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On relation between Hamada and Boozer magnetic coordinate system

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Abstract

The relation between the Hamada and the Boozer magnetic coordinate system is clarified by deriving them from a general magnetic field coordinate system. The coordinate transformation from the Hamada to the Boozer coordinate system is performed using a transformation function, which is easily calculated with the knowledge of the magnetic field strength only. In non-axisymmetric systems, the Fourier spectrum of the magnetic field strength $|\vec{B}|$ in the Hamada coordinate system is broader than that in the Boozer coordinate system and the leading modes of $|\vec{B}|$ in the Hamada coordinate system significantly deviate from a simple model field in comparison with the Boozer coordinate system. The Boozer coordinate system is suitable for numerical calculations, especially, for neoclassical and orbit calculations in non-axisymmetric systems.

Keywords: Magnetic coordinate system, Hamada coordinate system, Boozer coordinate system, Straight magnetic field line coordinate system, coordinate transformation, non-axisymmetric system, Fourier spectrum

§1. Introduction

A suitable choice of the magnetic coordinate system is important to derive and describe analytical expressions of MHD stabilities etc., especially in non-axisymmetric toroidal systems. The Hamada[1] and the Boozer[2] coordinate system are often used for studies on MHD stabilities,[3] orbit equations,[2] and neoclassical theories.[4,5] When these magnetic coordinate systems are utilized for numerical calculations of them, Fourier spectra in poloidal and toroidal directions are required to be as narrow as possible to obtain highly accurate results with less labor. For this requirement it may be necessary to transform a coordinate system to another one according to the situation under consideration. It is useful to understand properties of the magnetic coordinate systems from a view point of numerical tractability.

The purposes of this paper are 1) to clarify the relation between the Hamada and Boozer coordinate systems, 2) to obtain a coordinate transformation formula between them, and 3) to investigate the numerical tractability of both coordinate systems. Through derivation of the general magnetic coordinate system, we can relate the Hamada coordinate system with the Boozer coordinate system by a transformation function, which is easily obtained with knowledge of the magnetic field strength only. The Fourier spectrum of the magnetic field strength $|\vec{B}|$ in the Hamada coordinate system is broader than that in the Boozer coordinate system and the leading modes of $|\vec{B}|$ in the Hamada coordinate system significantly deviate from a simple model field used in neoclassical and orbit calculations in comparison with the Boozer coordinate system. Thus, the Boozer coordinate system is suitable for numerical calculations, especially, neoclassical and orbit calculations in non-axisymmetric systems.

The organization of this paper is as follows. Expressions of equilibrium quantities in a general magnetic coordinate system are given in §.2. General relations holding regardless of a choice of any magnetic coordinate system are also given. Section 3 gives the coordinate transformations from a general magnetic coordinate system to the Hamada or the Boozer coordinate system. An approach to construct the Hamada or the Boozer coordinate system due to the coordinate transformation with limited information and one due to the field line tracing are discussed. Relation between the Hamada and the Boozer coordinate system is shown in §.4 by introducing a transformation function. The numerical tractability is also discussed. Section 5 is devoted to conclusion.

§2. Expressions of equilibrium quantities in a general magnetic coordinate system

An MHD equilibrium is given by

$$\nabla P = \vec{J} \times \vec{B}, \tag{1}$$

$$\vec{J} = \nabla \times \vec{B}, \tag{2}$$

$$\nabla \cdot \vec{B} = 0. \tag{3}$$

These equations lead to

$$\nabla \cdot \vec{J} = 0, \tag{4}$$

$$\vec{B} \cdot \nabla P = 0, \tag{5}$$

$$\vec{J} \cdot \nabla P = 0. \tag{6}$$

We assume that the toroidal MHD equilibrium given by Eqs.(1)-(6) has nested magnetic flux surface having a single magnetic axis, each of which is specified by ρ:

$$P = P(\rho). \tag{7}$$

From Eqs.(3), (5), and (7), there is a potential function v such that

$$\vec{B} = \nabla\rho \times \nabla v. \tag{8}$$

Similarly, from Eqs.(4), (6), and (7), we can introduce a potential function w as follows:

$$\vec{J} = \nabla w \times \nabla\rho. \tag{9}$$

By using Eqs.(2) and (9), another expression of $\vec{B}$ is obtained by introducing a function u:

$$\vec{B} = \nabla u + w\nabla\rho. \tag{10}$$

Due to Eqs.(7), (8), and (9), Eq.(1) becomes

$$\frac{dP}{d\rho} = \vec{B} \cdot \nabla w = \vec{J} \cdot \nabla v. \tag{11}$$

Also, from Eqs.(8)-(10), we see

$$\vec{B}\cdot\vec{B} = \vec{B}\cdot\nabla u, \tag{12}$$
$$\vec{J}\cdot\vec{B} = \vec{J}\cdot\nabla u. \tag{13}$$

Each flux surface specified by $\rho = \text{const}$ is doubly periodic both in poloidal and toroidal directions. Let θ and ζ be the poloidal and toroidal angle variables in a magnetic coordinate system (ρ, θ, ζ), respectively, where $\mathcal{D}(\theta, \zeta) = [0, 2\pi)$. The functions u, v, and w must give single-valued $\vec{B}$ and $\vec{J}$ for the angle variables θ and ζ. Therefore, we can put them as follows:

$$v = \frac{d\Phi_T}{d\rho}\theta - \frac{d\Phi_P}{d\rho}\zeta + \tilde{v}(\rho,\theta,\zeta), \tag{14}$$
$$w = -\frac{dI_T}{d\rho}\theta - \frac{dI_P}{d\rho}\zeta + \tilde{w}(\rho,\theta,\zeta), \tag{15}$$
$$u = I_T\theta + I_P\zeta + \tilde{u}(\rho,\theta,\zeta) \tag{16}$$

where $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ are periodic functions with respect to θ and ζ with the period 2π. $2\pi\Phi_T(\rho)$ and $2\pi\Phi_P(\rho)$ are the toroidal and poloidal flux inside a flux surface ρ. $2\pi I_T(\rho)$ is the toroidal current inside the flux surface ρ and $2\pi I_P(\rho)$ is the poloidal current outside the flux surface ρ.

