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Averaged Resistive MHD Equations

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Abstract

Averaged resistive MHD equations are derived by using the Lagrangian formalism. The assumption is that the perturbation is of single toroidal harmonics on the special magnetic coordinates Jacobian of which is independent of the toroidal angle. The metrics contained in the obtained resistive equations are those appearing in the averaged equilibrium equations. The local stability conditions are also derived.

KEYWORDS: resistive MHD equations, averaged method, Mercier criterion, Lagrangian, resistive interchange, local stability

§1. Introduction.

The averaging method, which reduces the three dimensional problem to the two dimensional one, proves to be a quite efficient method to describe MHD equilibrium and stability in the helical torus. In the description of the ideal equilibrium problem, the averaging method can be formulated without any ordering assumptions.[1] In the time dependent problem, however, the averaging formulation is derived on the so-called stellarator ordering.[2] Such an approximation is not consistent, if we want to apply to the 3D equilibrium obtained without using such ordering. In Ref.1, the variational principle for the ideal MHD stability problem is derived without using the stellarator ordering. If we restrict our attention to the linearized problem, we can also derive the resistive equations consistent to the equilibrium.

In this paper, we will formulate the averaging approach to the linearized resistive MHD stability problem consistent to the 3D equilibrium without ordering assumptions. This is done by using the variational principle such as Lagrangian formulation,[3] we can choose the approximation functional space to describe the approximate solution to the problem. The averaging approach is characterized as the method on the approximation of the functional space with single toroidal harmonics.

In the next section, we briefly discuss the averaged equilibrium relations with respect to the toroidal angle on a certain curvilinear coordinate system. In §3 the averaged version of the linearized MHD equations are derived, by using the Lagrangian for the linearized MHD equations and posing the condition for the perturbation. The resistive ballooning equations are obtained in §4 by applying the ballooning approximation or the obtained set of equations, and the local stability conditions[4,5] are obtained in §5. The conclusions are stated in §6. In the Appendices the construction of the coordinates used in this paper as well as the relation used in this paper and equations in Ref.5 is discussed.

§2. Averaged Equilibrium.

We shall consider the MHD equilibrium satisfying the magnetostatic equations

$$\boldsymbol{J}\times\boldsymbol{B}=\nabla P,\qquad \boldsymbol{J}=\nabla\times\boldsymbol{B},\qquad \nabla\cdot\boldsymbol{B}=0. \tag{1}$$

and the perturbation around this equilibrium.

We shall consider the curvilinear coordinate system (X, Y, ζ), which is related to the cylindrical coordinates (r, z, ϕ) by the relations,

$$r = r(X, Y, \zeta), \quad z = z(X, Y, \zeta), \quad \phi = \zeta + \Phi_1(X, Y, \zeta), \tag{2}$$

$$\langle r \rangle_\zeta = X, \qquad \langle z \rangle_\zeta = Y, \qquad \langle \Phi_1 \rangle_\zeta = 0, \tag{3}$$

where bracket with the subscript ζ stands for the average with respect to ζ. The conditions in eqs.(3) are not requisite, but they are put in order to determine the coordinates uniquly. In the coordinates $(X, Y, \zeta) \equiv (x^1, x^2, x^3)$, the induction density (H^X, H^Y, H^ζ) or (H^1, H^2, H^3), contravariant component of the magnetic field multiplied by the Jacobian $\sqrt{g}$ is assumed to be independent of the toroidal angle $\zeta \equiv x^3$:

$$H^\alpha = \sqrt{g} B^\alpha = H^\alpha(X, Y). \tag{4}$$

Such coordinates can be constructed, if the magnetic surfaces exist. Since we consider the plasma region with nested magnetic surfaces, the magnetic surface is $\Psi_p(X,Y)$, and the magnetic field can be represented as.

$$\boldsymbol{B} = \nabla\Psi_p(X, Y) \times \nabla\zeta + H^\zeta(X, Y)\nabla X \times \nabla Y. \tag{5}$$

The procedure to construct such coordinates is discussed in Appendix A. Note that there are many variants of systems corresponding to the choice of Φ_1, although the averaged equilibrium equations have the same form (but different metrics) for all these variants of the coordinate system.

