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Vlasov Equation in the Stochastic Magnetic Field

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Abstract

We investigate the Vlasov equation in the stochastic magnetic field as a stochastic Liouville equation and derive the equation for the ensemble-averaged distribution function. The term resulting from the stochastic magnetic field has the derivatives with respect to both the velocity and the real space coordinates, which is a contrast to both the real space diffusion as seen in the guiding center picture and the velocity space diffusion as in the quasi-linear theory of the Vlasov equation including the electric field fluctuaions. We find that this term retains the mass and energy conservation properties of the original Lorentz force due to the stochastic magnetic field and yields the additional force in the momentum equation. This additional force produced by the stochastic field gives the drift velocity which corresponds to the familiar real space diffusion of the guiding center in the stochastic field. The finite Larmor radius effect on the diffusion is also estimated.

Keywords: stochastic magnetic field, stochastic Liouville equation

§1. Introduction

Particle diffusion due to the turbulent magnetic field is regarded as one of the mechanism of anomalous transport observed in magnetically confined plasmas.[1] Actually clear correlation between the confinement improvement and the reduction of magenetic fluctuations has been found experimentally as in the L-H transitions in tokamak plasmas.[2] Turbulent magnetic fields are generated from both the errors of external coils and the instabilities of the plasma itself. From the reason of its practical importance as well as its own interesting features as a problem of statistical physics, many theoretical studies on anomalous transport due to turbulent magenetic fields has been done.[3−5] Most of them are based on the picture of guiding centers streaming along the magnetic field lines. In this work we start from the Vlasov equation including the stochastic magnetic fields and consider only the collisionless case. The advantages of this treatment are that the finite larmor radius effects are directly included and that the stochastic field effects on the fluid equations are straightly obtained by taking moments of the Vlasov equation. Here we regard the Vlasov equation as a stochastic Liouville equation[6] due to the stochastic component of the magnetic field and derive the kinetic equation for the ensemble-averaged distribution function. This procedure is analogous to that in deriving Fokker-Planck equation from the Langevin equation. The term resulting from the stochastic magnetic field has the derivatives with respect to the velocity and the real space coordinates, which is different from both the real space diffusion as seen in the guiding center picture[4,5] and the velocity space diffusion as in the quasi-linear theory of the Vlasov equation including the electric field fluctuaions.[7] However it will be shown that this term retains the mass and energy conservation properties which the original Lorentz force term of the stochastic magnetic field has. Then we will find that the additional force due to the stochastic field appearing in the momentum equation gives the drift velocity which corresponds to the familiar real space diffusion of the guiding center in the stochastic field. It will be shown how the diffusion induced by the stochastic magnetic field is reducuced by the finite Larmor radius effect.

This paper is organized as follows. In §2, the general treatment of the stochastic Liouville equations are explained by following ref.6. In §3, we apply the theory given in §2 to the case of the guiding center streaming along the stochastic magnetic field lines and obtain the familar diffusion term in the collisionless limit. In §4, we treat the Vlasov equation in the stochastic magnetic field and derive the equation for the ensemble-averaged distribution function. There we examine the conservation properties of the term derived from the stochastic field and its relation to the guiding center diffusion. The finite Larmor radius effect on the diffusion is also estimated. Finally, discussion is given in §5.

§2. Stochastic Liouville Equations

Here the general theory of stochastic Liouville equations is briefly explained by following ref.6. A stochastic Liouville equation is written as

$$\frac{\partial \hat{f}(\boldsymbol{z},t)}{\partial t} = (L_0 + \tilde{L})\hat{f} \tag{2.1}$$

where $\boldsymbol{z}$ represents a point in the phase space considered here, $\hat{f}$ the distribution function as time t, L_0 the unperturbed part of the Liouville operator, and $\tilde{L}$ the perturbation part. Since $\tilde{L}$ include stochastic functions, the distribution function $\hat{f}$ is also stochastic.

Taking an ensemble average of (2.1) yields

$$\partial f/\partial t = L_0 f + \langle \tilde{L}\hat{f} \rangle \tag{2.2}$$

where $f \equiv \langle \hat{f} \rangle$ is the ensemble-averaged distribution function. One of our main purposes is to derive the equaition for the averaged distribution function f in the form of

$$\partial f/\partial t = (L_0 + \Gamma) f \tag{2.3}$$

in other words, by comparing (2.2) and (2.3), to obtain the operator Γ which satisfies

$$\langle \tilde{L}\hat{f} \rangle = \Gamma f. \tag{2.4}$$

Using the interaction representation

$$\hat{g}(t) = e^{-L_0 t}\hat{f}(t) \tag{2.5}$$

we have

$$\partial \hat{g}(t)/\partial t = \Omega(t)\hat{g} \tag{2.6}$$

with

$$\Omega(t) = e^{-L_0 t}\tilde{L}(t)e^{L_0 t}. \tag{2.7}$$

Specifying the non-stochastic initial distribution function $g(0) = f(0)$, we obtain the formal solution of (2.6) as

$$\begin{aligned} \hat{g}(t) &= \left[1 + \int_0^t dt_1 \Omega(t_1) + \int_0^t dt_1 \int_0^{t_1} dt_2 \Omega(t_1)\Omega(t_2) + \cdots \right] f(0) \\ &\equiv \exp_O \left(\int_0^t dt' \Omega(t') \right) \end{aligned} \tag{2.8}$$

where an ordered exponential is defined. Here we should note that $\Omega(t)$'s are not generally commutable operators.