Here, we will investigate properties of MHD equilibrium, which hold regardless of a choice of any magnetic coordinate system. Substitution of Eqs.(15) and (16), respectively, into Eqs.(11) and (12) yields

$$\vec{B}\cdot\nabla\tilde{w} = \frac{dP}{d\rho} + \frac{dI_T}{d\rho}\vec{B}\cdot\nabla\theta + \frac{dI_P}{d\rho}\vec{B}\cdot\nabla\zeta, \tag{17}$$
$$\vec{B}\cdot\nabla\tilde{u} = |\vec{B}|^2 - I_T\vec{B}\cdot\nabla\theta - I_P\vec{B}\cdot\nabla\zeta. \tag{18}$$

These equations are called the magnetic differential equation.[6] The magnetic differential equation on an unknown single-valued periodic function F:

$$\vec{B}\cdot\nabla F = S \tag{19}$$

has the following necessary-sufficient condition (solvability condition) with respect to the single-valued periodic function S:[7]

$$\int S d\tau = 0, \text{ for a volume enclosed by a flux surface,} \tag{20}$$

$$\oint S\frac{dl}{B} = 0, \text{ for a closed field line} \tag{21}$$

where $d\tau$ and dl are a volume element and a line element along a closed magnetic field line, respectively. These conditions ensure the existence of an bounded single-valued periodic solution F. Thus, in order for a magnetic coordinate system (ρ, θ, ζ) to exist and express an MHD equilibrium just as Eqs.(8)-(16), the solvability conditions for Eqs.(17) and (18) must be satisfied. The solvability conditions for Eq.(17) gives the following equations:

$$\begin{aligned}\frac{dP}{dV} &= -(2\pi)^2\left[\frac{dI_P}{dV}\frac{d\Phi_T}{dV} + \frac{dI_T}{dV}\frac{d\Phi_P}{dV}\right] = -(2\pi)^2\frac{d\Phi_T}{dV}\left[\frac{dI_P}{dV} + \iota\frac{dI_T}{dV}\right], \\ \oint\frac{dl}{B} &= \frac{N}{2\pi}\frac{dV}{d\Phi_T}\end{aligned} \tag{22}$$

where V is the volume inside a flux surface ρ, $\iota = d\Phi_P/d\Phi_T$ is the rotational transform, and N is the number of rotation in the toroidal direction for a closed magnetic field line. Similarly, the solvability conditions for Eq.(18) gives

$$\begin{aligned}\left\langle|\vec{B}|^2\right\rangle &= (2\pi)^2\frac{d\Phi_T}{dV}\left[I_P + \iota I_T\right], \\ \oint B dl &= 2\pi N\left[I_P + \iota I_T\right]\end{aligned} \tag{23}$$

where the flux-surface average:

$$\langle A\rangle = \frac{d}{dV}\int A d\tau \tag{24}$$

have been used.

Equation (8) indicates that a magnetic field line is determined as the intersection of a flux surface: $\rho =$ const with a surface: $v =$ const. Then, as is clear from Eq.(14) in a magnetic coordinate system where $\tilde{v}$ vanishes, magnetic field lines are expressed as straight lines. Similarly, from Eqs.(9) and (15) current lines are expressed as straight lines in a magnetic coordinates without $\tilde{w}$. An adequate coordinate transformation with respect to periodic coordinates makes $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ vanish up to two of them. Especially, we will consider the transformation which vanishes two of $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$. The properties of these magnetic coordinate systems are as follows.

A) a magnetic coordinate system with $\tilde{v} = \tilde{w} = 0$.

In this coordinate system, both magnetic field lines and current lines are straight. The Jacobian is a surface quantity as understood from in Eq.(11). This coordinate system corresponds to the Hamada coordinate.

B) a magnetic coordinate system with $\tilde{v} = \tilde{u} = 0$.

In this coordinate system, Only magnetic field lines are straight. The Jacobian is expressed by both surface quantities and $|\vec{B}|^2$ as understood from Eq.(12). This coordinate system corresponds to the Boozer coordinate.

C) a magnetic coordinate system with $\tilde{w} = \tilde{u} = 0$.

In this coordinate system, Only current lines are straight. The Jacobian is expressed by both surface quantities and $\vec{B} \cdot \vec{J}$ as understood from Eq.(13).

§3. Coordinate transformations from a general magnetic coordinate system to the Hamada and the Boozer coordinate system

A) Transformation to the Hamada coordinate system

We perform the coordinate transformation from the magnetic coordinate system (ρ, θ, ζ) to the Hamada magnetic coordinate system $(\rho, \theta_H, \zeta_H)$. Since $\tilde{v}$ and $\tilde{w}$ vanish in the Hamada coordinate system, we will consider the following transformation:

$$\begin{aligned} v &= \frac{d\Phi_T}{d\rho}\theta_H - \frac{d\Phi_P}{d\rho}\zeta_H, \\ w &= -\frac{dI_T}{d\rho}\theta_H - \frac{dI_P}{d\rho}\zeta_H. \end{aligned} \tag{25}$$

The inverse transformation is given by

$$\begin{aligned} \theta_H &= \theta + \left(\frac{d\Phi_T}{d\rho}\right)^{-1}\tilde{v} + \frac{d\Phi_P}{d\rho}G, \\ \zeta_H &= \zeta + \frac{d\Phi_T}{d\rho}G \end{aligned} \tag{26}$$

where

$$G(\rho, \theta, \zeta) \equiv -\frac{\dfrac{dI_T}{d\Phi_T}\tilde{v} + \tilde{w}}{\dfrac{d\Phi_T}{d\rho}\left[\dfrac{dI_P}{d\rho} + \iota\dfrac{dI_T}{d\rho}\right]}. \tag{27}$$

