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(Received – Jan. 18, 1993)

NIFS-213  Mar. 1993

RESEARCH REPORT
NIPS Series

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Thermal and Electric Oscillation
driven by Orbit Loss in Helical Systems

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Abstract
Coupled oscillation of the radial electric field, the heating power and the plasma temperature in helical systems is analysed. This oscillation is caused by the influence of the radial electric field on the direct loss of injected fast ions. The damped oscillations of the radial electric filed, temperature and fast-ion loss are predicted. Conditions for the mode and the dependence of the frequency are discussed.

keywords: radial electric field, direct orbit loss, oscillation, energy confinement time, slowing down time, torsatron/heliotron

PACS Index: 52.25.Fi, 52.55.Hc, 52.50.Gj
Recently the important role of the radial electric field on the confinement of the toroidal plasma is widely recognized. This is especially true for the plasmas in the toroidal helical devices such as torsatron/heliotron. This is because the trajectory of the helically trapped particles is strongly influenced by the radial potential difference which is of the order of the plasma temperature. Detailed work has been performed, specifying the magnetic configurations, on the motion of particles under the given electric field[1,2]. Recent progress has been focused on understanding the influence of the particle loss on the establishment of the radial electric field, including the analysis of the selfconsistent treatment of the radial electric field and the loss cone loss[3-5]. Majority of these selfconsistent analyses for helical plasmas, however, have been limited to the problems of the stationary solutions. The study on the dynamics, which is originated from the mutual influences between the loss cone loss and the electric field, has been studied associated with the H-mode physics in tokamaks[6], and requires further efforts. Oscillations of the loss of the fast ions has been reported on Heliotron-E device experimentally by Zushi and coworkers[7]. This phenomena, which we call the Zushi-oscillation, casts a strong motivation for the investigations. This is because the possible stationary operation is considered to be one of the main advantage of helical systems in proceeding to the future step of the fusion research.

In this article, we report the analytic study on the dynamics of the coupling between the radial electric field,
plasma temperature, and loss cone loss in helical plasmas, and show the existence of the damped oscillations. The analysis is limited to the point model, simplifying the transport problems, in order to show the qualitative nature of the possible thermal and electric oscillations in helical plasmas.

We study the plasmas heated by the neutral beam injection with the injection energy $W_b$. The loss cone boundary is a functional of the potential profile. Owing to this relation, the heating power couples to the plasma potential. The heating power dictates the evolution of the plasma temperature. At the same time, the plasma temperature strongly influences the plasma potential. This closes a link, in helical plasmas, between the plasma temperature profile and radial electric field through the loss cone loss of injected fast ions. We, in the following, study the dynamics of this link of processes, showing the possible oscillations. (The plasma density is influential on the radial electric field as well. However, in order to keep the simplicity and clarity of the argument, we here neglect the coupling with the evolution of the density profile.)

The relation of the radial electric field and plasma temperature has been studied in helical systems. For the NBI heated plasmas, a simple formula,

$$E_r = h_i T_i'/e + \delta E,$$  \hspace{1cm} (1)

works for the wide range of parameters[4], where the coefficient
$h_1$ is the neoclassical coefficient (close to 3.5). The term $\delta E$ is the contribution of the direct loss of fast ions. The expression for $\delta E$ is given as\cite{3,4}

$$\delta E = -\Gamma_b \left( \frac{\partial \Gamma_i}{\partial \varepsilon_r} \right)^{-1}, \quad (2)$$

where $\Gamma_b$ is the particle flux of fast ions, and $\Gamma_i$ is that of the bulk ions. For the parameters of the plasma in present experiments, we have

$$\frac{\partial \Gamma_i}{\partial \varepsilon_r} = eD_1 n/T \quad (3)$$

where $D_1$ is the neoclassical ion diffusivity, $n$ is the plasma density, and we take $T_i = T_e = T$ for the simplicity. Integrating $\delta E$, we have the contribution of the change $\delta E$ to the potential difference, $\delta \phi$, as

