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Prandtl Number of Toroidal Plasmas

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Abstract

Theory of the L-mode confinement in toroidal plasmas is developed. The Prandtl number, the ratio between the ion viscosity and the thermal conductivity is obtained for the anomalous transport process which is caused by the self-sustained turbulence in the toroidal plasma. It is found that the Prandtl number is of order unity both for the ballooning mode turbulence in tokamaks and for the interchange mode turbulence in helical system. The influence on the anomalous transport and fluctuation level is evaluated. Hartmann number and magnetic Prandtl number are also discussed.

Keywords: Prandtl number, Hartmann number, magnetic Prandtl number, Anomalous transport, Self-sustained turbulence, Tokamak, Helical systems, Collisionless skin depth

1. Introduction

Recently progress has been made in the understanding on the anomalous transport phenomena in toroidal plasmas. One of the characteristic features of the transport is that the energy confinement time τ_E decreases as the heating power P increases[1]. In order to explain this phenomena, a theory was developed on the self-sustained turbulence, analyzing the plasma state in which the anomalous transport and fluctuations enhance each other[2,3]. In this new approach, it was found that the ratio between the transport coefficients, such as λ/χ and μ/χ (λ: the current-diffusivity, χ: the thermal conductivity, μ: the ion viscosity), are the key parameters to determine the absolute values of the anomalous transport coefficients $\{\mu, \lambda, \chi\}$. The coefficients $\{\mu, \lambda, \chi\}$ were derived for high temperature plasmas with the approximation that $\mu/\chi \simeq 1$ and $\lambda/\chi \simeq (\delta/a)^2$ (δ: the collisionless skin depth, a: the plasma minor radius). The self-consistent determination of the ratio μ/χ, i.e., the Prandtl number, is necessary.

The importance of the Prandtl number has also been pointed out. In tokamaks, various kind of the improved confinement phenomena have been found. In these discharges, the energy confinement time was found to be better than the value expected for the L-mode plasmas[4]. The mechanism of the pinch of the plasma particles, and associated improved confinement have been theoretically discussed[5].

In this article, we develop the theory of the self-sustained turbulence, and calculate the Prandtl number self-consistently.

The result is obtained both for tokamaks and helical plasmas. It is confirmed that the ratio μ/χ and $(a/\delta)^2\lambda/\chi$ are close to unity.

2. Analysis of Tokamak Plasma

2.1 Basic Equations

We study the circular tokamak with the toroidal coordinates (r, θ, ζ). The reduced set of equations[6] is employed. The transport coefficients, which are derived by renormalizing the ballooning mode turbulence, were obtained in Ref.[3]. The transport coefficients $\{\mu, \lambda, \chi\}$ are calculated as

$$\mu = \Sigma \frac{|k_{1\perp}\Phi_1|^2}{2B^2} \frac{k_{1\perp}^2}{K_1} \tag{1}$$

$$\lambda = \delta^2 \mu_0 \mu_e \tag{2}$$

$$\mu_e = \Sigma \frac{|k_{1\perp}\Phi_1|^2}{2B^2} \frac{1}{K_1} \frac{\tau_{u1}}{\tau_{j1}} \left[k_{1\perp}^2 + \frac{i A_1 k_{1\theta} \nabla p_0}{\tau_{u1}\tau_{p1} B} \right] \tag{3}$$

and

$$\chi = \Sigma \frac{|k_{1\perp}\Phi_1|^2}{2B^2} \frac{1}{K_1} \frac{\tau_{u1}}{\tau_{p1}} \left[k_{1\perp}^2 + \frac{g\xi k_{1\parallel}^2}{\tau_{u1}\tau_{j1}} \right] \tag{4}$$

where

$$K_1 = \tau_{u1} k_{1\perp}^2 + g\xi \frac{k_{1/\!/}^2}{\tau_{j1}} + \frac{iA_1 k_{1\theta}\nabla p_0}{\tau_{p1}\ B} \qquad (5)$$