Taking an ensemble average of (2.8), we have

$$g(t) \equiv \langle \hat{g}(t) \rangle = \Phi(t) f(0) \tag{2.9}$$

where the relaxation operator $\Phi(t)$ is defined by

$$\Phi(t) \equiv \left\langle \exp_O \left(\int_0^t dt' \Omega(t') \right) \right\rangle . \tag{2.10}$$

From (2.5) and (2.9), we obtain

$$f(t) = e^{L_0 t} \Phi(t) f(0). \tag{2.11}$$

The cumulant operator $K(t)$ is defined as

$$\begin{aligned} \Phi(t) &\equiv \left\langle \exp_O \left(\int_0^t dt' \Omega(t') \right) \right\rangle \\ &\equiv \exp_P \left\{ \int_0^t dt_1 \langle \Omega(t_1) \rangle_c + \int_0^t dt_1 \int_0^{t_1} dt_2 \langle \Omega(t_1) \Omega(t_2) \rangle_c + \cdots \right\} \\ &\equiv \exp_P K(t). \end{aligned} \tag{2.12}$$

Here the expansion of $\exp_P K(t)$ in series is defined in the complicated manner, which depends on the properties of the operator $\Omega(t)$, and the suffix P specifies a prescription for the ordering of $\Omega(t)$'s. A cumulant $\langle \Omega(t_1) \cdots \Omega(t_n) \rangle_c$ can be expressed by a sum of certain products of moments of lower and the same order. For example, the lowest-order cumulants may be written as

$$\begin{aligned} \langle \Omega(t) \rangle_c &= \langle \Omega(t) \rangle \\ \langle \Omega(t_1) \Omega(t_2) \rangle_c &= \langle \Omega(t_1) \Omega(t_2) \rangle - P \langle \Omega(t_1) \rangle \langle \Omega(t_2) \rangle . \end{aligned}$$

Finally, the equation for $f(t)$ is obtained from (2.11) and (2.12) as

$$\partial f / \partial t = (L_0 + \Gamma) f \tag{2.13}$$

with

$$\Gamma \equiv e^{L_0 t} \frac{dK(t)}{dt} e^{-L_0 t}. \tag{2.14}$$

Assuming that $\langle \tilde{L} \rangle = 0$, we find

$$\langle \Omega(t) \rangle_c = \langle \Omega(t) \rangle = 0$$

$$\langle \Omega(t_1)\Omega(t_2) \rangle_c = \langle \Omega(t_1)\Omega(t_2) \rangle.$$

If higher than the second order cumulants vanish, $\Omega(t)$ may be called Gaussian and then we have

$$K(t) = \int_0^t dt_1 \int_0^{t_1} dt_2 \langle \Omega(t_1)\Omega(t_2) \rangle. \tag{2.15}$$

Substituting (2.15) into (2.14) and using (2.7), we obtain

$$\Gamma = \int_0^\infty d\tau \langle \tilde{L}(t) e^{L_0 \tau} \tilde{L}(t-\tau) e^{-L_0 \tau} \rangle. \tag{2.16}$$

where the upper boundary of the integral is taken as infinity instead of t since we assume that t is much larger than the correlation time τ_c of $\Omega(t)$.

Futhermore, in the case where $\tilde{L}$ does not depend on time t explicitly, (2.16) is written as

$$\begin{aligned} \Gamma &= \int_0^\infty d\tau \langle \tilde{L} e^{L_0 \tau} \tilde{L} e^{-L_0 \tau} \rangle \\ &= \int_0^\infty d\tau \langle \Omega(0)\Omega(-\tau) \rangle. \end{aligned} \tag{2.17}$$

Thus Γ is given by the time integration of the correlation of $\Omega(t)$. In deriving (2.16) or (2.17), we neglected the contributions from cumulants of higher than the second order in (2.12), which is not valid if the stochastic perturbations are too large. Actually, (2.17) has the form similar to the diffusivity of the quasi-linear theory. Renormlization theories[8–11] such as Dupree's one[8] and direct-interaction approximation[9,10] suggest that, for large perturbations, the unperturbed propagator $e^{L_0 \tau}$ in (2.16) and (2.17) should be replaced by the renormalized propagator $e^{(L_0 + \Gamma)\tau}$, from which Γ is defined in the recursive manner. Discussion relating to the validity of use of (2.16) or (2.17) will be found in §5.

§3. A Guiding Center Streaming along the Stochastic Magnetic Field Line

As a simple example of the application of the procedure given in the preceding section, we treat a guiding center streaming freely along the stochastic magnetic field line. Here we do not consider the effects of collisions and the finite Larmor radius of the particle gyro-motion. For simplicity, assuming that the guiding center has a constant velocity v along the field line, the probability distribution of the particle satisfies the following stochastic Liouville equation

$$\frac{\partial \hat{f}(\boldsymbol{x},t)}{\partial t} = (L_0 + \tilde{L})\hat{f} \tag{3.1}$$

where

$$L_0 = -v\boldsymbol{b}_0 \cdot \frac{\partial}{\partial \boldsymbol{x}} \tag{3.2}$$

$$\tilde{L} = -v\tilde{\boldsymbol{b}} \cdot \frac{\partial}{\partial \boldsymbol{x}}. \tag{3.3}$$

Here $\hat{f}(\boldsymbol{x},t)$ represents the probability density at time t at the point $\boldsymbol{x}$ in the real space. The unperturbed part of the Liouville operator L_0 corresponds to the guiding center motion along the unperturbed uniform magnetic field line, the direction of which is denoted by $\boldsymbol{b}_0$. The perturbation part $\tilde{L}$ represents the effect of the stochastic magnetic field component, and $\tilde{\boldsymbol{b}}$ the stochastic variation in the direction of the magnetic field.