We shall introduce the vectors

$$\bar{\boldsymbol{r}}(X, Y, \zeta) \equiv X\boldsymbol{e}_R(\zeta) + Y\boldsymbol{e}_Z, \tag{6}$$

$$\bar{\boldsymbol{B}} = \frac{1}{\sqrt{g_0}} H^\alpha \bar{\boldsymbol{r}}_\alpha, \tag{7}$$

$$\underline{\boldsymbol{B}} = \langle B_\alpha \rangle_\zeta \underline{\nabla} x^\alpha, \tag{8}$$

$$\bar{\boldsymbol{J}} = \frac{1}{\sqrt{g_0}} \langle \sqrt{g} J^\alpha \rangle_\zeta \bar{\boldsymbol{r}}_\alpha, \tag{9}$$

$$\underline{\nabla} = \underline{\nabla} x^\alpha \frac{\partial}{\partial x^\alpha} \equiv \boldsymbol{e}_r \frac{\partial}{\partial X} + \boldsymbol{e}_z \frac{\partial}{\partial Y} + \frac{1}{X}\boldsymbol{e}_\phi \frac{\partial}{\partial \zeta}, \tag{10}$$

and $\mathcal{J}_* \equiv \langle \sqrt{g} \rangle_\zeta / \sqrt{g_0}$. Here, $\langle \cdots \rangle_\zeta$ means the average with respect to ζ, and $\sqrt{g_0} \equiv X$ is the Jacobian in the cylindrical coordinates. The summation convention is used. Also we introduce the tensor

$$\underline{\underline{\mathsf{G}}} \equiv \sqrt{g_0} \left\langle \frac{g_{\alpha\beta}}{\sqrt{g}} \right\rangle_\zeta \underline{\nabla} x^\alpha \underline{\nabla} x^\beta, \tag{11}$$

so that $\underline{\boldsymbol{B}} = \underline{\underline{\mathbf{G}}} \cdot \overline{\boldsymbol{B}}$. In these notations, the equilibrium equations averaged with respect to the toroidal angle ζ can be written in the form[1)]

$$\overline{\boldsymbol{J}} \times \overline{\boldsymbol{B}} = \mathcal{J}_* \underline{\nabla} P, \qquad \overline{\boldsymbol{J}} = \underline{\nabla} \times \underline{\boldsymbol{B}}, \qquad \underline{\nabla} \cdot \overline{\boldsymbol{B}} = 0. \tag{12}$$

If we define

$$\overline{B}^2 \equiv \frac{\overline{\boldsymbol{B}} \cdot \underline{\boldsymbol{B}}}{\mathcal{J}_*} \equiv \frac{1}{\langle \sqrt{g} \rangle_\zeta} \langle \sqrt{g} B^2 \rangle_\zeta, \tag{13}$$

we can obtain from eqs.(12) the relation for the averaged current vector

$$\overline{\boldsymbol{J}} = P' \frac{\underline{\boldsymbol{B}} \times \underline{\nabla} \psi}{\overline{B}^2} + \sigma \overline{\boldsymbol{B}}, \tag{14}$$

with

$$\sigma \equiv \frac{\overline{\boldsymbol{J}} \cdot \underline{\boldsymbol{B}}}{\mathcal{J}_* \overline{B}^2} = \frac{\mathrm{d}F}{\mathrm{d}\Psi_\mathrm{p}} + \frac{F}{\overline{B}^2} \frac{\mathrm{d}P}{\mathrm{d}\Psi_\mathrm{p}} = \frac{\mathrm{d}F}{\mathrm{d}\Psi_\mathrm{p}} + \sigma_1 P', \tag{15}$$

where $\underline{B}_\zeta = F(\psi)$.

§3. Linearized Equations.

If the time dependence of the form $\exp(qt)$ is assumed for the perturbation, the Lagrangian for the linearized resistive MHD equation can be written in the form[3)]

$$\mathcal{L}_\mathrm{R} = \int \{ \mathcal{L}_0[\boldsymbol{\xi}_+, \boldsymbol{a}_+] - \mathcal{L}_0[\boldsymbol{\xi}_-, \boldsymbol{a}_-] + \mathcal{M}[\boldsymbol{\xi}_-, \boldsymbol{a}_+] - \mathcal{M}[\boldsymbol{\xi}_+, \boldsymbol{a}_-] \} \mathrm{d}\tau, \tag{16}$$

with

$$\begin{aligned} \mathcal{L}_0[\boldsymbol{\xi}, \boldsymbol{a}] = {} & q^2 \rho \boldsymbol{\xi}^2 + [\boldsymbol{Q} - \nabla \times (\eta \boldsymbol{a})] \cdot [\boldsymbol{Q} + \boldsymbol{J} \times \boldsymbol{\xi} - \nabla \times (\eta \boldsymbol{a})] \\ & + (\boldsymbol{\xi} \cdot \nabla P) \nabla \cdot \boldsymbol{\xi} + \gamma_s P (\nabla \cdot \boldsymbol{\xi})^2 + q \eta \boldsymbol{a}^2, \end{aligned} \tag{17}$$