$\tau_{u1} = \tau(1)+\mu k_{1\perp}^2$, $\tau_{j1} = \tau(1)+\mu_e k_{1\perp}^2$, $\tau_{p1} = \tau(1)+\chi k_{1\perp}^2$, and $\tau(1)$ is defined by $\partial Y_1/\partial t = \tau(1)Y_1$. Parameters $g\xi$ and A_1 are defined as $g\xi = e^2B^2/m_e m_i$, $A_1 p_1 = (B/m_i n_i)[\nabla(2r\cos\theta/R)\times\hat{\zeta}]\cdot\nabla p_1$. The suffix 1 denotes the back-ground turbulence. In these equations following notation is used: m_i is the ion mass, n_i is the ion density, m_e is the electron mass, E is the electric field, B is the main magnetic field, p is the plasma pressure, J is the current and σ is the conductivity. The static potential is given by Φ.

Using these renormalized transport coefficients, the set of model equations for the nonlinear waves driven by the pressure gradient is given in the following form.

Equation of motion:

$$n_i m_i \{d(\nabla_\perp^2\Phi)/dt - \mu\nabla_\perp^4\Phi\} = B^2\nabla_{/\!/} J + B\nabla p\times\nabla(2r\cos\theta/R)\cdot\hat{\zeta} \qquad (6)$$

The Ohm's law:

$$E + v\times B = \frac{1}{\sigma} J - \lambda\nabla_\perp^2 J \qquad (7)$$

and the energy balance equation

$$\frac{d}{dt}p + \frac{1}{B}[\phi, p_0] = \chi \nabla_\perp^2 p \tag{8}$$

where the tilde denoting the test wave is suppressed. The velocity is given as

$$v = -\nabla\phi \times \vec{b}/B \tag{9}$$

where $\vec{b}$ is the unit vector in the direction of the main magnetic field. The time derivative of the quantity Y, dY/dt, is given as

$$dY/dt = \partial Y/\partial t + [\phi_0, Y]/B \tag{10}$$

where the bracket [] denotes the Poisson's bracket: $[f,g] = (\nabla f \times \nabla g)\cdot\vec{b}$. Since the nonlinear interaction through the E×B nonlinearity is renormalized, the static component ϕ_0 is used in the Poisson bracket.

2.2 Ballooning Transformation and Dispersion Relation

The ballooning transformation[7)] is employed as

$$\phi(r, \theta, \zeta) = \Sigma \exp(-im\theta + in\zeta) \int \phi(\eta) \exp\{im\eta - inq\eta\} d\eta, \tag{11}$$

(q is the safety factor) since we are interested in the microscopic modes. The dispersion relation, which is derived from the partial differential equations (6)-(8), is reduced to the ordinary differential equation as[8)]

$$\frac{d}{d\eta}\frac{F}{\hat{\gamma}+\Xi F+\Lambda F^2}\frac{d\Phi}{d\eta}+\frac{\alpha[\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta]\Phi}{\hat{\gamma}+XF}$$

$$-(\hat{\gamma}+MF)F\Phi = 0. \qquad (12)$$

In writing Eq.(12), we use the normalization: $r/a \to \hat{r}$, $t/\tau_{Ap} \to \hat{t}$, $\chi\tau_{Ap}/a^2 \to \hat{\chi}$, $\mu\tau_{Ap}/a^2 \to \hat{\mu}$, $\tau_{Ap}/\mu_0\sigma_c a^2 \to 1/\hat{\sigma}$, $\lambda\tau_{Ap}/\mu_0 a^4 \to \hat{\lambda}$, $\gamma\tau_{Ap}\to\hat{\gamma}$. Notation is employed as: $\tau_{Ap} \equiv a\sqrt{\mu_0 m_i n_i}/B_p$, $\Xi = n^2q^2/\hat{\sigma}$, $\Lambda = \hat{\lambda}n^4q^4$, $X = \hat{\chi}n^2q^2$, $M = \hat{\mu}n^2q^2$. Other notation is standard: $B_p=Br/qR$, $\varepsilon=r/R$, R is the major radius, β is the pressure divided by the magnetic pressure ($\beta=\mu_0 n(T_e+T_i)/B^2$), γ is the growth rate, s is the shear parameter, $s = r(dq/dr)/q$, $F=1+(s\eta-\alpha\sin\eta)^2$, κ is the average well, $\kappa\equiv-(r/R)(1-1/q^2)$, α denotes the normalized pressure gradient, $\alpha=q^2\beta'/\varepsilon$ and $\beta'\equiv d\beta/d\hat{r}$.