For simplicity, we put

$$\boldsymbol{b}_0 = \boldsymbol{e}_z, \qquad \tilde{\boldsymbol{b}} = \tilde{b}_x(z)\boldsymbol{e}_x + \tilde{b}_y(z)\boldsymbol{e}_y \tag{3.4}$$

where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$ are unit vectors in the x, y and z-directions, respectively. Here we assumed that the stochastic variables $\tilde{b}_x$ and $\tilde{b}_y$ are functions of z alone, which do not depends explicitly on x, y and t. From (3.2)–(3.4) and (2.7), we obtain

$$\Omega(t) = -v\tilde{\boldsymbol{b}}(z+vt) \cdot \frac{\partial}{\partial \boldsymbol{x}}. \tag{3.5}$$

We find from (3.4) and (3.5) that in this case $\Omega(t)$'s are commutable operators,

$$[\Omega(t_1), \Omega(t_2)] \equiv \Omega(t_1)\Omega(t_2) - \Omega(t_2)\Omega(t_1) = 0. \tag{3.6}$$

Assuming that $\tilde{\boldsymbol{b}}$ is a homogeneous Gaussian stochastic vector variable with a zero mean, we have

$$\langle \tilde{\boldsymbol{b}} \rangle = 0, \qquad \langle \tilde{\boldsymbol{b}}(z_1)\tilde{\boldsymbol{b}}(z_2) \rangle = \langle \tilde{\boldsymbol{b}}(z_1 - z_2)\tilde{\boldsymbol{b}}(0) \rangle \tag{3.7}$$

which is used with (2.15) to yield

$$\begin{aligned} K(t) &= v^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \langle \tilde{\boldsymbol{b}}(z + vt_1)\tilde{\boldsymbol{b}}(z + vt_2) \rangle : \frac{\partial^2}{\partial \boldsymbol{x} \partial \boldsymbol{x}} \\ &= v^2 \int_0^t d\tau (t - \tau) \langle \tilde{\boldsymbol{b}}(v\tau)\tilde{\boldsymbol{b}}(0) \rangle : \frac{\partial^2}{\partial \boldsymbol{x} \partial \boldsymbol{x}}. \end{aligned} \tag{3.8}$$

Then we obtain from (2.14) and (3.8)

$$\begin{aligned} \Gamma &= v^2 \int_0^t d\tau \langle \tilde{\boldsymbol{b}}(v\tau)\tilde{\boldsymbol{b}}(0) \rangle : \frac{\partial^2}{\partial \boldsymbol{x} \partial \boldsymbol{x}} \\ &= |v| \mathsf{D}_m : \frac{\partial^2}{\partial \boldsymbol{x} \partial \boldsymbol{x}} \end{aligned} \tag{3.9}$$

which can be also derived from (2.17) and (3.5) easily. Here, as in (2.16), we took the upper boundary of the integral as infinity instead of t by assuming that t is much larger than the correlation time τ_c which is defined as the correlation length of $\tilde{\boldsymbol{b}}$ in the z-direction divided by $|v|$. In (3.9) we defined the diffusion tensor D_m of the stochastic field line by

$$\mathsf{D}_m \equiv \int_0^\infty dz \langle \tilde{\boldsymbol{b}}(z)\tilde{\boldsymbol{b}}(0) \rangle. \tag{3.10}$$

Substituting (3.2) and (3.9) into (2.13), we finally obtain the equation for the ensemble-averaged probability density $f \equiv \langle \hat{f} \rangle$ as

$$\frac{\partial f}{\partial t} + v\boldsymbol{b}_0 \cdot \frac{\partial f}{\partial \boldsymbol{x}} = |v| \mathsf{D}_m : \frac{\partial^2 f}{\partial \boldsymbol{x} \partial \boldsymbol{x}}. \tag{3.11}$$

Thus we see that the diffusivity due to the stochastic magnetic field is given by $|v|\mathsf{D}_m$, which coincides with the familiar result in the collisionless case.[4,5]

§4. Stochastic Vlasov Equation

Here we apply the procedure described in §2 to the following stochastic Vlasov equation :

$$\frac{\partial \hat{f}(\boldsymbol{x},\boldsymbol{v},t)}{\partial t} = (L_0 + \tilde{L})\hat{f} \tag{4.1}$$

where

$$L_0 = -\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}} - (\boldsymbol{v}\times\boldsymbol{\omega}_0)\cdot\frac{\partial}{\partial \boldsymbol{v}} \tag{4.2}$$

$$\tilde{L} = -(\boldsymbol{v}\times\tilde{\boldsymbol{\omega}})\cdot\frac{\partial}{\partial \boldsymbol{v}}. \tag{4.3}$$

Here $\hat{f}(\boldsymbol{x},\boldsymbol{v},t)$ represents the distribution function in the phase space, $\boldsymbol{\omega}_0$ and $\tilde{\boldsymbol{\omega}}$ are defined by

$$\boldsymbol{\omega}_0 = \frac{q\boldsymbol{B}_0}{mc}, \qquad \tilde{\boldsymbol{\omega}} = \frac{q\tilde{\boldsymbol{B}}}{mc}, \tag{4.4}$$

where $\boldsymbol{B}_0$ is the unperturbed magentic field, $\tilde{\boldsymbol{B}}$ the stochastic magnetic field component, q and m the electric charge and mass of the particle, respectively. The electric field is neglected here although we will discuss its effects later in §5. As in §3, assuming that the unperturbed field $\boldsymbol{B}_0$ is uniform and that $\tilde{\boldsymbol{B}}$ is perpendicular to $\boldsymbol{B}_0$, we put

$$\boldsymbol{\omega}_0 = \omega_0\boldsymbol{e}_z \;\; (\omega_0 \equiv qB_0/mc = \text{const}), \qquad \tilde{\boldsymbol{\omega}} = \tilde{\omega}_x(\boldsymbol{x})\boldsymbol{e}_x + \tilde{\omega}_y(\boldsymbol{x})\boldsymbol{e}_y \tag{4.5}$$

where we retained the dependence of $\tilde{\boldsymbol{\omega}}$ (or $\tilde{\boldsymbol{B}}$) on the perpendicular spatial coordinates (x, y) as well as the parallel coordinate z, which is inevitable when we take account of the finite Larmor radius effect.