$$\mathcal{M}[\boldsymbol{\xi}, \boldsymbol{a}] = \boldsymbol{J} \times \boldsymbol{\xi} \cdot \nabla \times (\eta \boldsymbol{a}). \tag{18}$$

where $\boldsymbol{\xi} \equiv \boldsymbol{\xi}_+ + \boldsymbol{\xi}_-$ stands for the plasma displacement, $\boldsymbol{a} \equiv \boldsymbol{a}_+ + \boldsymbol{a}_-$ is the electric displacement which is related to the perturbed magnetic field $\boldsymbol{b}$ by the relation $\boldsymbol{a} = \nabla \times \boldsymbol{b}/q$, and $\boldsymbol{Q} \equiv \nabla \times (\boldsymbol{\xi} \times \boldsymbol{B})$. The equations of motion for the physical variables $\boldsymbol{\xi}$ and $\boldsymbol{a}$ can be written in the form

$$\frac{\partial \mathcal{L}_0[\boldsymbol{\xi}, \boldsymbol{a}]}{\partial \boldsymbol{\xi}} - \frac{\partial \mathcal{M}[\boldsymbol{\xi}, \boldsymbol{a}]}{\partial \boldsymbol{\xi}} = 0, \quad \frac{\partial \mathcal{L}_0[\boldsymbol{\xi}, \boldsymbol{a}]}{\partial \boldsymbol{a}} + \frac{\partial \mathcal{M}[\boldsymbol{\xi}, \boldsymbol{a}]}{\partial \boldsymbol{a}} = 0. \tag{19}$$

Since the eigenvalue q is, in general, a complex number, the vectors $\boldsymbol{\xi}$ and $\boldsymbol{a}$ are also complex. For the ideal case, we can choose $\boldsymbol{\xi}_+$ as real, and $\boldsymbol{\xi}_-$ for the as pure imaginary. Then we can recover the Lagrangian for the ideal MHD equations

$$\mathcal{L}_\mathrm{I}[\boldsymbol{\xi}] = \int \{ q^2 \rho |\boldsymbol{\xi}|^2 + |\boldsymbol{Q}|^2 + \boldsymbol{J} \times \boldsymbol{\xi}^* \cdot \boldsymbol{Q} + (\boldsymbol{\xi} \cdot \nabla P)(\nabla \cdot \boldsymbol{\xi}^*) + \gamma_s P |\nabla \cdot \boldsymbol{\xi}|^2 \} \mathrm{d}\tau. \tag{20}$$

First, we shall consider eq.(20) for the ideal MHD. We consider the perturbation with single toroidal harmonics n; the contravariant component of the plasma displacement ξ^α is assumed to have the ζ-dependence of the form

$$\boldsymbol{\xi} = \bar{\xi}^\alpha \boldsymbol{r}_\alpha, \qquad \bar{\xi}^\alpha = \bar{\bar{\xi}}^{\alpha,c}(X,Y)\cos(n\zeta) + \bar{\bar{\xi}}^{\alpha,s}(X,Y)\sin(n\zeta). \tag{21}$$

The toroidal mode number n is assumed to satisfy the relations $2n \neq mN$, for not so large integer m, N being the toroidal period number in the equilibrium. In other word, we assume that the integral $\oint A\exp(\pm 2in\zeta)\mathrm{d}\zeta$ can be ignored for any equilibrium quantity A. We shall introduce the vectors $\bar{\boldsymbol{\xi}} \equiv \bar{\xi}^\alpha \bar{\boldsymbol{r}}_\alpha$, and $\bar{\boldsymbol{Q}} = \underline{\nabla} \times (\bar{\boldsymbol{\xi}} \times \bar{\boldsymbol{B}})$.

Substituting expressions for $\boldsymbol{\xi}$ into eq.(20), and carrying out the integration with respect to the toroidal angle ζ under the assumption for n stated above, we obtain the Lagrangian in the form

$$\mathcal{L}_1[\bar{\boldsymbol{\xi}}] = \int \Big\{ q^2 \rho \bar{\boldsymbol{\xi}}^* \cdot \underline{\underline{\mathbf{M}}} \cdot \bar{\boldsymbol{\xi}} + \bar{\boldsymbol{Q}}^* \cdot \underline{\underline{\mathbf{G}}} \cdot \bar{\boldsymbol{Q}} + \bar{\boldsymbol{J}} \times \bar{\boldsymbol{\xi}}^* \cdot \bar{\boldsymbol{Q}} + (\bar{\boldsymbol{\xi}} \cdot \underline{\nabla} P) \underline{\nabla} \cdot (\mathcal{J}_* \bar{\boldsymbol{\xi}}^*)$$
$$+ \frac{\gamma_s P}{\mathcal{J}_*} \left| \underline{\nabla} \cdot (\mathcal{J}_* \bar{\boldsymbol{\xi}}) \right|^2 + \gamma_s P \bar{\boldsymbol{\xi}}^* \cdot \underline{\underline{\mathbf{Y}}} \cdot \bar{\boldsymbol{\xi}} \Big\} \mathrm{d}\tau_0, \tag{22}$$

