta$ is spatially uniform. Changing the variation of $\delta\mathbf{B}$ in eqs.(98) and (99) to a variation of the more general quantity $\mathbf{q}$, as $\delta\mathbf{q}$, we may have the following expressions for the group of relaxation theories that lead to eqs.(100) and (101) as the Euler-Lagrange equation from the volume integral term for arbitrary variations of $\delta\mathbf{q}$:

$$\delta F[\delta\mathbf{q}] = \int \delta\mathbf{q} \cdot \{ \nabla \times (\eta\mathbf{j}) - \frac{\alpha}{2}\mathbf{A}\} \mathrm{d}v = 0 . \tag{104}$$

$$\delta^2 F[\delta\mathbf{q}] = \int \delta\mathbf{q} \cdot \{ \nabla \times (\eta\delta\mathbf{j}) - \frac{\alpha}{2}\delta\mathbf{A}\} \mathrm{d}v > 0 . \tag{105}$$

We adopt $\delta\mathbf{A}$ for $\delta\mathbf{q}$ in eqs.(104) and (105), and then we obtain the followings,

$$\delta F[\delta\mathbf{A}] = \int \delta\mathbf{A} \cdot \{ \nabla \times (\eta\mathbf{j}) - \frac{\alpha}{2}\mathbf{A}\} \mathrm{d}v = 0 , \tag{106}$$

$$\delta^2 F[\delta\mathbf{A}] = \int \delta\mathbf{A} \cdot \{ \nabla \times (\eta\delta\mathbf{j}) - \frac{\alpha}{2}\delta\mathbf{A}\} \mathrm{d}v > 0 . \tag{107}$$

Using eqs.(2) and (3), $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the followings from eq.(106),

$$\delta F[\delta \mathbf{A}] = \frac{1}{2}\int\{ \eta(\delta \mathbf{j}\cdot\mathbf{B} + \mathbf{j}\cdot\delta\mathbf{B}) - \alpha\delta\mathbf{A}\cdot\mathbf{A}\}\mathrm{d}v - \oint(\eta\delta\mathbf{A}\times\mathbf{j} + \frac{1}{2}\delta\mathbf{B}\times\mathbf{B})\cdot\mathrm{d}\mathbf{s} = 0 . \tag{108}$$

We see from comparison between the volume integral term of eq.(108) and eqs.(10) and (92) that $F$ in eq.(108) is the fuctional defined by $2F$ = - $\mu_o(\mathrm{d}K/\mathrm{d}t)$ - $\alpha Z/2$. Since the first term of $\mathrm{d}K/\mathrm{d}t$ in eq.(10) is the dissipation rate of the total helicity $K$ due to the resistivity $\eta$ and has the negative value, we therefore come to the following another thought [III-D] for the relaxed state that leads to eqs.(100) and (101):

[III-D] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structure yields the minimum dissipation rate of $K$ with $Z = Z_R$, which is expressed by

$$\textit{the minimum} \ |\frac{\mathrm{d}K}{\mathrm{d}t}| \ \textit{state} \text{ with } Z = Z_R , \tag{109}$$

where the functional $F$ is now defined by $F$ = - $\mathrm{d}K/\mathrm{d}t$ - $\alpha Z/2$, $Z_R$ is the value of $Z$ measured at the time of the relaxed state just after the relaxation phase, and $Z$ is, of course, not the invariant for time. The mathematical expressions of eq.(109) in the thought [III-D] lead us to the volume integral terms of eqs.(106) and (107), and therefore we obtain eqs.(100) and (101). The reciprocity of the variational calculus gives us the following thought [III-E] equivalent to the thought [III-D] given by eq.(109):

[III-E] The relaxed state of the resistive MHD plasma after the nonlinear relaxation phase is the state whose internal structures contain the maximum value of $Z$ with $\mathrm{d}K/\mathrm{d}t = \mathrm{d}_t K_R$, which is expressed by

$$\textit{the maximum } Z \textit{ state} \text{ with } \frac{\mathrm{d}K}{\mathrm{d}t} = \mathrm{d}_t K_R , \tag{110}$$

where $\mathrm{d}_t K_R$ is the value of $\mathrm{d}K/\mathrm{d}t$ measured at the time of the relaxed state just after the relaxation phase, and $\mathrm{d}K/\mathrm{d}t$ is, of course, not the invariant for time. The mathematical expressions of eq.(110) in the thought [III-E] lead us to the volume integral terms of eqs.(106) and (107), and therefore we obtain eqs.(100) and (101).

We now have obtained the third group of thoughts { from [III-A] to [III-E] } that lead to the same relaxed state of eq.(101) [ or eq.(16) in the low $\beta$ plasma limit ].