Without the stochastic field, the particle initially placed at $(\boldsymbol{x},\boldsymbol{v})$ in the phase space is found at $(\bar{\boldsymbol{x}}(t),\bar{\boldsymbol{v}}(t))$ at time t. Here $(\bar{\boldsymbol{x}}(t),\bar{\boldsymbol{v}}(t))$ is the solution of the ordinary differential equations

$$\frac{d\bar{\boldsymbol{x}}}{dt} = \bar{\boldsymbol{v}}, \qquad \frac{d\bar{\boldsymbol{v}}}{dt} = \bar{\boldsymbol{v}}\times\boldsymbol{\omega}_0$$

with the initial conditions

$$\bar{\boldsymbol{x}}(0) = \boldsymbol{x}, \qquad \boldsymbol{v}(0) = \boldsymbol{v}$$

and they are given by

$$\bar{x}(t) \quad = \quad x + (v_x/\omega_0)\sin\omega_0 t - (v_y/\omega_0)(\cos\omega_0 t - 1)$$

$$
\begin{aligned}
\bar{y}(t) &= y + (v_x/\omega_0)(\cos\omega_0 t - 1) + (v_y/\omega_0)\sin\omega_0 t \\
\bar{z}(t) &= z + v_z t \\
\bar{v}_x(t) &= v_x \cos\omega_0 t + v_y \sin\omega_0 t \\
\bar{v}_y(t) &= -v_x \sin\omega_0 t + v_y \cos\omega_0 t \\
\bar{v}_z(t) &= v_z.
\end{aligned}
\tag{4.6}
$$

Using (4.6) we have for an arbitrary function $F(\boldsymbol{x}, \boldsymbol{v})$

$$
e^{-L_0 t} F(\boldsymbol{x}, \boldsymbol{v}) e^{L_0 t} = F(\bar{\boldsymbol{x}}(t), \bar{\boldsymbol{v}}(t)). \tag{4.7}
$$

In order to make the analyses easier, it is convenient to use the cylindrical coordinates $(v_\perp, v_\theta, v_z)$ in the velocity space. Then we have

$$
\begin{aligned}
L_0 &= -v_z \frac{\partial}{\partial z} - v_\perp \left(\cos\theta \frac{\partial}{\partial x} + \sin\theta \frac{\partial}{\partial y} \right) + \omega_0 \frac{\partial}{\partial \theta} \\
\tilde{L} &= -v_z \left([\tilde{\omega}_x \sin\theta - \tilde{\omega}_y \cos\theta] \frac{\partial}{\partial v_\perp} + [\tilde{\omega}_x \cos\theta + \tilde{\omega}_y \sin\theta] \frac{1}{v_\perp} \frac{\partial}{\partial \theta} \right) \\
&\quad + v_\perp [\tilde{\omega}_x \sin\theta - \tilde{\omega}_y \cos\theta] \frac{\partial}{\partial v_z}
\end{aligned}
\tag{4.8}
$$

and

$$
\begin{aligned}
\bar{x}(t) &= x - (v_\perp/\omega_0)[\sin(\theta - \omega_0 t) - \sin\theta] \\
\bar{y}(t) &= y + (v_\perp/\omega_0)[\cos(\theta - \omega_0 t) - \cos\theta] \\
\bar{v}_\perp(t) &= v_\perp \\
\bar{\theta}(t) &= \theta - \omega_0 t.
\end{aligned}
\tag{4.9}
$$

Furthermore making use of the following formula for arbitrary operators A and B

$$
e^{-A} B e^{A} = B + [B, A] + \frac{1}{2!}[[B, A], A] + \frac{1}{3!}[[[B, A], A], A] + \cdots \tag{4.10}
$$

we can derive the following relations

$$
e^{-L_0 t} \frac{\partial}{\partial v_\perp} e^{L_0 t} = \frac{\partial}{\partial v_\perp} + \frac{1}{\omega_0} \left([\sin(\theta - \omega_0 t) - \sin\theta] \frac{\partial}{\partial x} - [\cos(\theta - \omega_0 t) - \cos\theta] \frac{\partial}{\partial y} \right)
$$

$$
e^{-L_0 t} \frac{\partial}{\partial \theta} e^{L_0 t} = \frac{\partial}{\partial \theta} + \frac{v_\perp}{\omega_0} \left([\cos(\theta - \omega_0 t) - \cos\theta] \frac{\partial}{\partial x} + [\sin(\theta - \omega_0 t) - \sin\theta] \frac{\partial}{\partial y} \right)
$$

$$e^{-L_0 t}\frac{\partial}{\partial v_z}e^{L_0 t} = \frac{\partial}{\partial v_z} - t\frac{\partial}{\partial z}. \tag{4.11}$$