where $\mathrm{d}\tau_0 \equiv \sqrt{g_0}\,\mathrm{d}X\mathrm{d}Y\mathrm{d}\zeta$, and

$$\underline{\underline{\mathbf{M}}} \equiv \frac{\left\langle \sqrt{g} g_{\alpha\beta} \right\rangle_\zeta}{\sqrt{g_0}} \underline{\nabla} x^\alpha \underline{\nabla} x^\beta, \tag{23}$$

$$\underline{\underline{\mathbf{Y}}} = \frac{1}{\sqrt{g_0}} \left\{ \left\langle \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}}{\partial x^\alpha} \frac{\partial \sqrt{g}}{\partial x^\beta} \right\rangle_\zeta - \frac{1}{\left\langle \sqrt{g} \right\rangle_\zeta} \frac{\partial \left\langle \sqrt{g} \right\rangle_\zeta}{\partial x^\alpha} \frac{\partial \left\langle \sqrt{g} \right\rangle_\zeta}{\partial x^\beta} \right\} \underline{\nabla} x^\alpha \underline{\nabla} x^\beta, \tag{24}$$

In the case of η=0, the potential energy in the Lagrangian becomes to the form given in Ref.1, provided the term containing **Y** is ignored. Hence, the energy principle given in Ref.1 is based on two assumptions: *(1)* the perturbation with single toroidal Fourier mode number, and *(2)* ignorance of the term **Y**. In Ref.1, where $\Phi_1=0$ is assumed, the ignorance of **Y** is an approximation. Since the ignored term gives positive contribution, the result gives always a slightly pessimistic. However, if the function Φ_1 is chosen so that the Jacobian is independent of ζ, $\sqrt{g} = \sqrt{g_0}\mathcal{J}_*$,

$$\underline{\underline{\mathbf{M}}} \equiv \mathcal{J}_*^2 \underline{\underline{\mathbf{G}}}, \qquad \underline{\underline{\mathbf{Y}}} \equiv 0. \tag{25}$$

In the following, we shall assume that this condition is satisfied. Thus, we can write the Lagrangian in the form

$$\mathcal{L}_0[\bar{\boldsymbol{\xi}}] = \int \Big\{ q^2 \rho \mathcal{J}_*^2 \bar{\boldsymbol{\xi}}^* \cdot \underline{\underline{\mathbf{G}}} \cdot \bar{\boldsymbol{\xi}} + \bar{\boldsymbol{Q}}^* \cdot \underline{\underline{\mathbf{G}}} \cdot \bar{\boldsymbol{Q}} + \bar{\boldsymbol{J}} \times \bar{\boldsymbol{\xi}}^* \cdot \bar{\boldsymbol{Q}}$$

$$+(\overline{\xi}\cdot\underline{\nabla}P)\underline{\nabla}\cdot(\mathcal{J}_*\overline{\xi}^*)+\frac{\gamma_s P}{\mathcal{J}_*}\left|\underline{\nabla}\cdot(\mathcal{J}_*\overline{\xi})\right|^2\}\mathrm{d}\tau_0. \tag{26}$$

Now we shall consider the resistive case. In addition to the contravariant component of the plasma displacement ξ^α, the covariant component of the electric displacement a_α are assumed to have the ζ-dependence of the form

$$\boldsymbol{a}=\overline{a}_\alpha\nabla x^\alpha, \qquad \overline{a}_\alpha=\overline{\overline{a}}{}_\alpha^{c}(X,Y)\cos(n\zeta)+\overline{\overline{a}}{}_\alpha^{s}(X,Y)\sin(n\zeta), \tag{27}$$

and introduce the vectors $\underline{\boldsymbol{a}}=\overline{a}_\alpha\underline{\nabla}x^\alpha$.