$$-e\delta \phi = \Delta_{layer} \left( T/D_1 n \right) \Gamma_b \quad (4)$$

where $\Delta_{layer}$ is the thickness of the layer where $\delta E$ has a substantial contribution. Noting the relation that $\Gamma_b = P_{loss} / (4\pi^2 a R W_{loss})$, where $P_{loss}$ is the loss power associated with the loss cone loss, $a$ and $R$ are minor and major radii, respectively, and $W_{loss}$ is the typical energy of loss ions, we have
\[ e \Phi = \frac{\Delta_{\text{layer}}}{4\pi^2 a R D_{\text{in}}} \frac{T}{W_{\text{loss}}} P_{\text{loss}} \]  

(5)

Summarizing Eqs. (1) and (5), the potential difference between the center and edge, \( \Phi_0 \), is given as

\[ e \Phi_0 = \left( -h_i T(0) - f_1 T_x \right) \frac{T(a)}{W_{\text{loss}}} \frac{P_{\text{loss}}}{P_{\text{in}}} \]  

(6)

The negative value of \( \Phi \) indicates that \( E_r \) is negative, i.e., the static potential is lower at the axis. In Eq. (6), \( f_1 \) is a numerical coefficient defined by \( a \Delta_{\text{layer}} n / 2 \tau_p D_{\text{in}}(a) \). \( \tau_p \) is the energy confinement time, \( n \) is the average density. The value \( T_x \) is the averaged temperature, which would be realized in the absence of the direct loss, given as

\[ n T_x = \tau_p P_{\text{in}} / 2 \pi^2 a^2 R, \]  

(7)

and \( P_{\text{in}} \) is the total injection power.

The loss cone boundary has been given in literatures[1,2]. Figure 1 illustrates the region of loss cone for the particles injected perpendicular to the magnetic field. In the case of the negative \( E_r \), which we consider here, the loss cone region for the deeply trapped particles penetrates in the the core region for a particular energy range as \( W_2 < W < W_1 \) [2].
\( W_1 = \frac{e(\phi(a) - \phi(r))}{\varepsilon_h(1-x^2) - \varepsilon_t(1+x)} \)  
(8-1)

and

\( W_2 = \frac{e(\phi(a) - \phi(r))}{\varepsilon_h(1-x^2) + \varepsilon_t(1-x)} \)  
(8-2)

where \( \varepsilon_h \) and \( \varepsilon_t \) are the helical and toroidal magnetic ripple at the plasma edge, respectively, and \( x = r/a \).

The energy range of the loss cone region at the magnetic axis, for present experimental parameters, lies between the injected energy and the bulk plasma temperature. (When the heating power is extremely high, the radial electric field would be so high that all the ions are trapped by the \( E \times B \) rotation, and loss cone disappears for the injected fast ions.)

For the simplicity, we consider the case where the deposition of the fast ions are localized at the center with the specific injection angle of the helically trapped particles. (The extension for the case of the diffused deposition and for the case of the mixed injection angles is straightforward.) The boundary for the loss cone is given as \( W_{\text{loss}} = W_1(0) \), i.e.,

\[ W_{\text{loss}} = -e\phi/(\varepsilon_h - \varepsilon_t) \]  
(9)

This relation holds for the general profile of the static potential for the case of \( E_r < 0 \) [2]. When the condition \( W_{\text{loss}} < W_b \)
holds, as is the case of present experiments, the heating by the fast ions takes place during the slowing down process from the energy $W_b$ to $W_{loss}$. Therefore the power which is used for the plasma heating, $P_{\text{heat}}$, is given as

$$P_{\text{heat}} = (1 - \frac{W_{loss}}{W_b})P_{\text{in}} \quad (10)$$

The relation $P_{\text{loss}} + P_{\text{heat}} = P_{\text{in}}$ means the energy conservation relation. Figure 2 illustrates the relation of the heating power as the function of the plasma temperature.