## § 3. Discussion and Summary

We have derived three groups of self-organization theories from the thought analysis in the previous section, i.e. the first group of thoughts { from [B] to [G] }, the second group of thoughts { from [II-A] to [II-F] }, and the third group of thoughts { from [III-B] to [III-E] }, all of which lead to the same relaxed state of the force-free field $\nabla \times \mathbf{B} = \pm\, \lambda \mathbf{B}$. The common hypothesis among all of these three groups of the self-organization theories is that the system of interest projected in a kind of functional space, where each point represents internal spatial profiles of the system, can approach sufficiently close to any point in the functional space through the turbulent relaxation phase like as the ergodic hypothesis, because we use the variational calculus with respect to the spatial variables $\mathbf{x}$ to obtain the self-organized relaxed state.

In the derivation of the thought [B] with eq.(17), we have pointed out clearly by using the results of the three-dimensional MHD simulation shown in Fig.3 of ref.[24] that there exists actually the magnetic energy relaxation process where fairly high dissipation of the total helicity, up to about 20 percent in one example, takes place and therefore the total helicity is no longer the invariant for time, and the system still relaxes to the force-free field of $\nabla \times \mathbf{B} = \pm\, \lambda \mathbf{B}$. This fact by the three-dimensional MHD simulation means that the concept of the total helicity invariant is not the essential physical condition necessary for the realization of the relaxed state $\nabla \times \mathbf{B} = \pm \lambda\mathbf{B}$ and therefore has to be removed from the self-organization theories of interest.

The four thoughts of [B] with eq.(17), [E] with eq.(27), [F] with eq.(30), and [III-B] with eq.(97) may be acceptable as the self-organization theories that can be deduced from the inspection of the relation between the two dissipation rate of equations for $W_m$ and $K$ due to the Ohm loss, i.e. eqs.(9) and (10), in order to find the internal spatial structure of the relaxed state. These four thoughts of [B], [E], [F] and [III-B] can lead to the correct relaxed state of the force-free field $\nabla \times \mathbf{B} = \pm \lambda\mathbf{B}$ with $K = K_R$ at the relaxed state, having no concern with the actural dissipation of the total helicity during the relaxation phase. These four thoughts of [B], [E], [F] and [III-B] are neither the energy principle nor the variational principle based on the invariant for time that leads to the state of equilibrium and the stability problems, but they are the theories to lead to the self-organized internal profiles as the dissipative structure to be realized during the nonlinear dissipative processes. These four thoughts of [B], [E], [F] and [III-B] declare simply that after traveling in the functional space through the turbulent relaxation phase the system may come to the state of $\nabla \times \mathbf{B} = \pm \lambda\mathbf{B}$ as the self-organized relaxed state and it may stay there for longer time compared with other points because of the relation between the two dissipation rate of equations for $W_m$ and $K$ due to the Ohm loss, eqs.(9) and (10).

Experimental MHD plasmas and/or three dimensional MHD simulations are, however, known to realize the self-organized relaxed state with finite pressure gradient and nonuniform profile of resistivity $\eta$ [2-11,44,45]. The four thoughts of [B], [E], [F], and [III-B] are disadvantageous to deal with these more general type of plasmas. For example, in the case of plasmas with spatially dependent resistivity $\eta$, it is rather difficult to prove directly that the self-organized relaxed state obtained by these four thoughts of [B], [E], [F], and [III-B] are followed by the self-similar decay phase without significant change of the spatial structure. This self-similar decay phase is one of the most important properties of the self-organized relaxed state, as was discussed at eq.(49) by the thought [II-A]. The disadvantage of the four thoughts of [B], [E], [F], and [III-B] for the dealing with the more general type of plasmas mentioned above results from their common origin that is deduced only from the relation between the two dissipation rate of equations for $W_m$ and $K$ due to the Ohm loss, eqs.(9) and (10), though there is no physical relation between the two quantities of $W_m$ and $K$ themselves. As is easily understood from the full set of basic equations for the three dimensional MHD simulations, i.e. the equations of mass, momentum, and energy ( or equivalently the entropy equation ) together with the Maxwell equations and Ohm's law, the nonlinear, dissipative system of the MHD plasma does evolve completely by these basic equations themselves without any interaction with the independent equation for the quantity "magnetic helicity" [21-26,44,45]. This fact means that the concept of the magnetic helicity is always used to interpret passively the physical process of interest and the equation of the magnetic helicity does not give any active effect to the physical process. In other words, it is natural to consider that the essential physical process for the realization of the self-organized relaxed state through the turbulent relaxation phase must be embedded in the laws themselves which are given by the basic equations and ruling the elements of the dissipative, nonlinear dynamical system of interest.