By using the expressions for $\boldsymbol{\xi}$ and $\boldsymbol{a}$, we can carry out the same procedure as the ideal case to obtain the averaged Lagrangian. However, the result of such a procedure is not so convenient, because there appears the metric tensor which is not directly related to the quantities appearing in the equilibrium equations. Therefore, we shall use the slightly different approach. We shall introduce the vectors $\boldsymbol{c}\equiv\boldsymbol{a}$ and the Lagrange multiplier $\boldsymbol{\lambda}$, and write the Lagrangian in the form

$$\mathcal{L}_0[\boldsymbol{\xi},\boldsymbol{a},\boldsymbol{c},\boldsymbol{\lambda}]=q^2\rho\boldsymbol{\xi}^2+[\boldsymbol{Q}-\nabla\times(\eta\boldsymbol{a})]\cdot[\boldsymbol{Q}+\boldsymbol{J}\times\boldsymbol{\xi}-\nabla\times(\eta\boldsymbol{a})]$$
$$+(\boldsymbol{\xi}\cdot\nabla P)\nabla\cdot\boldsymbol{\xi}+\gamma_s P(\nabla\cdot\boldsymbol{\xi})^2+q\eta\boldsymbol{c}\cdot\boldsymbol{a}+\boldsymbol{\lambda}\cdot(\boldsymbol{c}-\boldsymbol{a}). \tag{28}$$

For $\boldsymbol{c}$ and $\boldsymbol{\lambda}$, too, we assume the ζ-dependence of the form

$$\sqrt{g}\boldsymbol{c}=\sqrt{g_0}\overline{c}^\alpha\boldsymbol{r}_\alpha, \qquad \overline{c}^\alpha=\overline{\overline{c}}{}^{\alpha,c}(X,Y)\cos(n\zeta)+\overline{\overline{c}}{}^{\alpha,s}(X,Y)\sin(n\zeta), \tag{29}$$

$$\sqrt{g}\boldsymbol{\lambda}=\sqrt{g_0}\overline{\lambda}^\alpha\boldsymbol{r}_\alpha, \qquad \overline{\lambda}^\alpha=\overline{\overline{\lambda}}{}^{\alpha,c}(X,Y)\cos(n\zeta)+\overline{\overline{\lambda}}{}^{\alpha,s}(X,Y)\sin(n\zeta), \tag{30}$$

and define the vectors $\overline{\boldsymbol{c}}=\overline{c}^\alpha\overline{\boldsymbol{r}}_\alpha$, and $\overline{\boldsymbol{\lambda}}=\overline{\lambda}^\alpha\overline{\boldsymbol{r}}_\alpha$. Substituting expressions for $\boldsymbol{\xi}$, $\boldsymbol{a}$, $\boldsymbol{c}$, and $\boldsymbol{\lambda}$ into the Lagrangian, and carrying out the integration with respect to the toroidal angle ζ, we obtain the Lagrangian in terms of $\overline{\boldsymbol{\xi}},\underline{\boldsymbol{a}},\overline{\boldsymbol{c}}$, and $\overline{\boldsymbol{\lambda}}$. Then eliminating $\overline{\boldsymbol{c}}$ and $\overline{\boldsymbol{\lambda}}$ from the Lagrangian by the use of the relations $\overline{\boldsymbol{c}}=\underline{\underline{\mathbf{G}}}^{-1}\cdot\underline{\boldsymbol{a}}$, and $\overline{\boldsymbol{\lambda}}=-q\eta\overline{\boldsymbol{c}}$, we finally obtain

$$\mathcal{L}_\mathrm{R}=\int\left\{\overline{\mathcal{L}}_0\left[\overline{\boldsymbol{\xi}}_+,\underline{\boldsymbol{a}}_+\right]-\overline{\mathcal{L}}_0\left[\overline{\boldsymbol{\xi}}_-,\underline{\boldsymbol{a}}_-\right]+\overline{\mathcal{M}}\left[\overline{\boldsymbol{\xi}}_-,\underline{\boldsymbol{a}}_+\right]-\overline{\mathcal{M}}\left[\overline{\boldsymbol{\xi}}_+,\underline{\boldsymbol{a}}_-\right]\right\}\mathrm{d}\tau_0, \tag{31}$$

