Model of the L-Mode Confinement in Tokamaks

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Abstract

Model of the L-mode confinement in tokamaks is developed based on the microscopic ballooning mode instability. The anomalous transport coefficients determine the stability below the critical beta against the ideal MHD ballooning instability. The current-diffusivity has strong destabilizing effect, while the thermal transport $\chi$ and ion viscosity tend to stabilize the mode. The stability boundary for the least stable microscopic ballooning mode determines the anomalous transport coefficients. The obtained formula is compared with experimental observations on the L-mode confinement. The predictions on (1) the radial shape of $\chi$, (2) the temperature-profile resilience, (3) the global confinement characteristics, such as the power degradation and the dependences on the plasma current, current profile and mass number, and (4) the ratio of the perturbative $\chi$ to the energy balance $\chi$ are consistent with experimental database.

§1 Introduction

The L-mode confinement has been observed in all tokamaks [1,2]. The microscopic fluctuations have been confirmed to play important roles for the anomalous transport[3]. The power degradation is modeled by the relation $\chi \propto T^{1.5}/ab^2$, which has been derived for drift-wave theories[4]. However, this form of $\chi$ contradicts to the radial form of $\chi$. Ohkawa's model, $\chi = \delta^2 v_A/qR$ ($\delta$ is the collisionless skin depth, $R$ is the major radius, $q$ is the safety factor and $v_A$ is the Alfvén velocity), is one of the few which explains that $\chi$ is large at high temperature and becomes larger towards the edge as well[5], but could not fully explain the dependences of $\chi_R$.

We here present a new model of the L-mode confinement based on the transport-MHD analysis. The shear-stabilized plasma can become unstable due to the existence of the fluctuation-driven transport[6,7]. We analyze the ballooning instability by taking into account of the anomalous cross-field transport. Below the critical beta of the ideal MHD mode, the microscopic ballooning
mode has a large growth rate due to the small but finite current-diffusivity \( \lambda \). This mode is destabilized by \( \lambda \) but is stabilized by other transport coefficients, \( \chi \) and the ion viscosity \( \mu \). If the anomalous transport is enhanced much by this mode, then the mode is stabilized. The marginal stability condition for the least stable mode determines the transport coefficients. Results on \( \chi \) are compared to the experimental observations and show agreements.

\[ \frac{d}{d\eta} \frac{F}{1+\Sigma\eta/\tau+\Delta\eta^2/\tau} \frac{d\phi}{d\eta} = \frac{\alpha[\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta]\phi}{1+\Sigma\eta/\tau} - \frac{\tau(\tau+\Sigma\eta)\phi}{\tau} = 0 \]  

(1)

Notations are: \( \Sigma=\eta^2q^2/\delta \), \( \Lambda=\chi n^4 q^4 \), \( \Xi=\chi n^2 q^2 \), \( \kappa=\alpha n^2 q^2 \), \( \gamma \) is the growth rate, \( F=1+(s\eta-\alpha\sin\eta)^2 \), \( \chi \) is the average well and \( \kappa=-(r/R)(1-1/q^2) \), \( B_p=Br/qR \), \( \alpha=q^2\rho'/\varepsilon \), \( \varepsilon=a/R \), \( \beta \) is the pressure divided by the magnetic pressure, and ' denotes the derivative with respect to \( r/a \). We use the normalizations: \( r/a\rightarrow r \), \( \tau/\tau_{Ap}\rightarrow \tau \), \( \tau_{Ap}/a^2\rightarrow \mu \), \( \tau_{Ap}/\mu_0 \sigma a^2\rightarrow \lambda \), \( \Delta\eta^2/\tau \rightarrow \lambda \), \( \tau_{Ap}/a^2\rightarrow \lambda \). If we neglect \( \Lambda, \xi \) and \( \mu \), the equation is reduced to the resistive ballooning equation, and the ideal MHD mode equation is recovered by further taking \( 1/\delta=0 \).