Note that the function G is a periodic function of θ and ζ. Introducing a periodic function $\tilde{u}_H$ as follows

$$u = I_T\theta_H + I_P\zeta_H + \tilde{u}_H(\theta_H, \zeta_H), \tag{28}$$

we see

$$\tilde{u}_H = \tilde{u} - I_T\left(\frac{d\Phi_T}{d\rho}\right)^{-1}\tilde{v} - \frac{d\Phi_T}{d\rho}\left(I_P + \iota I_T\right)G. \tag{29}$$

Substituting Eqs.(25) and (28) into Eqs(8)-(10), we obtain the expressions of $\vec{B}$ and $\vec{J}$ in the Hamada coordinate system.

$$
\begin{aligned}
\vec{B} &= \frac{d\Phi_T}{d\rho}\nabla\rho\times\nabla\theta_H - \frac{d\Phi_P}{d\rho}\nabla\rho\times\nabla\zeta_H,\\
\vec{J} &= \frac{dI_T}{d\rho}\nabla\rho\times\nabla\theta_H + \frac{dI_P}{d\rho}\nabla\rho\times\nabla\zeta_H,\\
\vec{B} &= I_T\nabla\theta_H + I_P\nabla\zeta_H + \nabla\tilde{u}_H.
\end{aligned}
\tag{30}
$$

Substituting Eq.(25) into Eq.(11) and using Eq.(22), we can obtain the Jacobian of the Hamada coordinate system:

$$
\sqrt{g_H} = \frac{1}{\nabla\rho\cdot\nabla\theta_H\times\nabla\zeta_H} = \frac{1}{(2\pi)^2}\frac{dV}{d\rho}. \tag{31}
$$

Thus, we can obtain the expressions of $\vec{B}$, $\vec{J}$, and the Jacobian in the Hamada coordinate system. The poloidal and toroidal angles and $\tilde{u}_H$ are given by Eqs.(26) and (29) in terms of ρ, θ, and ζ, respectively.

Hereafter, we will show some useful relations and give other methods to construct the Hamada coordinates. The Jacobian in the Hamada coordinates $\sqrt{g_H}$ is related with $\sqrt{g} = (\nabla\rho\cdot\nabla\theta\times\nabla\zeta)^{-1}$ in the (ρ,θ,ζ) coordinate system by using Eq.(26):

$$
\vec{B}\cdot\nabla G = \frac{1}{\sqrt{g_H}} - \frac{1}{\sqrt{g}}\left[1 + \left(\frac{d\Phi_T}{d\rho}\right)^{-1}\frac{\partial\tilde{v}}{\partial\theta}\right]. \tag{32}
$$

Substitution of Eqs.(25) and (28) into Eq.(12) and usage of Eq.(23) give

$$
\vec{B}\cdot\nabla\tilde{u}_H = |\vec{B}|^2 - \left\langle|\vec{B}|^2\right\rangle. \tag{33}
$$

Thus, we can understand that even if we do not know some of information on the (ρ,θ,ζ) coordinate system, we can construct the Hamada coordinate system as long as we can see $\sqrt{g}$, $\tilde{v}$, and $|\vec{B}|^2$. Indeed, solving Eq.(32) gives the function $G(\rho,\theta,\zeta)$. The functions G and $\tilde{v}$ allow us to obtain $\theta_H(\rho,\theta,\zeta)$ and $\zeta_H(\rho,\theta,\zeta)$ through Eq.(26). Solving Eq.(33), we can see $\tilde{u}_H(\rho,\theta,\zeta)$ and, consequently, $\tilde{u}_H(\rho,\theta_H,\zeta_H)$.

Moreover, from Eq.(30) we see

$$
\vec{B}\cdot\nabla\theta_H = (2\pi)^2\frac{d\Phi_P}{dV},\quad \vec{B}\cdot\nabla\zeta_H = (2\pi)^2\frac{d\Phi_T}{dV}. \tag{34}
$$

As has been shown in Ref.1, the field line integral of Eq.(34)

$$\zeta_H = (2\pi)^2 \frac{d\Phi_T}{dV} \int_0 \frac{dl}{|\vec{B}|} \tag{35}$$

can construct the Hamada coordinate system. Here, we will show a usefull method combined with the algorithm given in Refs. 8 and 9. Using the field line integral given by Eq.(35) and the periodicity, we can see $A(\psi, \zeta_H) = \sum A_{m,n}(\psi) \exp(\mathrm{i}[n + \iota m]\zeta_H)$ where A is an any scalar periodic function such as $|\vec{B}|$, R, ϕ, and Z ((R, ϕ, Z) is the cylindrical coordinates). Then, the Fourier integral of $A(\psi, \zeta_H)$ together with an adequate window function gives the Fourier amplitude $A_{m,n}$. The function $\tilde{u}_H$ is also obtained by the field line integral of Eq.(33).

B) Transformation to the Boozer coordinate system

Since $\tilde{v}$ and $\tilde{u}$ vanish in the Boozer coordinate system $(\rho, \theta_B, \zeta_B)$, we will consider the following coordinate transformation:

$$\begin{aligned} v &= \frac{d\Phi_T}{d\rho}\theta_B - \frac{d\Phi_P}{d\rho}\zeta_B, \\ u &= I_T\,\theta_B + I_P\,\zeta_B. \end{aligned} \tag{36}$$

The inverse transformation is given by

$$\begin{aligned} \theta_B &= \theta + \left(\frac{d\Phi_T}{d\rho}\right)^{-1} \tilde{v} + \frac{d\Phi_P}{d\rho} G, \\ \zeta_B &= \zeta + \frac{d\Phi_T}{d\rho} G \end{aligned} \tag{37}$$

where

$$G(\rho, \theta, \zeta) \equiv - \frac{I_T \left(\dfrac{d\Phi_T}{d\rho}\right)^{-1} \tilde{v} - \tilde{u}}{\dfrac{d\Phi_T}{d\rho}\left[I_P + \iota I_T\right]}. \tag{38}$$

