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Vo Hong Anh and Nguyen Tien Dung

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A SYNERGETIC TREATMENT OF THE VORTICES BEHAVIOUR OF A PLASMA WITH VISCOSITY

VO HONG ANH*

National Institute for Fusion Science
Nagoya 464-01

and

NGUYEN TIEN DUNG

Department of Theoretical and Computational Physics
Vietnam National Atomic Energy Commission
59 Ly Thuong Kiet, Hanoi – Vietnam

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* On leave of absence from Vietnam National Atomic Energy Commission,
59 Ly Thuong Kiet, Hanoi, Vietnam

ABSTRACT

The known system of nonlinear partial differential equations (PDE) describing vortical motions of an ideal electron-ion plasma with viscosity in the presence of a slightly inhomogeneous magnetic field[1) is reduced to a Lorentz type system of 3 ordinary differential equations (ODE) the numerical solution of which with a set of a values for real plasma physical parameters shows the occurrence of a state with strange attractors that means the beginning of the vortices formation as an essentially nonlinearity effect.

1. Basic Equations

The content of the present paper represents an attempt to consider the vortices formation in a plasma as a nonlinear phenomenon by using a procedure different from the traditional linearization of the governing equations with subsequent consideration of the interaction between "modes" of the soliton. We mean the simplification procedure typical for the so called synergetic approach[2)] based on nonlinearity as an essential general feature of quite different phenomena.

We proceed from the known system of PDE for the relative electron density perturbation n and the self-consistent electric potential ϕ describing the wave behaviour of a plasma with ion viscosity μ placed under a slightly inhomogenerous magnetic field B_0[1)] which is of the form:

$$\frac{\partial n}{\partial t}+\frac{Kc}{B_0}\frac{\partial \phi}{\partial y}=\frac{c}{B_0}I(n,\phi)\ ,$$

$$\frac{c}{B_0\Omega_i}\left\{\frac{\partial}{\partial t}+V^*\frac{\partial}{\partial y}\right\}\Delta\phi-V_0\frac{\partial n}{\partial y}-\mu\frac{c}{B_0\Omega_i}\nabla^4\left(\phi+\frac{T_i}{e}n\right)$$
$$=\frac{c^2}{B_0\Omega_i}\left[i\left(\Delta\phi,\phi\right)-\frac{T_i}{e}\operatorname{div} I\left(n,\nabla\phi\right)\right]\ , \tag{1}$$

where

$$I(n,\phi)=\tan\frac{\partial n}{\partial x}\frac{\partial \phi}{\partial y}-\frac{\partial n}{\partial y}\frac{\partial \phi}{\partial x}\ ,\ K\equiv L_n^{-1}=\frac{\nabla n_0}{n_0}\ ,$$

$$\Omega_1=\frac{eB_0}{m_i c}\ ,\ V^*=\frac{-KT_i}{m_i\Omega_i}\ ,\ V_0=\frac{g}{\Omega_i}\ ,\ g=\frac{T_i+T_i}{m_i R_B}\ ,\ R_B^{-1}=\frac{\nabla B_0}{B_0}\ .$$

The magnetic field B_0 is directed along the Z axis and is nonuniform along the X axis along which the plasma density is nonuniform as well. The most important assumptions in obtaining the system (1) are (i) that the electron motion is a pure drift (the motion along magnetic field is ignored) and (ii) that the wave perturbations are self-localized what makes it possible to introduce the constants K and

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Key Words : Electron-Ion Plasma, Vortical Motion, Viscosity, Synergetic Approach, Strange Attractor.

g . Aiming to investigate the development of nonlinear regime in different conditions concerning external forces and internal plasma parameters, instead of the direct numerical solution of the system (1)[1] we first try to simplify it in a manner proposed by E.N. Lorentz[3] when considering the vorticity equation governing the atmosphere. For this purpose, we introduce the following expansion for ϕ and n :

$$\phi = \sum_{m=0,\, n=1}^{\infty} \phi_{nm} \sin(mkx) \sin(nly)$$

$$n = \sum_{m=0,\, n=1}^{\infty} N_{nm} \cos(mkx) \sin(nly) \ . \tag{2}$$

Following Lorentz procedure, these series are to be cut off as follows:

$$\begin{aligned} \phi &= X \sin kx \sin ly \\ n &= Y \sin ly \cos kx + Z \sin 2kx \ . \end{aligned} \tag{3}$$