The set of general thoughts { [I], [II] } with eqs.(32)-(35) comes merely from the following observation using a kind of thought experiment: The nonlinear dissipation itself is supposed to induce changes of the internal spatial structure of the system. On the way of nonlinear, dissipative, dynamical evolution, the system will pass through and stay longer time at the state such that receives the least change of its own spatial structure from the nonlinear dissipative processes and therefore yields the minimum dissipation rate of the global quantity $W$ for the instantaneously containing value of $W$. Such that state must be supposed to be recognized as the self-organized relaxed state in the whole dynamical evolution of the system, if the relaxed state has a peculeiar spatial structure, not like as trivial uniform one. The peculiar spatial structure of the self-organized relaxed state must result from the laws themselves which give the functional form of the dissipation and determine the relation between $dW/dt$ and $W$, as is seen from the theoretical process from eq.(36) to eq.(49). This thought that the peculiar spatial structure of the relaxed state must result from the laws giving the functional form of the dissipation may be confirmed by the fact that the three thouhts of [II-A], [II-C], and [II-E] for the different quantities $W_m$, $K$, and $Z$ with the dissipation due to the common Ohm loss do lead to the same relaxed state given by eq.(44) and/or eq.(47), as was shown in the subsection 2.2 of the second group of the self-organization theories.

The state described by eq.(44) and/or eq.(47) does represent more general self-organized relaxed state that yields the minimum dissipation rate and is proved directly to be followed by the self-similar decay phase without significant change of its own spatial structure, as was shown at eq.(49), and also satisfies the experimental fact of $\mathbf{j} = 0$ near the wall where the resistivity $\eta$ goes to infinity, as was mentioned after eq.(47). It is natural to consider that in experimental devices, the relaxed state of the force-free fields $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with a constant $\lambda$ is rather a special example of the more general self-organized relaxed states of eq.(44) that takes place in a peculiar situation where the spatially uniform resistivity profile and the low $\beta$ plasma limit have become to be assumed as a result, as was shown at eqs.(50)-(59) [37]. Since there exists actually the magnetic energy relaxation process where the total helicity is no longer invariant for time, we cannot say that the relaxed state is determined by the initial total helicity, having no concern with the initial spatial profiles. Instead of this, we can say that the attractors $\mathbf{q}^*(W, \mathbf{x})$ of the dissipative structure, which are derived as the Euler-Lagrange equations and represent the internal structures of the self-organized relaxed state, are determined essentially by the laws themselves which give the functional forms of the dissipation rate, having no concern with the initial spatial profiles. And also we can say that the spatial profiles of the self-organized relaxed state are determined uniquely by the Euler-Lagrange equations and the resultant quantities measured at the time of the relaxed state, such as the magnetic energy, the toroidal flux, and the toroidal current flux, together with the boundary conditions, having no concern with the initial spatial profiles.

In conclusion, the thought analysis on the self-organization theories presented here leads us to the followings: The self-organized relaxed state, as the attractor of the dissipative structue [39], of the resistive MHD plasma is formulated as the state such that yields the minimum dissipation rate due to the Ohm loss of the global auto- and/or cross-correlations of $\mathbf{j}$, $\mathbf{B}$, and $\mathbf{A}$ for their own instantaneous containing values of the global correlations. All of these formulation for the state with the minimum dissipatin rate of these global correlations due to the common Ohm loss lead to the same Euler-Lagrange equation of eq.(44) and/or eq.(47) which represents more general self-organized relaxed state that is proved directly to be followed by the self-similar decay phase without significant change of its own spatial structure, as was shown at eq.(49), and also satisfies the experimental fact of $\mathbf{j} = 0$ near the wall where the resistivity $\eta$ goes to infinity, as was mentioned after eq.(47). In experimental devices, the relaxed state of the force-free fields $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with a constant $\lambda$ must be considered to be rather a special example of the more general self-organized relaxed states of eq.(44) that takes place in a peculiar situation where the spatially uniform resistivity profile and the low $\beta$ plasma limit have become to be assumed as a result. The formulation to find the internal structure of the self-organized relaxed state as the dissipative structure is extended to the more general nonlinear, dissipative dynamical system as follows: The attractors $\mathbf{q}^*(W_{ji}, \mathbf{x})$ of the dissipative structure are the states such that yield the minimum dissipation rate $dW_{ji}/dt$ for their own instantaneously containing values of the global correlations $W_{ji}$, where $W_{ji}(t) = \int q_j(t,\mathbf{x})q_i(t,\mathbf{x})\, d\mathbf{x}$ , as was discussed after eqs.(32)-(35). This formulation is neither the energy principle nor the variational principle based on the invariant for time such that leads to the equilibrium state and the stability problems. The attractors $\mathbf{q}^*(W_{ji}, \mathbf{x})$ of the dissipative structure are determined uniquely by the laws giving the functional forms of the dissipation of interest and the relations between $dW_{ji}/dt$ and $W_{ji}$ [37].


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