Equation (1) predicts that the current-diffusive ballooning mode has a large growth rate even with very small value of \( \lambda \). We take \( 1/\delta=0 \) for simplicity. The growth rate of the short wave length mode, driven by \( \lambda \) term, is given analytically as \( \tau \sim \chi 1/5(nq)^4/5\alpha^2/5\zeta-2/5 \) [9]. This large growth rate is confirmed by the numerical calculation [10].

As the transport coefficients are increased much, the stabilizing effects by \( \chi \) and \( \beta \) overcome the destabilizing effect of \( \lambda \).
The stability boundary is derived by setting $\tau=0$ in Eq. (1). For the ballooning mode which is destabilized by the normal curvature, not by the geodesic curvature, i.e., $1/2+\alpha s$, the stability boundary for the least stable mode is obtained as [9]

$$\alpha^{3/2} = f(s) \sqrt{\alpha} z^{3/2}$$ (2)

where $f(s)=\sqrt{6} s$ or $1.7 (s \to 0)$.

§3 Transport Coefficient

Based on the stability analysis, we derive the formula for the anomalous transport coefficient. When the mode amplitude and the associated anomalous transport is small, Eq. (1) predicts the instability. When the mode develops and the transport coefficient reaches the condition Eq. (2) for given pressure gradient $\alpha$, the self-sustaining turbulent state is realized. This state is thermodynamically stable: The excess growth of the mode and enhanced transport coefficients lead to the damping of the mode. When the mode amplitude and transport coefficients are below Eq. (2), the mode continues to grow.

From Eq. (2) we express $z$ in terms of the Prandtl numbers $\mu/\alpha$ and $\lambda/\alpha$ (note that $\lambda$ is proportional to the electron viscosity in the plasma frame[8]) as $z = \alpha^{3/2}(\lambda/\alpha) \sqrt{\alpha/\mu}/f(s)$. The ratio $\lambda/\alpha$ and $\mu/\alpha$ are given to be constant. We have $\lambda/\alpha = e^2/a^2$ and $\mu/\alpha \approx 1$ for electrostatic perturbations[11]. The formula of $z$ is finally obtained in an explicit form as

$$z = f(s)^{-1} q^2 (RB'/r)^{3/2} e^2 v_A/R.$$ (3)

The typical perpendicular wave number of the most unstable mode satisfies $k_0 = 1/\sqrt{\alpha}$. The typical correlation time of the mode is estimated as $\tau_c = 1/\tau$, $\tau = \sqrt{\alpha/6} \sqrt{(v_A/qR)}$.

§4 Comparison with Experiments

This form of $z$ is consistent with experimental results known for the L-mode. In the following, we show the predictions from
our theory and compare them with database, by choosing $n_i = n_e$ and $T_i = T_e$ for the simplicity.

(i) $\tau$ has the dimensional dependence of $[T]^{1/5}/[a][B]^{2}$.
Note that $\tau$ is enhanced not by the local value of $T$ but by the gradient of the pressure.

(ii) The density and $q$ profiles govern the radial profile of $\tau$. Equation (3) indicates that $\tau$ increases towards the edge for the usual plasma profiles in the L-mode. Figure 1 shows a typical example of the predicted $\tau$ profile.

(iii) The dependence on the dimensionless quantities is predicted as $\tau \sim q^2/f(s)/\sqrt{A}$, where $A$ is the ion mass number. This is consistent with the experimental study on the local $\tau$ that $\tau \sim B_p^{-1} \rho_s^{-2}$ with $1 < y < 2$ and $0 < z < 1$ [12], and with the favourable mass dependence.