Using (4.6)–(4.9) and (4.11) to evaluate (2.7), we obtain

$$\Omega(t) = W(t) + Z(t) + X(t) \tag{4.12}$$

where

$$\begin{aligned}
W(t) &\equiv -v_z\Bigg([\tilde{\omega}_x(\bar{\boldsymbol{x}}(t))\sin(\theta-\omega_0 t) - \tilde{\omega}_y(\bar{\boldsymbol{x}}(t))\cos(\theta-\omega_0 t)]\frac{\partial}{\partial v_\perp} \\
&\quad + [\tilde{\omega}_x(\bar{\boldsymbol{x}}(t))\cos(\theta-\omega_0 t) + \tilde{\omega}_y(\bar{\boldsymbol{x}}(t))\sin(\theta-\omega_0 t)]\frac{1}{v_\perp}\frac{\partial}{\partial\theta}\Bigg) \\
Z(t) &\equiv v_\perp[\tilde{\omega}_x(\bar{\boldsymbol{x}}(t))\sin(\theta-\omega_0 t) - \tilde{\omega}_y(\bar{\boldsymbol{x}}(t))\cos(\theta-\omega_0 t)]\left(\frac{\partial}{\partial v_z} - t\frac{\partial}{\partial z}\right) \\
X(t) &\equiv \frac{v_z}{\omega_0}\Bigg([\tilde{\omega}_x(\bar{\boldsymbol{x}}(t))(\cos\omega_0 t - 1) - \tilde{\omega}_y(\bar{\boldsymbol{x}}(t))\sin\omega_0 t]\frac{\partial}{\partial x} \\
&\quad + [\tilde{\omega}_x(\bar{\boldsymbol{x}}(t))\sin\omega_0 t + \tilde{\omega}_y(\bar{\boldsymbol{x}}(t))(\cos\omega_0 t - 1)]\frac{\partial}{\partial y}\Bigg).
\end{aligned} \tag{4.13}$$

As in (3.7), assuming that the stochastic magnetic field $\tilde{\boldsymbol{B}}$ has a zero mean and is stochastically homogeneous, we can put

$$\langle\tilde{\boldsymbol{\omega}}(\boldsymbol{x})\rangle = 0, \qquad \langle\tilde{\boldsymbol{\omega}}(\boldsymbol{x}_1)\tilde{\boldsymbol{\omega}}(\boldsymbol{x}_2)\rangle = \langle\tilde{\boldsymbol{\omega}}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\tilde{\boldsymbol{\omega}}(0)\rangle. \tag{4.14}$$

From (2.17), (4.12) and (4.13), we find

$$\Gamma = \Gamma_W + \Gamma_Z + \Gamma_{WZ} + \Gamma_{WX} + \Gamma_{ZX} \tag{4.15}$$

where

$$\begin{aligned}
\Gamma_W &= \int_0^\infty d\tau\langle W(0)W(-\tau)\rangle \\
\Gamma_Z &= \int_0^\infty d\tau\langle Z(0)Z(-\tau)\rangle \\
\Gamma_{WZ} &= \int_0^\infty d\tau\langle W(0)Z(-\tau) + Z(0)W(-\tau)\rangle \\
\Gamma_{WX} &= \int_0^\infty d\tau\langle W(0)X(-\tau)\rangle \\
\Gamma_{ZX} &= \int_0^\infty d\tau\langle Z(0)X(-\tau)\rangle.
\end{aligned} \tag{4.16}$$

Here Γ_W represents the perpendicular diffusion in the velocity space, which contains the second-order derivatives with respect to $v_\perp$ and θ. We find parallel derivatives $\partial/\partial v_z$ and $\partial/\partial z$ in Γ_Z. In Γ_{WZ}, the perpendicular derivatives $(\partial/\partial v_\perp, \partial/\partial\theta)$ are coupled to the parallel derivatives $(\partial/\partial v_z, \partial/\partial z)$. Similarly Γ_{WX} and Γ_{ZX} contain the cross derivatives $(\partial/\partial v_\perp, \partial/\partial\theta) \times (\partial/\partial x, \partial/\partial y)$ and $\partial/\partial v_z \times (\partial/\partial x, \partial/\partial y)$, respectively. Thus Γ has the velocity diffusion terms as well as the cross terms consisting of the velocity and real space coordinate derivatives. However we should note that Γ does not include the real space diffusion given by the second-order derivatives with respect to the real space coordinates. This is a remarkable contrast to (3.9) although we will find later that a real space diffusion similar to (3.9) is derived from (4.15) in the indirect manner.

We define the correlation tensor $\mathsf{C}(\tau)$ as

$$\begin{aligned} \mathsf{C}(\tau) &= \sum_{i,j=x,y} C_{ij}(\tau)\boldsymbol{e}_i\boldsymbol{e}_j \\ &= \langle\tilde{\boldsymbol{\omega}}(\bar{\boldsymbol{x}}(0)-\bar{\boldsymbol{x}}(-\tau))\tilde{\boldsymbol{\omega}}(0)\rangle/\omega_0^2 \\ &= \langle\tilde{\boldsymbol{b}}(\bar{\boldsymbol{x}}(0)-\bar{\boldsymbol{x}}(-\tau))\tilde{\boldsymbol{b}}(0)\rangle \end{aligned} \tag{4.17}$$

where we used $\tilde{\boldsymbol{\omega}}/\omega_0 = \tilde{\boldsymbol{B}}/B_0 = \tilde{\boldsymbol{b}}$. In (4.16) time integration terms such as $\int_0^\infty d\tau\mathsf{C}(\tau)\cos\omega_0 t$ are included. If we assume that the gyration period ω^{-1} is much smaller than the correlation time τ_c for $\mathsf{C}(\tau)$, the integrands oscillate rapidly and these integrals become negligibly small. With this assumption, we find Γ_W, Γ_Z and Γ_{WZ} to neglected. Then the remaining terms are given by