Substituting (3) into (1) and introducing new variables:

$$\frac{c}{B_0} X \to X, \ \frac{1}{2} Y \to Y, \ Z \to Z \ ,$$

one obtains after some algebra:

$$\begin{aligned} \dot{X} &= gY - 2\mu X \\ \dot{Y} &= -rX - ZX \\ \dot{Z} &= -YX \end{aligned} \tag{4}$$

Here $r=K/2$, and a dot denotes a time derivative. This is the well known Lorentz type system of 3 ODE describing nonlinear evolution of many phenomena of different nature depending on the character of parameters entering the equations. In our case the parameter that will e paid the main attention is the ion viscosity coefficient μ . The most attractive feature of the Lorentz equations is simplicity that does not harm the nonlinearity as the essence of Nature.

2. Numerical Integration of the Lorentz Type Equations. State with Strange Attractors

To put the problem on a computer one has to evaluate the coefficients g , r and μ . The parameters value interval for current plasma systems can be taken as[4] : $T_i \approx 10^3 \div 10^4 K$, $n_e \approx 10^{16} \div 10^{18} cm^{-3}$, $m_i \approx A \cdot 1{,}66 \cdot 10^{-24} g$, the Coulomb logarithm $L \approx 10 \div 20$. The coefficient μ can be evaluated using the formula[5] :

$$\mu \approx 0{,}96\,\frac{T_i}{m_i}\,\tau_{ei} \approx \frac{6{,}17\cdot 10^{25}(T_i[KeV])^{5/2}[cm^2]}{n_e[cm^{-3}]\,Z^{*3}A^{1/2}(L/10)\,e}\;.$$

Here τ_{ei} is the electron-ion collision time, and simplicity one can set the ion charge $Z^*=1$ and the mass number $A=1$.

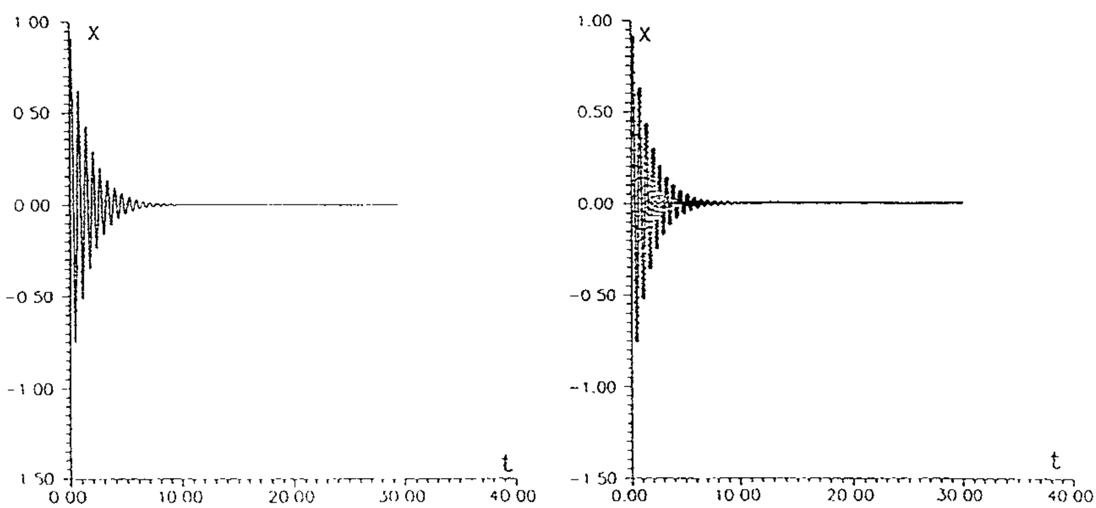
For illustration purpose we choose:

$$n_e = 10^{16}cm^{-3},\ T_i = 10^3K,\ L_n = 0{,}05 \quad \text{what gives:}$$
$$\mu \approx 0.6,\ g \approx 10,\ r = 1/(2L_n) \approx 10,\ R_B \approx 10L_n = 0.5\,.$$

Now the system (4) takes the form ready for numerical solution:

$$\dot{X} = 10Y - 1.2X\,,$$
$$\dot{Y} = -\,10X - ZX,$$
$$\dot{Z} = -\,XY \qquad . \tag{5}$$