Note that the inverse transformation of the angles has the same form as that in the Hamada coordinate system except for the definition of G. Introducing a periodic function $\tilde{w}_B$:

$$w = -\frac{dI_T}{d\rho}\theta_B - \frac{dI_P}{d\rho}\zeta_B + \tilde{w}_B(\theta_B, \zeta_B), \tag{39}$$

we have

$$\tilde{w}_B = \tilde{w} + \frac{dI_T}{d\Phi_T}\tilde{v} + \frac{d\Phi_T}{d\rho}\left[\frac{dI_P}{d\rho} + \iota \frac{dI_T}{d\rho}\right] G. \tag{40}$$

Substituting Eqs.(36) and (39) into Eqs(8)-(10), we obtain the expressions of $\vec{B}$ and $\vec{J}$ in the Boozer coordinate system.

$$\begin{aligned}
\vec{B} &= \frac{d\Phi_T}{d\rho}\nabla\rho\times\nabla\theta_B - \frac{d\Phi_P}{d\rho}\nabla\rho\times\nabla\zeta_B, \\
\vec{J} &= \frac{dI_T}{d\rho}\nabla\rho\times\nabla\theta_B + \frac{dI_P}{d\rho}\nabla\rho\times\nabla\zeta_B + \nabla\tilde{w}_B\times\nabla\rho, \\
\vec{B} &= I_T\nabla\theta_B + I_P\nabla\zeta_B + \tilde{w}_B\nabla\rho.
\end{aligned} \tag{41}$$

Substituting Eq.(36) into Eq.(12) and using Eq.(23), we can obtain the Jacobian of the Boozer coordinate system:

$$\sqrt{g_B} = \frac{1}{\nabla\rho\cdot\nabla\theta_B\times\nabla\zeta_B} = \frac{d\Phi_T}{d\rho}\frac{I_P+\iota I_T}{|\vec{B}|^2}. \tag{42}$$

We can obtain the expressions of $\vec{B}$, $\vec{J}$, and the Jacobian in the Boozer coordinate system. The poloidal and toroidal angles and $\tilde{w}_B$ are given by Eqs.(37) and (40) in terms of ρ, θ, and ζ, respectively.

The Jacobian in the Boozer coordinate system $\sqrt{g_B}$ is related with $\sqrt{g}$ in the (ρ,θ,ζ) coordinate system by using Eq.(37):

$$\vec{B}\cdot\nabla G = \frac{1}{\sqrt{g_B}} - \frac{1}{\sqrt{g}}\left[1+\left(\frac{d\Phi_T}{d\rho}\right)^{-1}\frac{\partial\tilde{v}}{\partial\theta}\right]. \tag{43}$$

We see that Eq.(43) is the same as Eq.(32) in the Hamada coordinate system by exchanging $\sqrt{g_B}$ for $\sqrt{g_H}$. Substitution of Eqs.(36) and (39) into Eq.(11) and usage of Eqs.(22) and (23) give

$$\vec{B}\cdot\nabla\tilde{w}_B = \frac{dP}{d\rho}\left[1-\frac{|\vec{B}|^2}{\left\langle|\vec{B}|^2\right\rangle}\right]. \tag{44}$$

As long as $\sqrt{g}$, $\tilde{v}$, and $|\vec{B}|^2$ are given, the Boozer coordinate system can be constructed by solving Eqs.(43) and (44), as well as the case in the Hamada coordinate system.

As well as in the Hamada coordinate system, from Eq.(41) we have

$$\vec{B}\cdot\nabla\theta_B = (2\pi)^2\frac{d\Phi_P}{dV}\frac{|\vec{B}|^2}{\left\langle|\vec{B}|^2\right\rangle}, \quad \vec{B}\cdot\nabla\zeta_B = (2\pi)^2\frac{d\Phi_T}{dV}\frac{|\vec{B}|^2}{\left\langle|\vec{B}|^2\right\rangle}. \tag{45}$$

The Boozer coordinate system can also be constructed by field line integrals of Eqs.(45) and (44).[8,9]

§4. Relation between the Hamada and the Boozer coordinate system

As has been shown in the previous section, the coordinate transformation from the magnetic coordinate system (ρ, θ, ζ) to the Hamada or the Boozer coordinate system can be performed by solving Eqs.(32) and (33) or Eqs.(43) and (44), if the Jacobian $\sqrt{g}$, $|\vec{B}|^2$, and $\tilde{v}$ are known in the (ρ, θ, ζ) system. Therefore, the coordinate transformation between magnetic coordinate systems where magnetic field lines are straight: $(\tilde{v} = 0)$ are done as follows:

$$
\begin{aligned}
\rho_N &= \rho_N(\rho), \\
\theta_N &= \theta + \frac{d\Phi_P}{d\rho} G, \\
\zeta_N &= \zeta + \frac{d\Phi_T}{d\rho} G,
\end{aligned}
\tag{46}
$$

where the coordinate transformation from the (ρ, θ, ζ) system to the $(\rho_N, \theta_N, \zeta_N)$ system is considered. The transformation function $G(\rho, \theta, \zeta)$ is determined by the following equation:

$$
\vec{B} \cdot \nabla G = \left(\frac{d\rho_N}{d\rho} \right)^{-1} \frac{1}{\sqrt{g_N}} - \frac{1}{\sqrt{g}}
\tag{47}
$$

where $\sqrt{g_N} = (\nabla \rho_N \cdot \nabla \theta_N \times \nabla \zeta_N)^{-1}$ is the Jacobian of the new magnetic coordinate system. Hence, according to the choice of new Jacobian $\sqrt{g_N}$ the transformation function G is determined by Eq.(47), which gives the angles in the new coordinate system through Eq.(46). Here, let us consider the coordinate transformation from the Boozer to the Hamada coordinate system putting $\rho_N = \rho_H$ and $\rho = \rho_B$. In this case, Eq.(47) becomes

$$
\vec{B} \cdot \nabla G = (2\pi)^2 \frac{d\rho_B}{dV} \left[1 - \frac{|\vec{B}|^2}{\left\langle |\vec{B}|^2 \right\rangle} \right]
\tag{48}
$$

or

$$
\left(\frac{\partial}{\partial \zeta_B} + \iota \frac{\partial}{\partial \theta_B} \right) G = \left[\frac{\left\langle |\vec{B}|^2 \right\rangle}{|\vec{B}|^2} - 1 \right]
\tag{49}
$$

and Eq.(46) becomes

$$
\begin{aligned}
\rho_H &= \rho_H(\rho_B), \\
\theta_H &= \theta_B + \iota G, \\
\zeta_H &= \zeta_B + G.
\end{aligned}
\tag{50}
$$