$$\begin{aligned} \Gamma_{WX} &= \omega_0\left[\left(\frac{\partial}{\partial v_y}D_{xx} - \frac{\partial}{\partial v_x}D_{yx}\right)\frac{\partial}{\partial x} + \left(\frac{\partial}{\partial v_y}D_{xy} - \frac{\partial}{\partial v_x}D_{yy}\right)\frac{\partial}{\partial y}\right] \\ \Gamma_{ZX} &= \omega_0\frac{\partial}{\partial v_z}\frac{1}{v_z}\left[(-v_yD_{xx}+v_xD_{yx})\frac{\partial}{\partial x} + (-v_yD_{xy}+v_xD_{yy})\frac{\partial}{\partial y}\right] \end{aligned} \tag{4.18}$$

which are summed up to give $\Gamma = \Gamma_{WX} + \Gamma_{ZX}$ in the greatly compact form as

$$\Gamma = \omega_0\frac{\partial}{\partial\boldsymbol{v}}\cdot\left(\frac{\boldsymbol{v}}{v_\parallel}\times\mathsf{D}\cdot\frac{\partial}{\partial\boldsymbol{x}}\right) \tag{4.19}$$

where we put $v_\parallel = \boldsymbol{v}\cdot\boldsymbol{b}_0 = v_z$ and

$$\mathsf{D} = \sum_{i,j=x,y} D_{ij}\boldsymbol{e}_i\boldsymbol{e}_j = v_\parallel^2\int_0^\infty d\tau\mathsf{C}(\tau). \tag{4.20}$$

The equation for the ensemble-averaged distribution function $f \equiv \langle \hat{f} \rangle$ is obtained from (2.3), (4.2) and (4.19) as

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + (\boldsymbol{v} \times \boldsymbol{\omega}_0) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \omega_0 \frac{\partial}{\partial \boldsymbol{v}} \cdot \left(\frac{\boldsymbol{v}}{v_\parallel} \times \mathsf{D} \cdot \frac{\partial f}{\partial \boldsymbol{x}} \right). \tag{4.21}$$

Here we consider the conservation properties of the term Γf deriving from the stochastic magnetic field. We find from (2.4)

$$\left\langle \int d\boldsymbol{v} A(\boldsymbol{v}) \tilde{L} \hat{f} \right\rangle = \int d\boldsymbol{v} A(\boldsymbol{v}) \Gamma f \tag{4.22}$$

for an arbitrary function of velocity $A(\boldsymbol{v})$ which is not stochastic. From (4.3) we see that the left-hand side of (4.21) should vanish in the case of $A(\boldsymbol{v}) = 1$ and $A(\boldsymbol{v}) = \frac{1}{2}mv^2$, which implies the conservation of mass (or number) and kinetic energy of particles by the stochastic operator $\tilde{L}$. These conservation properties must be retained in the operator Γ as is seen from (4.22). Actually, we can easily show from (4.19) that

$$\int d\boldsymbol{v} \Gamma f = \int d\boldsymbol{v} \frac{1}{2} m v^2 \Gamma f = 0. \tag{4.23}$$

It should be noted that these conservation properties are not satisfied generally by the real space diffusion type operator consisting of the sencond-order derivatives with respect to the real space coordinates, and therefore that we cannot allow that type operator as Γ.

When we put $A(\boldsymbol{v}) = m\boldsymbol{v}$ in (4.22), we obtain the average force per unit volume due to the stochastic magnetic field as

$$\boldsymbol{F} \equiv \int d\boldsymbol{v} m \boldsymbol{v} \Gamma f = -\frac{qB_0}{c} \int d\boldsymbol{v} \frac{\boldsymbol{v}}{v_\parallel} \times \mathsf{D} \cdot \frac{\partial f}{\partial \boldsymbol{x}}. \tag{4.24}$$

Then the average drift velocity $\boldsymbol{v}_F$ produced by this force is given by

$$n \boldsymbol{v}_F \equiv \frac{c}{qB_0} \boldsymbol{F} \times \boldsymbol{b}_0 = -\int d\boldsymbol{v} \mathsf{D} \cdot \frac{\partial f}{\partial \boldsymbol{x}} \tag{4.25}$$

where $n \equiv \int d\boldsymbol{v} f$ is the density. We see that the particle flux (4.25) is driven by the gradient of the distribution function in the real space, which corresponds to the real space diffusion as given by (3.9).

From (4.6) and (4.9), we have the relative displacement of the particle in the time interval τ as

$$\begin{aligned}\boldsymbol{r}(\tau) &\equiv \bar{\boldsymbol{x}}(0) - \bar{\boldsymbol{x}}(-\tau) \\ &= v_z\tau\boldsymbol{e}_z + (v_\perp/\omega_0)([\sin(\theta+\omega_0\tau) - \sin\theta]\boldsymbol{e}_x - [\cos(\theta+\omega_0\tau) - \cos\theta]\boldsymbol{e}_y)\end{aligned} \tag{4.26}$$

from which the parallel and perpendicular distances are given by

$$\begin{aligned} r_\parallel(\tau) &= v_\parallel \tau \\ r_\perp(\tau) &= 2(v_\perp/\omega_0)|\sin(\omega_0\tau/2)|. \end{aligned} \tag{4.27}$$