Equation (12) constitutes the nonlinear dispersion relation. Since the nonlinear interactions are renormalized in the coefficients, μ, λ and χ, the marginal stability condition (γ=0) gives the relation between the renormalized transport in the stationary state. From Eq.(12) with γ=0 for the least stable mode, $\hat{\chi}$ was expressed in terms of the Prandtl numbers $\hat{\mu}/\hat{\chi}$ and $\hat{\mu}_e/\hat{\chi}$ as[2,3)] (note the relation $\hat{\lambda} = (\delta/a)^2\hat{\mu}_e$)

$$\hat{\chi} = f(s)^{-1}\alpha^{3/2}(\delta/a)^2(\hat{\mu}_e/\hat{\chi})\sqrt{\hat{\chi}/\hat{\mu}}. \qquad (13)$$

By the spirit of the mean field approach, we substitute the parameters of the least stable mode in the numerator and denominators in the renormalized transport coefficients (1)-(3). Since the term $|k_{1\perp}\Phi_1|^2 k_{1\perp}{}^2/[2B^2K_1]$ is common to all coefficients, we have the estimates

$$\frac{\mu_e}{\mu} = \frac{\tau_{u1}}{\tau_{j1}}\left[\ 1 + \frac{iA_1 k_{1\theta}\nabla p_0}{k_{1\perp}{}^2\tau_{u1}\tau_{p1}B}\right] \tag{14}$$

and

$$\frac{\chi}{\mu} = \frac{\tau_{u1}}{\tau_{p1}}\left[\ 1 + \frac{g\xi k_{1\parallel}{}^2}{k_{1\perp}{}^2\tau_{u1}\tau_{j1}}\right] \tag{15}$$

where the least stable mode is substituted in the fluctuation mode in the right hand side of Eqs.(14) and (15). In the self-sustained state, $\tau \sim 0$ holds. The relations $\tau_{u1} \simeq \mu k_{1\perp}{}^2$, $\tau_{j1} \simeq \mu_e k_{1\perp}{}^2$ and $\tau_{p1} \simeq \chi k_{1\perp}{}^2$ hold. With the help of the ballooning transformation and the normalization, we have

$$\frac{iA_1 k_{1\theta}\nabla p_0}{k_{1\perp}{}^2\tau_{u1}\tau_{p1}B} \rightarrow \frac{\alpha n^2 q^2 \langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle}{\hat{\mu}\hat{\chi}\langle F^3\rangle(nq)^6} \tag{16}$$

and

$$\frac{g\xi k_{1\parallel}{}^2}{k_{1\perp}{}^2\tau_{u1}\tau_{j1}B} \rightarrow \frac{(\mu_e/\lambda)\langle|\partial/\partial\eta|^2\rangle}{\hat{\mu}\hat{\mu}_e\langle F^3\rangle(nq)^6} \tag{17}$$

(Note the relation $k_{\parallel\perp}{}^2 \to Fn^2q^2$.) The mode number n is set to be that for the least stable mode, and the average $\langle\cdots\rangle$ is defined as

$$\langle\cdots\rangle \equiv \int\cdots\Phi(\eta)^2 d\eta \left[\int\Phi(\eta)^2 d\eta\right]^{-1} \tag{18}$$

where $\Phi(\eta)$ is the eigenfunction of Eq.(12) with $\hat{\gamma}=0$ for the least stable mode. The parallel derivative is evaluated as

$$|\partial/\partial\eta|^2 \equiv \int(\partial\Phi/\partial\eta)^2 d\eta \left[\int\Phi^2 d\eta\right]^{-1} \tag{19}$$