We mention that the variable X represents the evolution of the potential ϕ, while Y and Z represent of the harmonics of the relative density perturbation n. Numerical solution of the system (5) is carried out using the method described in Ref.3. The initial state is taken in the vicinity of the equilibrium steady state with $(X_0, Y_0, Z_0) = (0, 0, 0)$. Graphs of X, Y, Z as functions of time (see Fig.1, e.g.) all show nonregular non-periodic character. As for the behavior of the trajectories in phase space (projections on the X - Y, X - Z, Y - Z planes, see Fig.2, e.g.), one can observe that they are confined in a finite region. For the first 1500 iterations the trajectory spirals outward from the vicinity of the initial point, then turns backward and spirals around the 0-point without attractor behavior that means the occurrence of an essentially nonlinear state with vortices formation.



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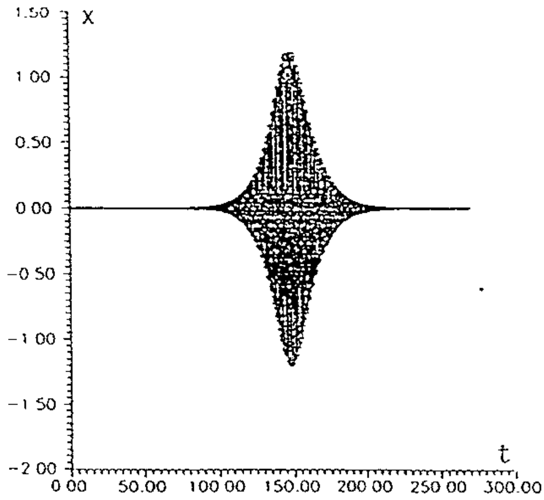

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Fig.1.

Numerical solution of the Lorentz type equations (5). Graph of X as a function of time showing nonregular nonperiodic character. Units are arbitrary. The dimensionless time increment $\Delta\ \tau = 0.01$ for (a) and $\Delta\ \tau = 0.09$ for (b,c) .Total number of iterations =3000

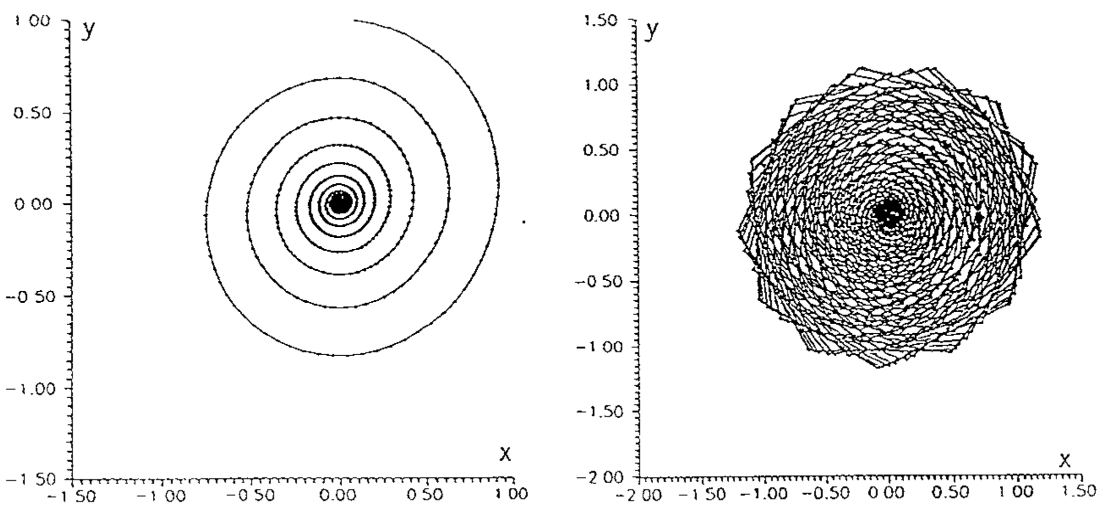

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Fig.2.

Numerical solution of the Lorentz type equations (5). Projection on the X-Y Plane for the first 3000 iterations showing the strange attractor behavior of the trajectory. The dimensionless time increment $\Delta\ \tau = 0.01$ for (a) (more fine structure) and $\Delta\ \tau = 0.09$ for (b) .

In conclusion, we note that such a synergetic, in our opinion, allows one to elucidate the real physical picture of the process and to follow its development with changing different plasma parameters (especially the viscosity coefficient μ) using a simple mathematical model, what will be the next step of this investigation.

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