It is seen from (4.17), (4.26) and (4.27) that the correlation tensor $\mathsf{C}(\tau)$ dampes due to the parallel motion and oscillates due to the perpendicular gyro-motion as τ increases. The parallel damping is much slower than the perpendicular oscillation because of the assumption employed to derive (4.18). Here we define $L_\parallel$ and $L_\perp$ as the parallel and perpendicular correlation lengths of the stochastic magnetic field. For the particle with the Larmor radius $\rho \equiv v_\perp/\omega_0$ much smaller than $L_\perp$, decorrelation due to the perpendicular motion can be neglected and the diffusion tensor (4.20) is rewritten as

$$\mathsf{D} = v_\parallel^2 \int_0^\infty d\tau \langle \tilde{\boldsymbol{b}}(v_\parallel \tau \boldsymbol{b}) \tilde{\boldsymbol{b}}(0)\rangle = |v_\parallel| \mathsf{D}_m \quad (\text{for } \ \rho < L_\perp) \tag{4.28}$$

where D_m is the diffusion tensor of the magnetic field line given by (3.10). This diffusion tensor has exactly the same form as in (3.9). However, for the particle with the Larmor radius much larger than $L_\perp$, the perpendicular decorrelation is so strong that the diffusion tensor (4.20) becomes smaller than (4.28) by a factor of $L_\perp/\rho$ and we have

$$\mathsf{D} \sim (L_\perp/\rho)|v_\parallel|\mathsf{D}_m \quad (\text{for } \ \rho > L_\perp) \tag{4.29}$$

When the Larmor radius $\rho_{th} \equiv v_{th}/\omega_0$ given by the thermal velocity $v_{th} \equiv \sqrt{T/m}$ is much smaller than $L_\perp$, we find from (4.25) and (4.28)

$$n\boldsymbol{v}_F = -\left(\frac{2}{\pi}\right)^{1/2} v_{th}\mathsf{D}_m \cdot \frac{\partial n}{\partial \boldsymbol{x}} \tag{4.30}$$

where we used the local Maxwellian distribution funtion with a constant temperature T for f. On the other hand, when the thermal Larmor radius $\rho_{th} \equiv v_{th}/\omega_0$ is much larger than $L_\perp$, (4.25) is roughly estimated using (4.29) as

$$n\boldsymbol{v}_F \sim -\left(\frac{L_\perp}{\rho_{th}}\right) v_{th} \mathsf{D}_m \cdot \frac{\partial n}{\partial \boldsymbol{x}} \tag{4.31}$$

which is smaller than (4.30) by a factor of $L_\perp/\rho_{th}$.

In Appendix, the expression of the particle flux unifying the both limits (4.30) and (4.31) is given by specifying the wavenumber spectral function of the stochastic magnetic field with the statistical isotropy in the perpendicular plane.

§5. Discussion

In the preceding section, we negelected such effects as collisions, electric fields, inhomogeneity of the unperturbed (or external) magnetic fields and explicit time dependence of the stochastic fields. Therefore these effects are required to be small for the results we obtained to be valid. Here we examine the conditions of the validity. Since the main contribution to the integration (2.17) or (4.20) for Γ comes from several correlation times of the stochastic field, the mean free path between the partcle collisions and the characteristic length of the inhomogeneous external magnetic field should be larger than the correlation length of the stochastic field. If collisional effects are significant, transport due to the stochastic field is expected to decrease compared to that in the collisionless case as in refs.4 and 5. When the effects of the inhomogeneities of the external magnetic field such as magnetic shear and curvature are not negligible, we must use the correct particle orbit in the inhomogeneous field instead of (4.6) for the path integral of (4.20).

In our calculations, the stochastic field should be regarded as static whithin the correlation time (which is defined as the correlation length of the stochastic field devided by the thermal velocity). The external electric field also need to be negligible although, when the external electric field is homogeneous and perpendicular to the external magnetic field, we can apply the same procedure as in §4 using the Galilean transformation to the frame moving with the $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity. Then we must take account of the $\boldsymbol{E} \times \boldsymbol{B}$ drift motion in the relative displacement $\boldsymbol{r}(\tau) \equiv \bar{\boldsymbol{x}}(0) - \bar{\boldsymbol{x}}(-\tau)$ appeared in the correlation tensor $\mathrm{C}(\tau)$ defined in (4.17). This shows the possibility of the reduction of the particle diffusion by the external perpendicular electric field through the enhancenment of the relative separation and the resultant decorrelation. If the fluctuations of the electric field, which were neglected in this work, exist, we should add well-known velocity space diffusion, which causes the energy transfer between particles and fields in contranst with the stochastic magnetic field case (see (4.23)).

In evaluating the diffusion tensors (3.10) and (4.20), we did not take accout of the modification of the integral paths due to the stochastic field. The effects of the perpendicular diffusion of the integral paths due to the stochastic field on (3.10) and (4.20) can be neglected if the stochastic field line diffuses perpendicularly less than $L_\perp$ during the parallel displacement of $L_\parallel$:

$$\langle \tilde{b}^2 \rangle^{1/2} < L_\perp / L_\parallel .$$

If we estimate roughly the parallel and perpendicular correlation lengths as $L_\parallel \sim R$ (the toroidal major radius) $\sim 10^0 - 10^1 m$ and $L_\perp \sim \rho_s (\equiv c\sqrt{m_i T_e}/eB) \sim 10^{-3} m$ with $T_e \sim 10^2 - 10^3 eV$ and $B \sim 10^0 T$, the above condition is given by $\langle \tilde{b}^2 \rangle^{1/2} < 10^{-4} - 10^{-3}$. This condition is likely to be almost satisfied in the present tokamak plasmas. However, for larger magnetic fluctuations with $\langle \tilde{b}^2 \rangle^{1/2} > L_\perp / L_\parallel$, we need to include the correction due to the stochastic field into the integral paths of (3.10) and (4.20), which requires more advanced treatment such as renormalization technique.[8–11)]

In order to derive (4.18), we used the drift ordering given by $(\omega_0 \tau_c)^{-1} = \rho / L_\parallel \ll 1$, which is considered to be valid in the experimental plasmas where the parallel correlation length is quite larger than the thermal Larmor radius. Since generally the perpendicular correlation length is shorter than the parallel one, it possibly happens that the Larmor radius is comparable to or larger than the perpendicular correlation length. Then the diffusion of the particles with such large Larmor radii due to the stochastic field is expected to be considerably reduced according to (4.29) and (4.31). Actually it is experimentally observed that the anomaly of transport of the energetic particles is relatively smaller than that of the particles with less energy and so is that of ions compared to that of electrons.[1)]

One may have wondered why the term resulting from the stochastic magnetic field has the form given in the right-hand side of (4.21) which is different from both the real space diffusion in (3.11) and the velocity space diffusion as seen in the case of the electric field fluctuaions. However this form is natural from the point of view of the conservation properties owned by the original stochastic magnetic field term as found in (4.23). The term we derived retains the mass and energy conservation. Then it was shown that the additional force due to the stochastic field appears in the momentum equation and that this force in turn gives the drift velocity leading to the familar type of diffusive flux. It is interesting as a future task how this force affects the instability or other problems when it is included in the momentum equation or in the Ohm's law.