Substituting Eqs.(16) and (17) into Eqs.(14) and (15), we have

$$\frac{\mu_e}{\mu} = \frac{\mu}{\mu_e}\left[1 + \frac{\alpha\langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle}{\hat{\mu}\hat{\chi}\langle F^3\rangle(nq)^4}\right] \tag{20}$$

and

$$\frac{\chi}{\mu} = \frac{\mu}{\chi}\left[1 + \frac{\hat{\mu}_e}{\hat{\lambda}}\frac{\langle|\partial/\partial\eta|^2\rangle}{\hat{\mu}\hat{\mu}_e\langle F^3\rangle(nq)^6}\right] \tag{21}$$

The normalized mode number N is introduced[2,3,8] as $N^4 = (nq)^4(\hat{\chi}\hat{\mu}/\alpha)$. By using N and Eq.(13), Eqs.(20) and (21) are rewritten as

$$\frac{\mu_e}{\mu} = \frac{\mu}{\mu_e}\left[1 + \frac{\langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle}{N^4\langle F^3\rangle}\right] \tag{22}$$

and

$$\frac{\chi}{\mu} = \frac{\mu}{\chi}\left[1 + \frac{\langle|\partial/\partial\eta|^2\rangle}{fN^6\langle F^3\rangle}\right] \tag{23}$$

2.3 Case of Normal-Curvature-Driven Mode

In the case that the magnetic shear is not strong, i.e., $1 \gg s-\alpha$, the mode is driven by the normal curvature. In this case the eigen mode structure $\phi(\eta)$ is approximated by the gaussian form as[3] $\phi(\eta) = \exp(-\eta^2/\eta_0{}^2)$.

For this mode structure, we have

$$\frac{\langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle}{\langle F^3\rangle} \simeq 1 - \frac{(1/2+\alpha-s-3s^2)}{4\eta_0{}^2} \tag{24}$$

and

$$|\partial/\partial\eta|^2 = 1/\eta_0{}^2 \tag{25}$$

In the weak shear limit, the least stable mode satisfies $N^4=1/2$, $N^2f=1$, and $\eta_0{}^2 = 8/3$. Using these numbers, and substituting Eqs.(24) and (25) into Eqs.(22) and (23), we have

$$\mu_e/\mu \simeq 1.5 \tag{26}$$

and

$$\chi/\mu \simeq 1.3. \tag{27}$$

2.4 Case of Geodesic-Curvature-Driven Mode

When the shear is strong, the driving mechanism by the geodesic curvature overcomes that by the normal curvature. When α is much smaller than s, the averages are estimated for the gaussian form of $\phi(\eta) = \exp(-\eta^2/\eta_0{}^2)$ as

$$\langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle = \kappa + (1+s\eta_0{}^2/4)\exp(-\eta_0{}^2/8) \tag{28}$$

and $\langle F^3\rangle = 1 + 3s^2\eta_0{}^2/4 + 9s^4\eta_0{}^4/16 + 15s^6\eta_0{}^6/64$ in the low α limit.

We estimate the Prandtl number for the case of $s\simeq1$. The eigenvalue equation (12) was solved numerically[8]. The least stable mode was given and satisfies

$$N \simeq 0.187. \tag{29}$$

The coefficient $1/f$ was given as $1/f \sim 2.5$. The solution $\phi(\eta)$ is illustrated in Fig.1. We have the fitting

$$\eta_0 \simeq 1.6\pi. \tag{30}$$

Using this solution, we have

$$\langle\{\kappa+\cos\eta+(s\eta-\alpha\sin\eta)\sin\eta\}\rangle \simeq 0.31 \tag{31}$$

$\langle F^3\rangle \simeq 4.1\times10^3$ and

$$\langle|\partial/\partial\eta|^2\rangle = 0.08. \tag{32}$$

Substituting these numbers, we have

$$\mu_e/\mu \simeq 1.03 \tag{33}$$

and

$$\chi/\mu \simeq 1.23 \tag{34}$$

From Eqs.(26), (27), (33) and (34), we see that the approximate formula $\mu/\chi\simeq1$ and $\mu_e/\mu\simeq1$ are valid both for the weak and strong shear cases.