From Eqs.(49) and (50) we can easily show

$$
\frac{\partial(\theta_H, \zeta_H)}{\partial(\theta_B, \zeta_B)} = \frac{\left\langle |\vec{B}|^2 \right\rangle}{|\vec{B}|^2}.
\tag{51}
$$

Eqs.(49) to (51) are very useful to transform expressions in the Hamada coordinate system to those in the Boozer coordinate system. Using Eqs.(49) and (50) we will consider a large-aspect ratio low-β helical plasma, in which we can put

$$|\vec{B}| = B_0 \left[1 - \varepsilon_t \cos\theta_B - \varepsilon_h \cos(L\theta_B - M\zeta_B)\right] \tag{52}$$

where ε_t ($\ll 1$) and ε_h ($\ll 1$) are an aspect ratio and a helical ripple, respectively, and L and M are the polarity and the toroidal pitch number of helical coils, respectively. Up to 1st order of ε_t and ε_h, $\left\langle |\vec{B}|^2 \right\rangle = B_0^2$ and solving Eq.(49) we see

$$G = \frac{2\varepsilon_t}{\iota} \sin\theta_B + \frac{2\varepsilon_h}{\iota L - M} \sin(L\theta_B - M\zeta_B). \tag{53}$$

By using this result, Eq.(50) becomes

$$\begin{aligned} \theta_H &= \theta_B + 2\varepsilon_t \sin\theta_B + \frac{2\iota\varepsilon_h}{\iota L - M} \sin(L\theta_B - M\zeta_B), \\ \zeta_H &= \zeta_B + \frac{2\varepsilon_t}{\iota} \sin\theta_B + \frac{2\varepsilon_h}{\iota L - M} \sin(L\theta_B - M\zeta_B). \end{aligned} \tag{54}$$

The result of a large-aspect ratio low-β tokamak plasma is obtained by setting $\varepsilon_h = 0$. To see the difference of angles in both coordinates, let us consider the angle α between the surface of ζ_H = const and the surface of ζ_B = const. From Eq.(54) $\tan\alpha = \mathrm{d}(R\zeta_B)/\mathrm{d}(r\sin\theta_B) \sim -2/\iota$ near the magnetic axis where $\varepsilon_t \gg \varepsilon_h$. Even near the magnetic axis, the gradient of the angle grid in the Hamada coordinates is large and this result is due to the toroidicity. Away from the magnetic axis, $\tan\alpha \sim -2/\iota + 2\varepsilon_h L/(\varepsilon_t M)$ around $\theta_B, \zeta_B \sim 0$ and the contribution of ε_h becomes large. For the Fourier spectrum of the magnetic field strength in the Hamada coordinate system, from solving Eq.(54) for θ_B and ζ_B and substituting them into Eq.(52), the factor $2M\varepsilon_t/\iota$ before $\sin\theta_H$ in the cosine function may be order unity except for the magnetic axis, and the Fourier spectrum becomes much broader. Hence, the Fourier spectra of $|\vec{B}|$ in both coordinate systems have a large difference, especially in the periphery.

Figure 1 shows Fourier spectra of $|\vec{B}|$ in both the Boozer ((a) and (c)) and the Hamada ((b) and (d)) coordinate system. Figs.1-(a) $\sim$ (b) and 1-(c) $\sim$ (d) correspond to a flux surface near the magnetic axis and a flux surface in the periphery, respectively. As a magnetic field an LHD vacuum magnetic field is used where $L = -2$, $M = -10$, the vacuum magnetic axis is not shifted, and the toroidally-averaged magnetic surfaces are horizontally elongated. As is seen from Figs.1-(a) $\sim$ (b), near the magnetic axis the difference of the Fourier spectra is small between the two coordinate systems and similar to those of the model magnetic field given by Eq.(52). However, in the periphery, as is indicated in Figs.1-(c) $\sim$ (d), the Fourier spectrum in the Hamada coordinate system is much broader than that in the Boozer coordinate system and the leading modes in the Hamada coordinate system are significantly different from those of the model field. In the Boozer coordinate system leading modes are closer to those of the model field. This is due to the difference of angle grids as shown in Figs.2-(a) $\sim$ (c), where the poloidal and toroidal angle grids are shown on four different flux surfaces. The magnetic coordinate system shown in Fig.2-(a) is the coordinate system where the toroidal angle is one of the cylindrical coordinates, used in the MAGN code.[10] The magnetic coordinate systems shown in Figs.2-(b) and (c) are the Boozer and the Hamada coordinate system, respectively. The angle grids of the Boozer coordinate system are similar to those of the MAGN coordinate system, whereas the angle grids of the Hamada coordinate system are significantly different from those of the Boozer and the MAGN coordinate system.

Hence, the Boozer coordinate system is suitable for numerical calculations, especially, for orbit and neoclassical calculations where only $|\vec{B}|$ is needed except for surface quantities, because the Fourier spectrum broadness of $|\vec{B}|$ is smaller than that in the Hamada coordinate system and the Fourier spectrum is similar to that of a simple model field.

§5. Conclusion

By clarifying the relation between the Hamada and the Boozer magnetic coordinate systems, the coordinate transformation formula from the former to the latter has been derived. Only one transformation function is needed, which is easily calculated with knowledge of the magnetic field strength $|\vec{B}|$. It has been shown that the Boozer coordinate system is suitable for numerical calculations, especially, for neoclassical and orbit calculations, because the Fourier spectrum of $|\vec{B}|$ is not broad and the leading modes are similar to a model magnetic field used in neoclassical and orbit calculations. It has been also indicated that the Hamada coordinate system can be obtained by the field line integral as well as the Boozer coordinate system.[8,9] The former needs the integral of $1/|\vec{B}|$ and the latter needs that of $|\vec{B}|$.