(iv) The point model argument of the energy balance, $\tau_B = a^2/\tau$ and $2 \pi a^2 \rho_e T = \tau_B P$, provides the scaling law

$$\tau_B = C a^{0.4} R_0^{1.2} I_p^{0.8} \rho_s^{-0.6} A^{0.5} f^{-0.4} \left( a_p / \sqrt{A} \right)^{0.6},$$  \hspace{1cm} (4)$$

where $C$ is a numerical coefficient and $a_p$ is the gradient scale length $(nT) / |\nabla(nT)|$. This result is consistent with the L-mode scaling law, including the dependences on $a$, $R$, $I_p$, $P$, $A$ and internal inductance[13]. Slight difference is seen in the the final term in the parenthesis. We notice, however, the density and the gradient scale length have a collinearity in database. In L-mode plasmas, the density profile is much steeper than the temperature gradient near edge and $I_p$ in Eq.(4) would be replaced by $a_p = |n_i / \nabla n_i|$. The high density plasma has more steeper edge density: Tsuji has reported that $a_p / \rho_e$ is an weak function of the density[14]. For some dataset of JT-60, Takizuka found the density dependence as $\tau_B(thermal) \sim n_e^{0.5}$, suggesting that the careful classification of the database with respect to the profile is necessary[15].

(v) Due to the dependence of $\tau$ on $\nabla T$, the temperature profile weakly depends on the location of the peak of the power deposition. The peaking factor, $T(\text{at } q=1) / \langle T \rangle$, is predicted to
scale $\sim q(a)^{0.6}$. These results explain the profile resilience.

(vi) Since $\tau \sim (V(nT)/n)^{1.5}$, the thermal diffusion coefficient
deduced from the pulse propagation, $\tau_{HP}$, is larger than that
evaluated by the power balance $\tau$. If $|Vn_i/n_i| << |\nabla T/T|$holds, for
instance, we have $\tau_{HP} = 2.5\tau$.

(vii) The typical wavelength and correlation time of the
mode are calculated. $k_{\perp} \rho_i$ is of the order of 0.1 or less. ($\rho_i$
is the ion gyroradius.) It should be noted that, though the
dimensional relation $k_{\perp} \sim [B]/[\sqrt{T}]$ holds, $k_{\perp}$ does not scale with the
local gyroradius. The collisionless skin depth would be more
relevant length. The typical time scale $\tau_c$ depends on average
temperature, not explicitly on $n_e$ and $B$.

(viii) The estimation $\pi/n \sim 1/k_{\perp}L_p$ shows that the mode
amplitude is larger near edge and is larger for the high heating
power.

§5. Summary and Discussion

The model of the L-mode in tokamaks was developed. The
stability of the microscopic ballooning mode is investigated
under the influence of the self-generated anomalous transport
coefficients $\chi$, $\lambda$ and $\mu$. The transport coefficient is determined
from the marginal stability condition for the least stable mode.
(We here note that the usual method for the estimation of $\chi$ by
$\tau/k_{\perp}^2$ for the most unstable mode gives the same results as $\chi \sim
\alpha^2/\varepsilon(\lambda/2)$.) The predictions as to $\chi$ are compared with
experiments. Major part of the observations on L-mode can be
explained by this model simultaneously.

We here use simplifications for the analytic insight. The
study on the case where the geodesic curvature drives the insta-
bility gives the similar results[10]. The result Eq. (3) contains
uncertainty in the numerical factor. Nonlinear simulation would
give the numerical factor and would examine the assumption for $\chi$
that the transport coefficients affecting the microscopic mode is
equated to that for the global quantity. Also necessary is the
study of the effects such as the diamagnetic drift for kinetic
corrections. These research are open for future study.
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References

Fig. 1  Predicted radial profile of $\chi$ as a function of $r/a$. The solid line shows Eq. (3) and the dashed line is $5 \times$ Eq. (3). Parameters are: $B=4T$, $R=3m$, $R/a=4$, $q(r)=1+2(r/a)^2$, $T(0)=10$ keV, $p(r)=p(0)h(r)$, $n(r)=n(0)\sqrt{h(r)}$, $h(r)=(1-(r/a)^2+A)$, $A=0.01$, and $A=1$. 
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