2.5 Fluctuation Amplitude

Equation (1) denotes the relation between the fluctuation amplitude and the transport coefficient. When the transport coefficient is given, the expected fluctuation level is derived from this relation. Employing the newly obtained Prandtl numbers, we estimate the level of fluctuations.

In the framework of the ballooning transformation, the term K_1 in the denominator is given as

$$K_1 = \mu k_{1\perp}{}^4\{(\hat{\mu}_e/\hat{\mu})^2 + (\hat{\chi}/\hat{\mu})^2 - 1\} \tag{35}$$

Substituting this relation into Eq.(1) with $\hat{\gamma}=0$, we have

$$\mu = \frac{|\Phi|^2}{2B^2\mu\{(\hat{\mu}_e/\hat{\mu})^2+(\hat{\chi}/\hat{\mu})^2-1\}} \tag{36}$$

The fluctuation amplitude is given as

$$e\Phi/T = \sqrt{2\{(\hat{\mu}_e/\hat{\mu})^2+(\hat{\chi}/\hat{\mu})^2-1\}}[eB\mu/T] \tag{37}$$

Substituting the relations Eqs.(26), (27), (33) and (34), we have

$$e\Phi/T = 2.48\ [eB\mu/T] \tag{38}$$

for the weak shear case and

$$e\Phi/T = 1.8\ [eB\mu/T] \tag{39}$$

for the high shear case ($s \simeq 1$).

The explicit formula of χ, Eq.(13), and Eqs.(26) and (27) gives

$$\hat{\chi} = 1.33\alpha^{1.5}f^{-1}(\delta/a)^2 \tag{40}$$

and

$$\hat{\mu} = \alpha^{1.5}f^{-1}(\delta/a)^2 \tag{41}$$

for the low shear case. The fluctuation amplitude of the nonlinearly driven modes is predicted, for the low shear case, in terms of the equilibrium plasma parameters as

$$e\Phi/T = 2.48\alpha^{1.5}f^{-1}(eB\delta^2/\tau_{Ap}T). \tag{42}$$

This equation can be rewritten as

$$e\Phi/T = 5q^2\sqrt{m_e/m_i}f(s)^{-1}(R\beta'/r)^{3/2}\beta^{-1}(\delta/R), \tag{43}$$

(note that $\beta' \equiv d\beta/d\hat{r}$). Dominant dependence is given by δ/R with the geometrical factors such as q, s and ε.

3. Toroidal Helical Plasmas

In toroidal helical plasmas, such as the torsatron/Heliotron devices, the configuration is characterized by the magnetic hill. In the system with magnetic hill, the interchange modes become unstable. The theory for the self-sustained turbulence has also been developed[2,3].

The renormalization similar to Eqs.(1)-(3) has been developed. With the same spirit of the mean field approximation, we have the evaluation as

$$\frac{\mu_e}{\mu} = \frac{\mu}{\mu_e}\left[1 + \frac{D_0 \hat{k}_\theta{}^2}{\hat{\mu}\hat{\chi}\hat{k}_\perp{}^6} \right] \tag{44}$$

and

$$\frac{\chi}{\mu} = \frac{\mu}{\chi}\left[1 + \frac{(v_A/\delta)^2 k_{/\!/}{}^2}{\mu\mu_e k_\perp{}^6} \right] \tag{45}$$

The notation D_0 is introduced

$$D_0 = -\beta' \Omega' \varepsilon^{-2}/2 \tag{46}$$

to denote the driving force due to the magnetic hill (Ω' is the average magnetic curvature). Normalized value $\hat{k}$ is defined as ak. The perpendicular wave number $k_\perp$ is defined as $k_\perp{}^2 = k_\theta{}^2 + k^2$, where k is the radial wave number.

By use of the shear parameter, the parallel wave number $k_{/\!/}$ is given as $sk_\theta x/qR$, where x is the distance from the mode rational surface. Owing to the Fourier transformation from x to k, we have $k_{/\!/}{}^2/k_\perp{}^2 = (sk_\theta/qRk_\perp)^2 |d/dk|^2$.