References

1) S.Hamada, Nucl.Fusion **2** (1962) 23.

2) A.H.Boozer, Phys.Fluids **23** (1980) 904.

3) J.L.Johnson and J.M.Greene, Plasma Physics **9** (1967) 611.

4) K.C.Shaing and J.D.Callen: Phys.Fluid **26** (1983) 3315.

5) K.C.Shaing, S.P.Hirshman and J.D.Callen: Phys.Fluid **29** (1986) 521.

6) M.D.Kruskal and R.M.Kulsrud, Phys.Fluids **1** (1958) 265.

7) W.A.Newcomb, Phys.Fluids **2** (1959) 362.

8) G.Kuo-Pertravic, A.H.Boozer, J.A.Rome, and R.H.Fowler, J.Comput.Phys. **51** (1983) 251.

9) J.A.Rome, J.Comput.Phys. **82** (1989) 348.

10) J.Todoroki, J.Phys.Soc.Jpn **58** (1989) 3979.

Figure Captions

Fig.1 Fourier spectra of $|\vec{B}|$ in both the Boozer and the Hamada coordinate system on a flux surface near the magnetic axis ((a) and (b)), and on a flux surface in the periphery ((c) and (d)), respectively.

Fig.2 The poloidal and toroidal angle grids in four different flux surfaces (a) in the coordinate system used in the MAGN code, (b) in the Boozer coordinate system, and (c) in the Hamada coordinate system.

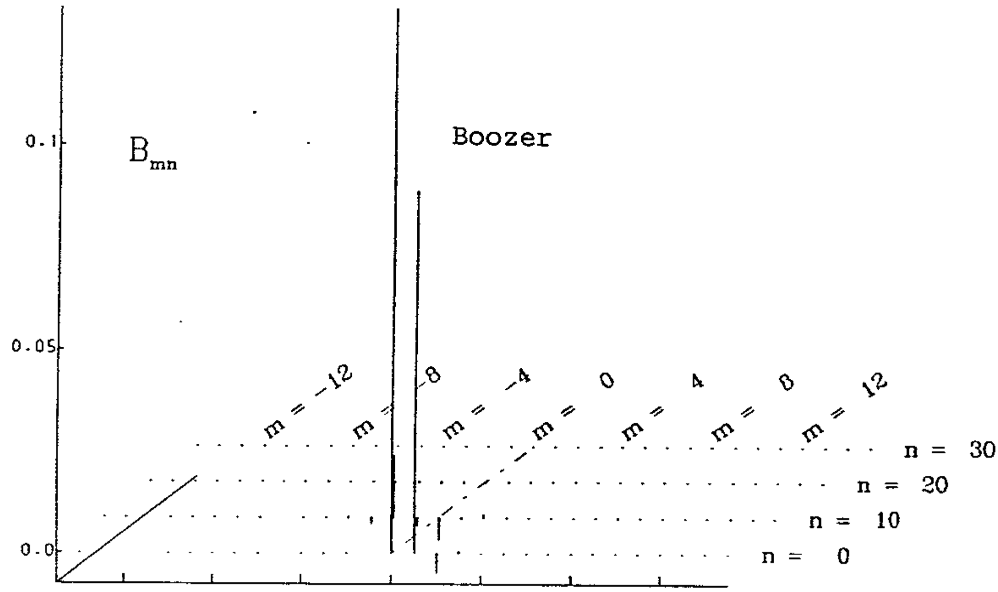

Fig.1-(a)   N.Nakajima et.al.

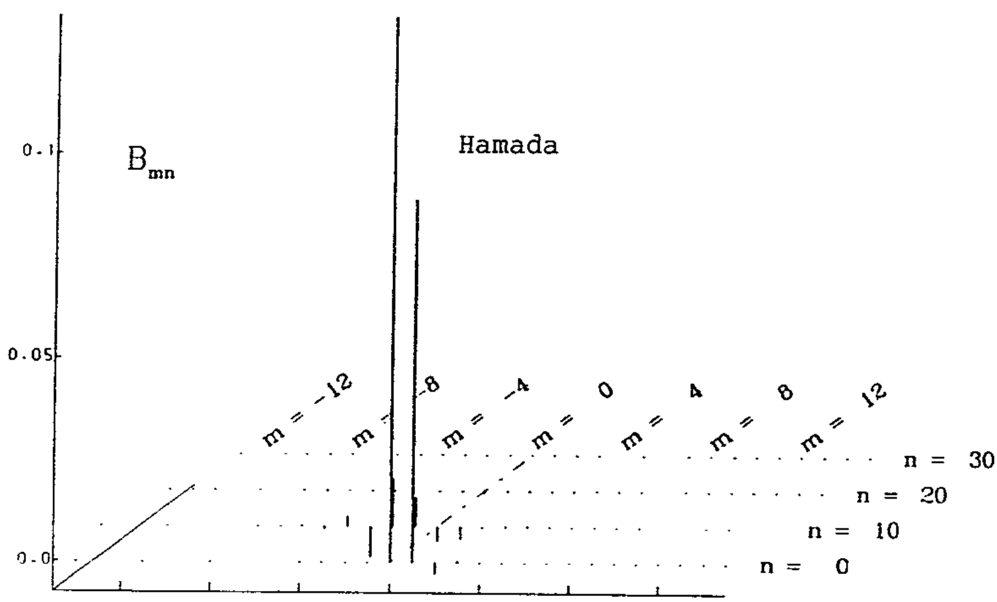

Fig.1-(b)   N.Nakajima et.al.