Since the bulk plasma heating occurs through the slowing down process of fast ions, there is a time delay between the deposited power and the power transferred to bulk plasma. The time evolution of the power which causes the temporal change of the plasma temperature per unit volume, $P_v$, is simply modelled as

$$\frac{dP}{dt} = (P_{\text{heat}}/V_p - P)/\tau_{\text{heat}} \quad (11)$$

where $\tau_{\text{heat}}$ is the typical slowing down time and $V_p$ is the plasma volume.

The evolution of the plasma temperature is given by the energy transport equation

$$\frac{dT}{dt} = (P/n - T)/\tau_E. \quad (12)$$

Equations (11) and (12) with equations (6), (9) and (10) constitute the set of basic equations.
We take the normalized form \( \hat{T} = T/W_b \), \( \hat{P} = \tau E P/nW_b \), and \( \tau = t/\tau E \). Equations (6) and (9)-(12) are rewritten as

\[
\frac{\partial \hat{T}}{\partial \tau} = \hat{P} - \hat{T} \tag{13-1}
\]

\[
\frac{\partial \hat{P}}{\partial \tau} = \sigma \left( [1 - CT - C_1 \hat{P}_{in}(a)] \hat{P}_{in} - \hat{P} \right) \tag{13-2}
\]

where \( \sigma = \tau E/\tau_{heat} \), \( \hat{P}_{in} = \tau E \hat{P}_{in}/nV_p W_b \), \( \hat{T}(a) = T(a)/W_b \), and \( C \) and \( C_1 \) are coefficients defined as

\[
C = h_i \frac{T(0)}{T} \frac{1}{\epsilon_h - \epsilon_t} \tag{14}
\]

and \( C_1 = C f_1/h_i \). For the torsatron/ heliotron configuration, such as Heliotron E and CHS, the term in Eq. (6), which is in proportion to \( f_1 \), has only a small contribution. We therefore neglect this term here.

Under the condition of our interest corresponding to the experimental conditions, which is shown in Fig. 1, the equation (13) has one fixed point. The stationary solution is given as

\[
\hat{T} = \hat{P} = \hat{P}_0 = \hat{P}_{in}/(1 + CP_{in}) \tag{15}
\]

The stability of the trajectory near this fixed point is studied by expanding \( \hat{T} \) and \( \hat{P} \) near \( \hat{P}_0 \) as \( \hat{T} = \hat{P}_0 + x \) and \( \hat{P} = \hat{P}_0 + y \). The linearized equation is given as
\[
\begin{align*}
\begin{bmatrix}
\frac{\partial x}{\partial \tau} \\
\frac{\partial y}{\partial \tau}
\end{bmatrix} &= \begin{bmatrix}
-1 & 1 \\
-\sigma C_p & -\sigma
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\end{align*}
\]  
(16)

by assuming that \( T(r) \) profile is fixed. Writing the time dependence as \((x, y) \sim (x_0, y_0) \exp(\lambda \tau)\), we have

\[
\lambda = -(1+\sigma)/2 \pm \sqrt{(1-\sigma)^2/4-\sigma C_p in}
\]  
(17)

This result shows that the oscillation around the stationary solution is possible if the condition

\[
4\sigma C_p in > (1-\sigma)^2
\]  
(18)

is satisfied. This relation is more easily satisfied for the case that the slowing down time is close to the energy confinement time, \( \sigma \approx 1 \), or the case of large \( C \) (i.e., \( \epsilon_t \) is closer to \( \epsilon_h \)). Since the real part of the eigenvalue \( \lambda \) is always negative, the oscillation is a damped oscillation. The oscillation is more clearly seen if \( |\text{Im} \lambda| > |\text{Re} \lambda| \), or

\[
C_p in > 1
\]  
(19)

In this case, the oscillation frequency, \( |\text{Im} \lambda/\tau_E| \), is approximately proportional to \( \sqrt{P_{in}/\tau_{heat}} \). If Eq.(19) is not satisfied, the damping rate is comparable to or larger than the oscillation frequency, and the oscillation would not be observed.
From Eq. (18), we see that the oscillation is possible only if \( \tau_R \) and \( \tau_{\text{heat}} \) is finite. The oscillation requires the relaxations both in the real space and the velocity space.