As in the previous section, we introduce a normalization as

$$H = (D_0 \hat{\lambda}/s^2 \hat{\chi})(\hat{\mu}\hat{\lambda}/s^2)^{-1/3} \tag{47}$$

$$z = \hat{k}/\hat{k}_0 \tag{48}$$

$$b = \hat{k}_\theta{}^2/\hat{k}_0{}^2 \tag{49}$$

and

$$\hat{k}_0 = (s^2/\hat{\mu}\hat{\lambda})^{1/6}. \tag{50}$$

The ratios μ_e/μ and χ/μ are then given as

$$\frac{\hat{\mu}_e}{\hat{\mu}} = \frac{\hat{\mu}}{\hat{\mu}_e}\left[1 + \frac{bH}{\langle (b+z^2)^3 \rangle}\right] \tag{51}$$

and

$$\frac{\hat{\chi}}{\hat{\mu}} = \frac{\hat{\mu}}{\hat{\chi}}\left[1 + \frac{b\langle |d/dz|^2 \rangle}{\langle (b+z^2)^3 \rangle}\right] \tag{52}$$

where the average $\langle \cdots \rangle$ is defined by the integral

$$\langle \cdots \rangle \equiv \int \cdots \Phi(z)^2 dz \left[\int \Phi(z)^2 dz\right]^{-1} \tag{53}$$

and $\Phi(z)$ is the eigenfunction of the least stable mode.

With the normalization, the eigenmode equation was given as[3)]

$$d^2\Phi/dz^2 + [(H-b^2) - z^6/b]\Phi(z) = 0. \qquad (54)$$

For a convenience, we rewrite Eq.(54) as

$$d^2\Phi/dy^2 + G[\ 1 - y^6\ ]\Phi(y) = 0. \qquad (55)$$

by using the notation $y = [b(H-b^2)]^{-1/6}z$ and $G = b^{1/3}(H-b^2)^{4/3}$. The solution of Eq.(55) is given in Fig.2.

For the least stable mode, H and b was given as[3)]

$$H \cong 1.6 \qquad (56)$$

and

$$b \cong 0.43. \qquad (57)$$

From the numerical calculation, the eigenfunction $\Phi(y)$ is obtained. The average $\langle\cdots\rangle$ is performed using the eigenfunction. We have the estimates $1/\langle(b+z^2)^3\rangle \simeq 6.1$ and

$$\langle |d/dy|^2\rangle \equiv \int(d\Phi/dy)^2dy\ \left[\int\Phi^2dy\right]^{-1} \simeq 0.86 \qquad (58)$$

Substituting these numbers, we have

$$\mu_e/\mu \simeq 2.3 \qquad (59)$$

and

$$\chi/\mu \simeq 2.0 \tag{60}$$

This result also confirms that the Prandtl numbers are close to unity for the self-sustained turbulence in the toroidal helical plasmas.

4 Hartmann Number and Magnetic Prandtl Number

Using the results on the anomalous transport coefficient, the Hartmann number[9] is calculated, which is an important parameter for the global instabilities[10]. Hartmann number $\boldsymbol{M}$ is defined as $\boldsymbol{M} = BL\sqrt{\sigma/m_i n_i \mu}$, where L is the typical scale length. Substituting plasma minor radius into L, we have the relation $\boldsymbol{M}^2 = (qR/a)^2(\hat{\sigma}/\hat{\mu})$. Using the formula of Eq.(40), we have

$$\boldsymbol{M} = (qR/\delta)\left[\sigma f \alpha^{-3/2}\right]^{1/2} \tag{61}$$

for the L-mode plasma. The dependence on the plasma parameter is explicitly rewritten as

$$\boldsymbol{M} \propto B^2 q^{-1} \varepsilon^{1/4} n^{-1/2} a^{3/2}. \tag{62}$$

The Hartmann number does not depend on the plasma temperature. It is in contrast to the classical estimation where the dependence $\mu \propto T^{-1/2}$ gave the dependence $\boldsymbol{M} \propto T$. Because the viscosity is enhanced by the anomalous transport but the conductivity is not, the Hartmann number is smaller than the neoclassical estimate.