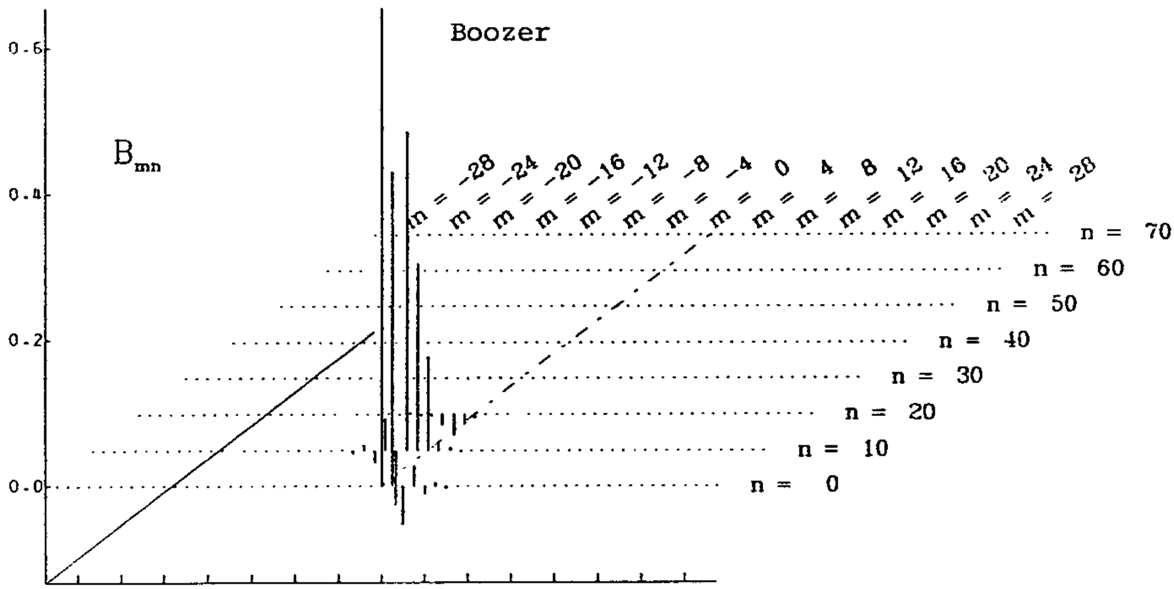

Fig.1-(c) N.Nakajima et.al.

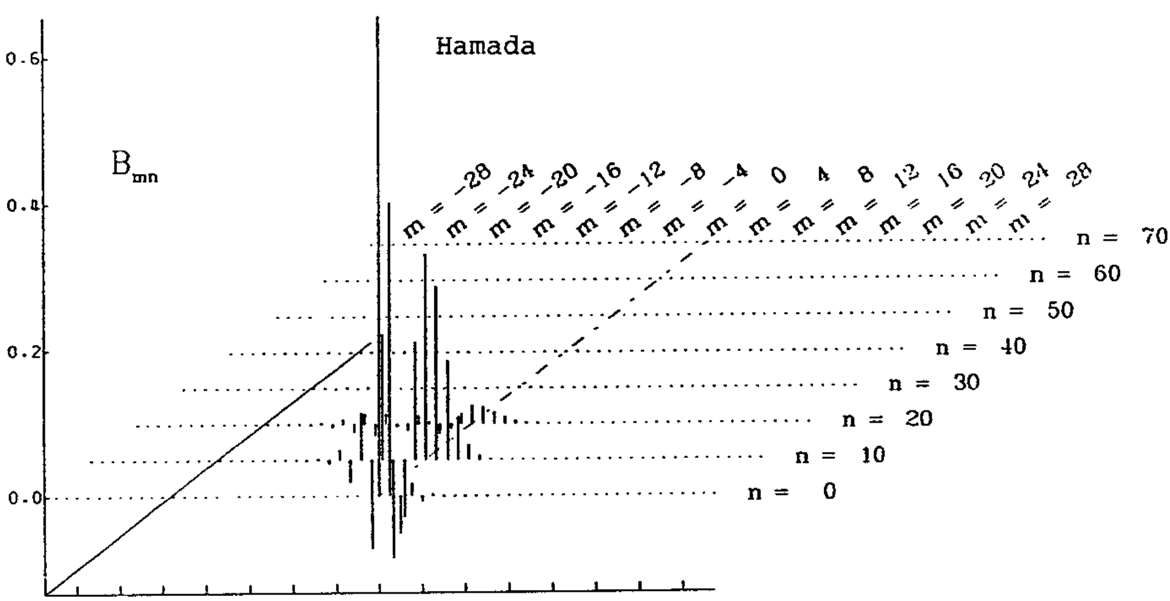

Fig.1-(d) N.Nakajima et.al.