Appendix:

Finite Lamor Fadius Dependence of the Diffusivity

If we assume that the stochastic magnetic field $\tilde{\boldsymbol{B}}$ is statistically isotropic in the plane perpendicular to the unperturbed magnetic field $\boldsymbol{B}_0$, the correlation tensor $\langle \tilde{\boldsymbol{b}}(\boldsymbol{r})\tilde{\boldsymbol{b}}(0)\rangle$ is expanded in the wavenumber space as

$$\langle \tilde{\boldsymbol{b}}(\boldsymbol{r})\tilde{\boldsymbol{b}}(0)\rangle = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} dk_\parallel \int_0^\infty k_\perp dk_\perp \int_0^{2\pi} d\phi \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) Q(k_\parallel, k\perp)\mathsf{P}(\phi) \tag{A.1}$$

where $(k_\perp, \phi, k_\parallel)$ are the cylindrical coordinates in the wavenumber vector space and the tensor $\mathsf{P}(\phi)$ is defined by

$$\mathsf{P}(\phi) = \mathsf{I}_\perp - \boldsymbol{n}_\perp(\phi)\boldsymbol{n}_\perp(\phi) \tag{A.2}$$

with

$$\begin{aligned} \mathsf{I}_\perp &\equiv \mathsf{I} - \boldsymbol{b}_0\boldsymbol{b}_0 \equiv \boldsymbol{e}_x\boldsymbol{e}_x + \boldsymbol{e}_y\boldsymbol{e}_y \\ \boldsymbol{n}(\phi) &\equiv \boldsymbol{k}_\perp/k_\perp \equiv \boldsymbol{e}_x\cos\phi + \boldsymbol{e}_y\sin\phi. \end{aligned} \tag{A.3}$$

Here $\mathsf{P}(\phi)$ appeared due to the solenoidal condition $\nabla\cdot\tilde{\boldsymbol{b}} = \nabla\cdot\tilde{\boldsymbol{B}}/B_0 = (\partial\tilde{B}_x/\partial x + \partial\tilde{B}_y/\partial y)/B_0 = 0$. We can write $\mathsf{P}(\phi)$ in the form of 2×2 matrix in the (x,y)-plane as

$$\mathsf{P}(\phi) = \begin{bmatrix} \sin^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \cos^2\phi \end{bmatrix}.$$

Using (4.17), (4.20), (4.26) and (A.1)–(A.3) and assuming that $\omega_0 \gg k_\parallel v_\parallel$, we have

$$\begin{aligned} \mathsf{D} &= \frac{v_\parallel^2}{2(2\pi)^2}\int_{-\infty}^{\infty} dk_\parallel \int_0^\infty k_\perp dk_\perp \frac{Q(k_\parallel, k_\perp)}{-ik_\parallel v_\parallel + 0} J_0\left(\frac{k_\perp v_\perp}{\omega_0}\right) \\ &\times \begin{bmatrix} J_0\left(\frac{k_\perp v_\perp}{\omega_0}\right) - J_2\left(\frac{k_\perp v_\perp}{\omega_0}\right)\cos 2\theta & -J_2\left(\frac{k_\perp v_\perp}{\omega_0}\right)\sin 2\theta \\ -J_2\left(\frac{k_\perp v_\perp}{\omega_0}\right)\sin 2\theta & J_0\left(\frac{k_\perp v_\perp}{\omega_0}\right) + J_2\left(\frac{k_\perp v_\perp}{\omega_0}\right)\cos 2\theta \end{bmatrix}. \end{aligned} \tag{A.4}$$

Now we assume that the spectral function $Q(k_\parallel, k_\perp)$ is written as

$$Q(k_\parallel, k_\perp) = Q_\parallel(k_\parallel)\exp\left(-\frac{1}{2}k_\perp^2 L_\perp^2\right) \tag{A.5}$$

where $L_\perp$ is interpreted as the perpendicular correlation length of the stochastic magnetic field. Substituting (A.4) and (A.5) into (4.25) and using the local Maxwellian distribution function for f, we obtain

$$n\boldsymbol{v}_F = \sqrt{\frac{2}{\pi}}\frac{v_{th}D_m}{[1+4(\rho_{th}/L_\perp)^2]^{1/2}}\frac{\partial n}{\partial \boldsymbol{x}_\perp} \tag{A.6}$$

where we used $\partial/\partial\boldsymbol{x}_\perp = \mathsf{I}_\perp \cdot \partial/\partial\boldsymbol{x}$ and the diffusivity tensor defined by (3.10) rewritten as $\mathsf{D}_m = D_m\mathsf{I}_\perp$ due to the assumption of the isotropy in the perpendicular plane given at the beginning of this appendix. In the limits of $\rho_{th}/L_\perp \ll 1$ and $\rho_{th}/L_\perp \gg 1$, (A.6) reduces to (4.29) and (4.31), respectively.

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