with

$$\overline{\mathcal{L}}_0[\overline{\boldsymbol{\xi}},\underline{\boldsymbol{a}}]=q^2\rho\mathcal{J}_*^2\overline{\boldsymbol{\xi}}\cdot\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{\xi}}+[\overline{\boldsymbol{Q}}-\eta\underline{\nabla}\times\underline{\boldsymbol{a}}]\cdot\underline{\underline{\mathbf{G}}}\cdot[\overline{\boldsymbol{Q}}-\eta\underline{\nabla}\times\underline{\boldsymbol{a}}]$$
$$+\overline{\boldsymbol{J}}\times\overline{\boldsymbol{\xi}}\cdot[\overline{\boldsymbol{Q}}-\eta\underline{\nabla}\times\underline{\boldsymbol{a}}]+(\overline{\boldsymbol{\xi}}\cdot\underline{\nabla}P)\underline{\nabla}\cdot(\mathcal{J}_*\overline{\boldsymbol{\xi}})$$
$$+\frac{\gamma_s P}{\mathcal{J}_*}\left[\underline{\nabla}\cdot(\mathcal{J}_*\overline{\boldsymbol{\xi}})\right]^2+q\eta\underline{\boldsymbol{a}}\cdot\underline{\underline{\mathbf{G}}}^{-1}\cdot\underline{\boldsymbol{a}}. \tag{32}$$

and

$$\overline{\mathcal{M}}\left[\overline{\boldsymbol{\xi}},\underline{\boldsymbol{a}}\right]=\eta\overline{\boldsymbol{J}}\times\overline{\boldsymbol{\xi}}\cdot\underline{\nabla}\times\underline{\boldsymbol{a}}. \tag{33}$$

From this Lagrangian we obtain the resistive MHD equations

$$q^2\rho \mathcal{J}_*^2 \underline{\underline{\mathbf{G}}}\cdot\overline{\xi}+\overline{\boldsymbol{Q}}\times\overline{\boldsymbol{J}}+\overline{\boldsymbol{B}}\times\underline{\nabla}\times(\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{Q}})+\mathcal{J}_*\underline{\nabla}\left[(\overline{\xi}\cdot\underline{\nabla}P)+\frac{\gamma_s P}{\mathcal{J}_*}\underline{\nabla}\cdot(\mathcal{J}_*\overline{\xi})\right]$$
$$+\eta\overline{\boldsymbol{J}}\times\underline{\nabla}\times\underline{\boldsymbol{a}}-\eta\overline{\boldsymbol{B}}\times\underline{\nabla}\times\left[\underline{\underline{\mathbf{G}}}\cdot\underline{\nabla}\times\underline{\boldsymbol{a}}\right]=0, \tag{34}$$

$$q\underline{\underline{\mathbf{G}}}^{-1}\cdot\underline{\boldsymbol{a}}-\underline{\nabla}\times(\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{Q}})+\eta\underline{\nabla}\times\left[\underline{\underline{\mathbf{G}}}\cdot\underline{\nabla}\times\underline{\boldsymbol{a}}\right]=0. \tag{35}$$

We shall write equations in terms of the perturbed magnetic field $\overline{\boldsymbol{b}}$ which is related to $\underline{\boldsymbol{a}}$ by the relation

$$q\underline{\boldsymbol{a}}\equiv\underline{\underline{\mathbf{G}}}\cdot\underline{\nabla}\times(\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{b}}). \tag{36}$$

Introducing the perturbed pressure p_1, we can write eqs.(34) and (35) in the form

$$q^2\rho \mathcal{J}_*^2 \underline{\underline{\mathbf{G}}}\cdot\overline{\xi}+\overline{\boldsymbol{b}}\times\overline{\boldsymbol{J}}+\overline{\boldsymbol{B}}\times\underline{\nabla}\times(\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{b}})+\mathcal{J}_*\underline{\nabla}p_1=0, \tag{37}$$

$$p_1+(\overline{\xi}\cdot\underline{\nabla}P)+\frac{\gamma_s P}{\mathcal{J}_*}\underline{\nabla}\cdot(\mathcal{J}_*\overline{\xi})=0, \tag{38}$$

$$\overline{\boldsymbol{b}}-\underline{\nabla}\times(\overline{\xi}\times\overline{\boldsymbol{B}})+\frac{\eta}{q}\underline{\nabla}\times\left[\underline{\underline{\mathbf{G}}}\cdot\underline{\nabla}\times(\underline{\underline{\mathbf{G}}}\cdot\overline{\boldsymbol{b}})\right]=0. \tag{39}$$

The metrics $\mathcal{J}_*$, and $\underline{\underline{\mathbf{G}}}$ represent the effects of the helical perturbation in equilibrium. For axisymmetric tokamak, these matrices are unity, and the equations are correct without approximation. Hence, eqs.(37)-(39) compose the basic set of the linearized resistive equations consistent to the averaging approximation. Only the metrics appearing in the averaged equilibrium equations are contained in these equations. This is because we choose the special coordinate system on which the Jacobian is independent of ζ.

§4. Resistive Ballooning Equations.