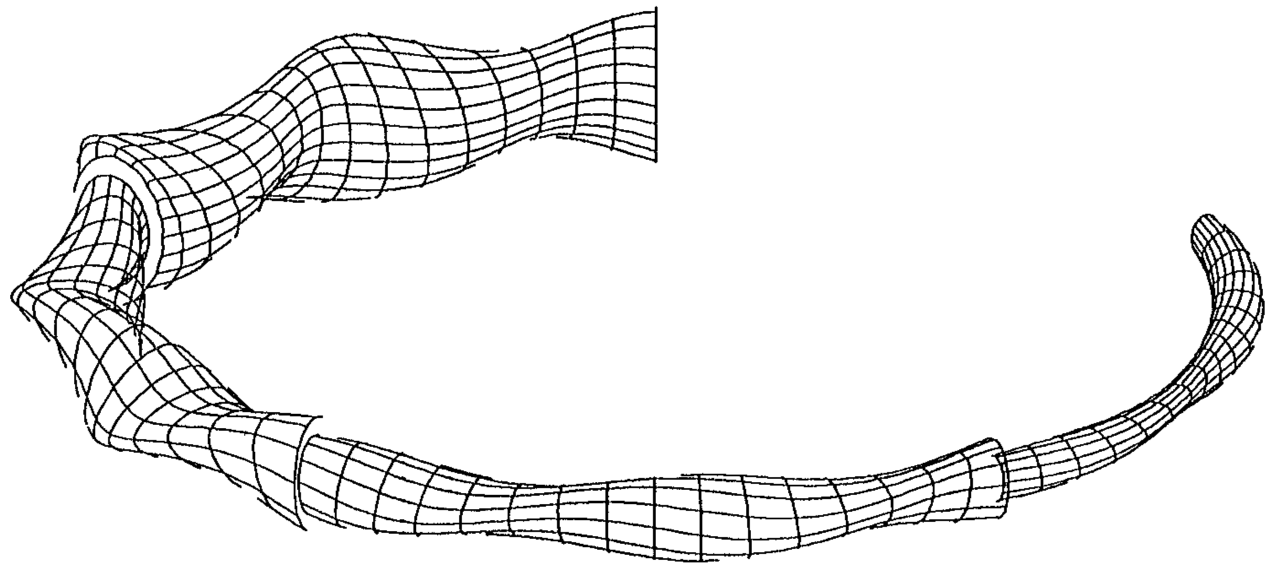

Fig.2-(a) N.Nakajima et.al.

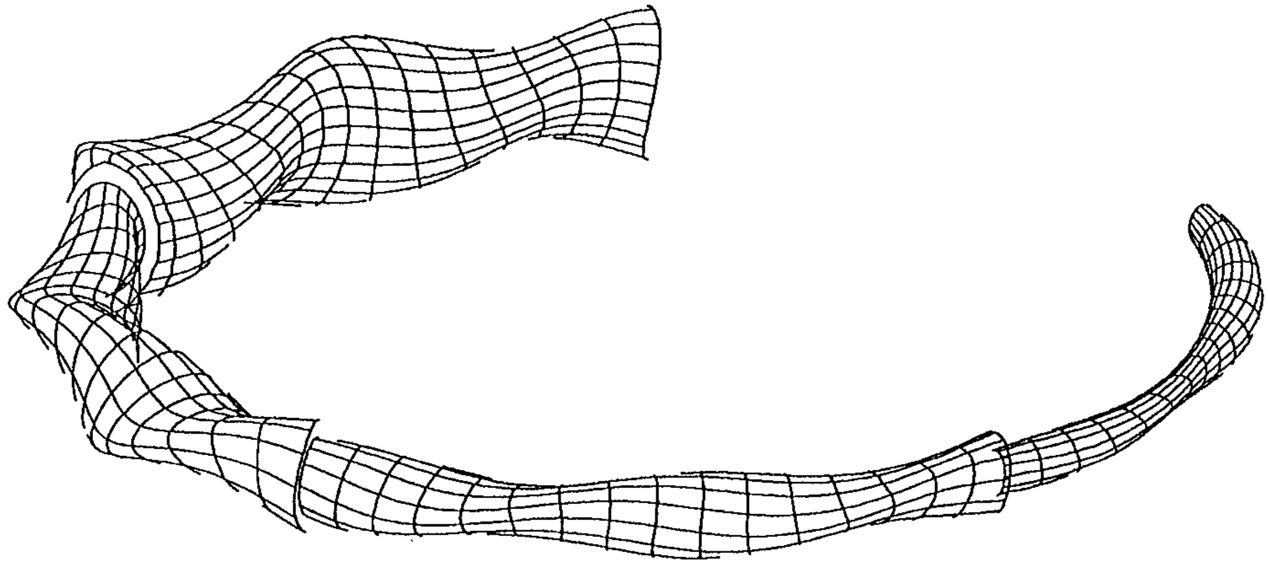

Fig.2-(b) N.Nakajima et.al.

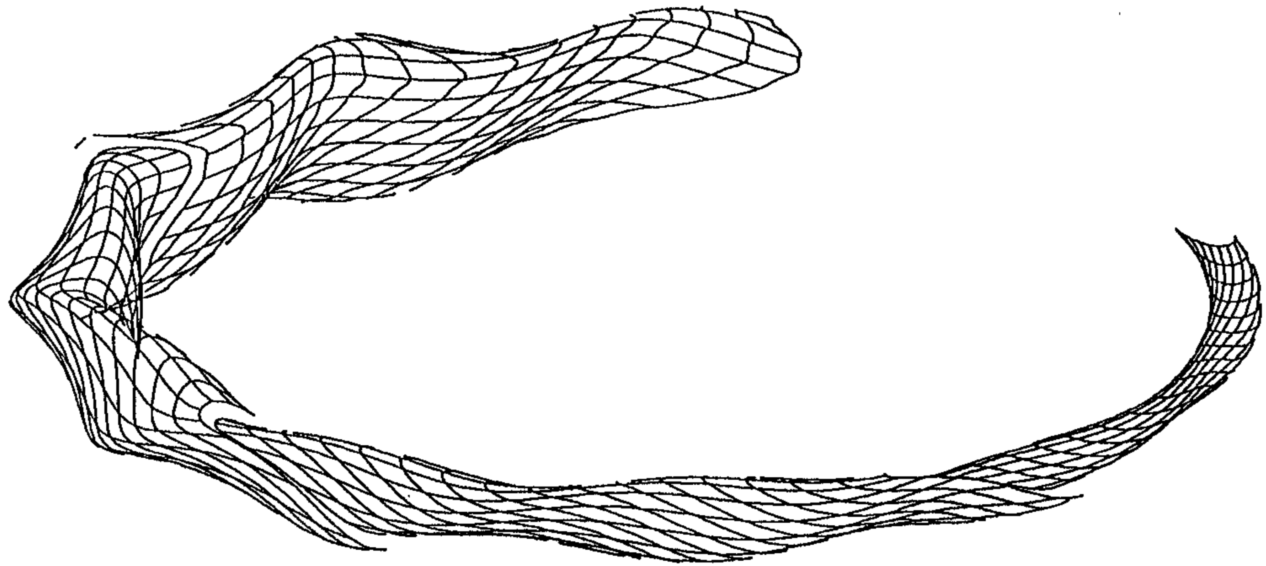

Fig.2-(c) N.Nakajima et.al.

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