The other important parameter is the magnetic Prandtl number, $P_M = \mu_0 \sigma \mu$, which may have a key role in the dynamo mechanism. It has been conjectured that the condition $P_M > 1$ may be related to the generation of dynamo effect[11]. The magnetic Prandtl number is rewritten as $P_M = \hat{\mu}\hat{\sigma}$. Substituting Eq.(40), we find that P_M is an increasing function of the plasma beta value. In the L-mode plasmas, P_M depends strongly on the plasma temperature, as

$$P_M \propto T^3. \tag{63}$$

P_M can exceed unity for the parameter of present day experiments. For the plasma of $T \sim 1$keV, $\mu_0 \sigma$ is in the range of $\sim 30 s/m^2$. The viscosity is estimated as $1 \sim 10$ m^2/s. The magnetic Prandtl number can easily be of the order of 100. The implication of this high magnetic Prandtl number requires future studies.

5. Summary and Discussion

In this article, the Prandtl number for the toroidal plasma is obtained. The ratio between the transport coefficients, which are obtained by renormalizing fluctuations, are calculated for the least stable mode. The result showed that μ_e/μ and χ/μ remain close to unity ($1 \sim 2$) for both tokamaks and toroidal helical plasmas. This confirmed the validity of the approximate estimate $\mu_e/\mu \sim 1$ and $\chi/\mu \sim 1$, which were used in the analysis of the L-mode plasma.

The analysis is also extended to estimate the fluctuation level of the self-sustained turbulence. As in the case of the thermal conductivity, the theoretical prediction on the fluctuation level is also affected by a numerical factor of order unity compared to the results in Refs.[2,3]. For the future comparison of the theoretical prediction with the experimental results, the Prandtl numbers can be used.

It is noted that the present theory is developed in the framework of the one point renormalization and mean field approach. The uncertainty in the numerical coefficients of order unity still exists. The numerical simulation would be necessary to obtain the accurate numerical factor for the transport coefficient and the Prandtl numbers.

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Figure Caption

Fig.1 Mode structure of the least stable mode for the ballooning mode in tokamaks. Parameters: n=26, s=1, q=3, ε=1/8, α=0.0258, $\hat{\mu}=\hat{\chi}=10^{-6}$, $\hat{\lambda}=10^{-10}$ and $1/\hat{\sigma}=0$.

Fig.2 Mode structure for the interchange mode in torsatron/ Heliotron plasmas.