As an application of the resistive equations, we shall derive the averaged version of the Mercier criterion for the ideal interchange mode[4] and the stability condition for the resistive interchange mode.[5]

For this purpose, assuming the ordering

$$q^2\sim\frac{\eta}{q}\sim\frac{1}{n^2}\ll 1, \tag{40}$$

we apply the ballooning approximation[6]

$$\overline{\xi}=\exp(inS)\left\{\overline{\xi}^{(0)}+\frac{1}{n}\overline{\xi}^{(1)}+\cdots\right\}, \tag{41}$$

etc. with $\overline{\boldsymbol{B}}\cdot\underline{\nabla}S=0$, to eqs.(37)-(39). In the lowest order, we obtain $\overline{\xi}^{(0)}\cdot\underline{\nabla}S=0$, and $\overline{\boldsymbol{b}}^{(0)}\cdot\underline{\nabla}S=0$, and $\underline{\boldsymbol{B}}\cdot\overline{\boldsymbol{b}}^{(0)}+\mathcal{J}_*p_1^{(0)}=0$. Therefore, if we introduce the quantities

$$\bar{e} = \frac{\underline{B}\times\underline{\nabla}S}{\mathcal{J}_*\overline{B}^2}, \qquad \underline{e} \equiv \underline{\underline{\mathbf{G}}}\cdot\bar{e}, \qquad \bar{e}^2 \equiv \mathcal{J}_*\bar{e}\cdot\underline{e}, \tag{42}$$

we can put

$$\bar{\boldsymbol{\xi}}^{(0)} = \Phi\bar{e} + \frac{\mu}{\mathcal{J}_*}\overline{\boldsymbol{B}}, \qquad \bar{\boldsymbol{b}}^{(0)} = \frac{\mathcal{J}_*}{\bar{e}^2}\Theta\bar{e} - \frac{1}{\overline{B}^2}\Psi\overline{\boldsymbol{B}}, \tag{43}$$

and $p_1^{(0)} = \Psi$.

From the next order equations, we obtain the following set of first order differential equations

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Phi = \mathcal{J}_*\left\{\frac{1}{\bar{e}^2} + \frac{\eta}{q}\overline{B}^2\right\}\Theta, \tag{44}$$

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Psi = -\mathcal{J}_*\frac{(\bar{e}\cdot\underline{\nabla}P)}{\bar{e}^2}\Theta - q^2\rho\,\mathcal{J}_*\overline{B}^2\mu, \tag{45}$$

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla}\mu = \mathcal{J}_*\left\{(\bar{e}\cdot\underline{\boldsymbol{K}}) - (\bar{e}\cdot\underline{\nabla}P)\left[\frac{1}{\gamma_s P} + \frac{1}{\overline{B}^2}\right]\right\}\Phi - \mathcal{J}_*\left\{\frac{1}{\gamma_s P} + \frac{1}{\overline{B}^2} + \frac{\eta}{q}\bar{e}^2\right\}\Psi, \tag{46}$$

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Theta = \mathcal{J}_*(\bar{e}\cdot\underline{\boldsymbol{K}})\Psi + q^2\rho\,\mathcal{J}_*\bar{e}^2\Phi, \tag{47}$$

where

$$\bar{e}\cdot\underline{\boldsymbol{K}} \equiv -\frac{1}{\mathcal{J}_*}\underline{\nabla}\cdot(\mathcal{J}_*\bar{e}) + \frac{\bar{e}\cdot\underline{\nabla}P}{\overline{B}^2}. \tag{48}$$

If we put

$$\bar{e} = \frac{\underline{\nabla}\psi}{|\underline{\nabla}\psi|^2} + \Lambda\bar{s}, \qquad \bar{s} = \frac{\underline{B}\times\underline{\nabla}\psi}{\mathcal{J}_*\overline{B}^2}, \tag{49}$$

Λ satisfies the equation

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Lambda = \iota\underline{\nabla}\cdot\left(\iota\frac{\overline{B}^\zeta\underline{\nabla}\psi}{|\underline{\nabla}\psi|^2}\right). \tag{50}$$

In order to obtain the stability criterion, we consider the asymptotic behavior of the solution of eqs.(44)-(47) at the large value of Λ. Assuming the ordering

$$q^2 \sim \frac{\eta}{q} \sim \frac{1}{\Lambda^2} \sim \varepsilon^2 \ll 1, \tag{51}$$

we expand,

$$\Phi = \Phi_0(u) + \varepsilon\Phi^{(1)}(u,\theta) + \cdots, \tag{52}$$

$$\Psi = \Psi_0(u) + \varepsilon\Psi^{(1)}(u,\theta) + \cdots, \tag{53}$$

$$\mu = \varepsilon^{-1}\{\mu^{(0)}(u,\theta) + \varepsilon\mu^{(1)}(u,\theta) + \cdots\}, \tag{54}$$

$$\Theta = \varepsilon^{-1}\{\Theta^{(0)}(u,\theta) + \varepsilon\Theta^{(1)}(u,\theta) + \cdots\}, \tag{55}$$

and

$$\overline{\boldsymbol{B}}\cdot\underline{\nabla} = \overline{\boldsymbol{B}}\cdot\underline{\nabla}\theta\frac{\partial}{\partial\theta} + \varepsilon\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Lambda\frac{\partial}{\partial u}, \tag{56}$$

where $\Lambda = u/\varepsilon$.