The ratio between the amplitudes of oscillation of the temperature and the heating power can be also obtained. From Eq. (6), we see that the potential difference is \( h_i T(0)/T \) times larger than the variation of the average temperature. This is also the case for the heating power. Substituting the eigenvalue Eq. (17) in to Eq. (16), we have the ratio between \( y \) and \( x \), i.e., \( y_0/x_0 \), is given by

\[
y_0/x_0 = (\sigma-1)/2 \pm \sqrt{(1-\sigma)^2/4-\alpha C_{\text{in}}^2}
\]  (20)

For the case that the oscillation can be observed, i.e., Eq. (19) holds and \( \sigma=1 \), \( y \) is larger than \( x \). This implies that the variation of the ion loss is more prominent than the change of the temperature.

In summary, the analytic theory of the thermal and electric oscillation in helical plasma is developed. The existence of the coupled oscillations of \( (T, E_r, W_{\text{loss}}, P_{\text{heat}}) \), caused by the coupling of the loss cone loss with the radial electric field, is predicted. This newly predicted coupled oscillation is found to be stable. This suggests that the oscillations appear as the postcursor to the sudden and large amplitude change of temperature change (such as the internal disruption). This is consistent with what Zushi and coworkers have observed on
Heliotron E. It is also noted, from Eq. (20), that the temporal variation is more prominent for the loss cone loss than for the plasma temperature. This explains one of the characteristics of the observed Zushi-oscillation, that the oscillation is prominent in the loss of energetic ions. The dependence of the oscillation frequency is derived, which is different from those in the conventional relaxation process of the pressure driven modes. If the relations $\tau_{\text{heat}} \propto T^{1.5}$ and $T \propto P_{\text{in}}^{-\mu}$ hold, then the frequency is proportional to $P_{\text{in}}^{1/2-3\mu/4}$. The frequency is higher for higher heating power in the L-mode plasmas.

When heating power is extremely high so that $\bar{W}_b < \bar{W}_2$ holds, or

$$\frac{\tau_{\text{heat}} P_{\text{in}}}{n \nu_p \bar{W}_b} > \frac{1}{C \varepsilon_{h} \varepsilon_{t}}$$  \hspace{1cm} (21)$$

the radial electric field would be so high that all the injected ions are trapped by the $E \times B$ rotation, and loss cone disappears for the fast ions. The extrapolation of this theory to the general injection angle would be straightforward. The variety in phenomena would be expected.

This oscillation requires both the time delays, i.e., real and velocity space diffusion. The analysis in this note is limited to the point model, which simplifies the diffusion process much. The study including the radial profiles, which would be necessary for the quantitative comparison with the Zushi-oscillation, is left for the future study.
One of the authors (K.I) acknowledges discussion with Dr. H. Zushi. This work is partly supported by the Grant-in-Aid for Scientific Research of MoE Japan.
References


Figure Captions

Fig. 1 Loss cone region of the perpendicularly injected fast ions in torsatron/heliotron configuration. The initial energy $W_b$ and the energy at the loss boundaries, $W_1$ and $W_2$, are also noted.

Fig. 2 Heating efficiency, $P_{\text{heat}}/P_{\text{in}}$, and the normalized loss power of the energy transport, $nTV_p/\tau_B P_{\text{in}}$ (dashed line), are plotted as a function of the temperature for the perpendicular injection. The crossing of two lines gives the stationary solution. When the temperature is very high and $W_2(0) > W_b$ holds, then the injected fast ions are confined by the $E\times B$ rotation, and the heating efficiency increases again.
Fig. 2

\[ \frac{nTV_p}{\tau_E P_{in}} \]

\[ P_{\text{heat}} / P_{in} \]
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