Fig.1

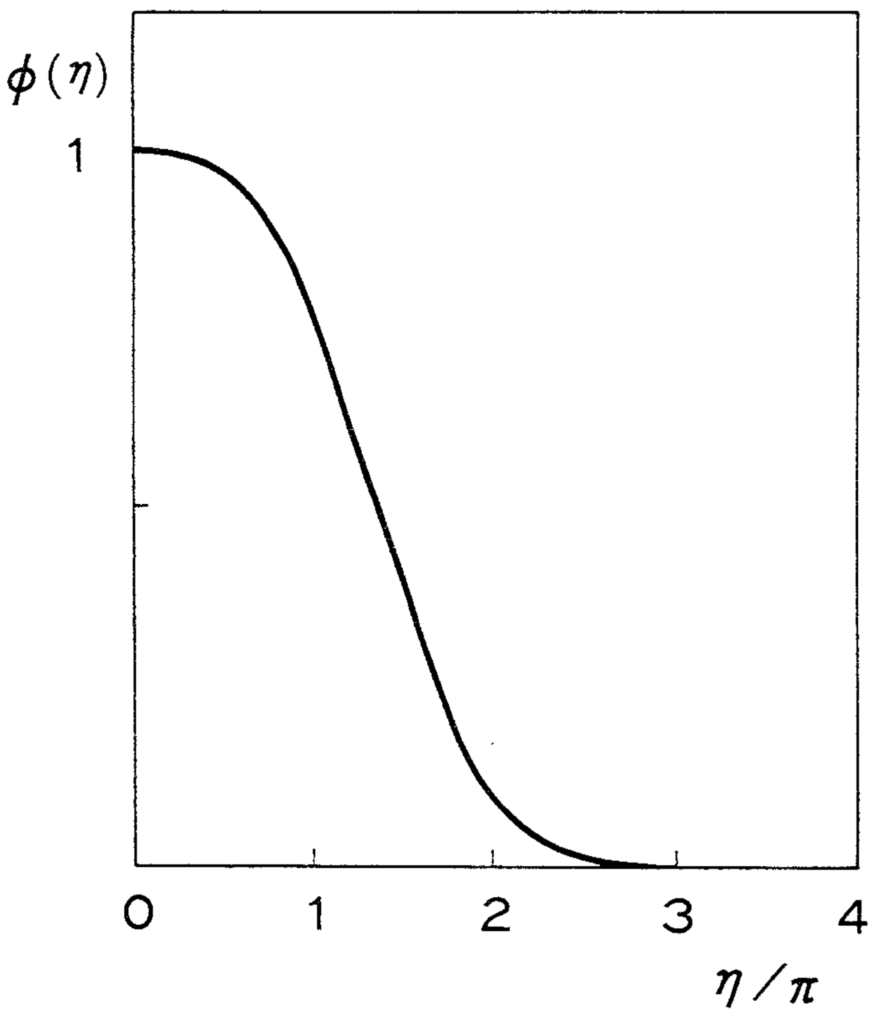
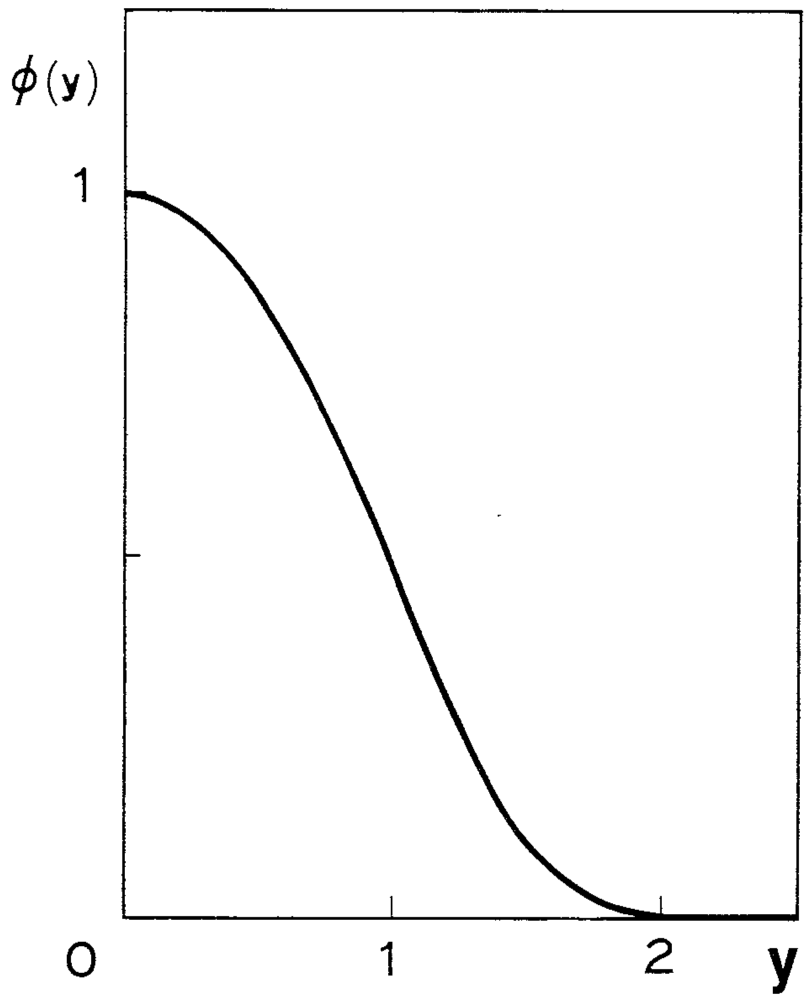


Fig. 2



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