From the lowest order equation, using the relation

$$\overline{\boldsymbol{e}}\cdot\underline{\boldsymbol{K}} = \Lambda\underline{K}_s + \underline{K}_\psi = -\frac{\Lambda}{\mathcal{J}_*}\overline{\boldsymbol{B}}\cdot\underline{\nabla}\sigma_1 + \underline{K}_\psi, \tag{57}$$

we obtain

$$\mu^{(0)} = \mu_0(u) - u\hat{\sigma}_1\Phi_0(u), \qquad \Theta^{(0)} = \Theta_0(u) - u\hat{\sigma}_1\Psi_0(u), \tag{58}$$

where

$$\hat{\sigma}_1 = \sigma_1 - \left\langle\frac{\sigma_1}{G_{ss}}\right\rangle \bigg/ \left\langle\frac{1}{G_{ss}}\right\rangle, \tag{59}$$

$$G_{ss} = \mathcal{J}_*\overline{s}\cdot\underline{\underline{\mathbf{G}}}\cdot\overline{s}, \tag{60}$$

and

$$\langle A\rangle = \oint\frac{A\mathcal{J}_*\mathrm{d}\theta}{\overline{B}^\theta}. \tag{61}$$

From the solvability conditions of the next order equations we obtain the following set of first order differential equations

$$S\frac{\mathrm{d}\Phi_0}{\mathrm{d}u} = g_0\frac{\Theta_0}{u^2} + \frac{\eta}{q}h_0\Theta_0 - \frac{\eta}{q}h_1u\Psi_0, \tag{62}$$

$$S\frac{\mathrm{d}\Psi_0}{\mathrm{d}u} = -P'g_0\frac{\Theta_0}{u^2} + q^2p\,h_1u\Phi_0 - q^2p\,h_0\mu_0, \tag{63}$$

$$S\frac{\mathrm{d}\mu_0}{\mathrm{d}u} = (f_1 - P'f_0)\Phi_0 + \frac{\eta}{q}h_1u\Theta_0 - \left\{f_0 + \frac{\eta}{q}u^2h_2\right\}\Psi_0, \tag{64}$$

$$S\frac{\mathrm{d}\Theta_0}{\mathrm{d}u} = f_1\Psi_0 - q^2p\,h_1u\mu_0 + q^2p\,h_2u^2\Phi_0, \tag{65}$$

where

$$h_0 = \langle\overline{B}^2\rangle, \qquad h_1 = \langle\hat{\sigma}_1\overline{B}^2\rangle, \qquad h_2 = \langle\hat{\sigma}_1^2\overline{B}^2 + G_{ss}\rangle, \tag{66}$$

$$g_0 = \left\langle\frac{1}{G_{ss}}\right\rangle, \qquad f_0 = \frac{\langle 1\rangle}{\gamma_s P} + \left\langle\frac{1}{\overline{B}^2}\right\rangle + \left\langle\frac{\hat{\sigma}_1^{\,2}}{G_{ss}}\right\rangle, \tag{67}$$

$$f_1 = \langle\underline{K}_\psi + \frac{1}{\mathcal{J}_*}\hat{\sigma}_1\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Lambda\rangle + P'\left\langle\frac{\hat{\sigma}_1^{\,2}}{G_{ss}}\right\rangle$$

$$= -\frac{V''}{\Psi_{\mathrm{p}}'} - S\left\langle\frac{\sigma_1}{G_{ss}}\right\rangle \bigg/ \left\langle\frac{1}{G_{ss}}\right\rangle + P'\left\langle\frac{1}{\overline{B}^2} + \frac{\hat{\sigma}_1^{\,2}}{G_{ss}}\right\rangle, \tag{68}$$

and $S = \langle\overline{\boldsymbol{B}}\cdot\underline{\nabla}\Lambda\rangle = 2\pi\iota'$.

§5. Local Stability Criterions.

In the ideal case (η=0), eqs.(62)-(65) can be reduced to the single differential equation

$$\frac{\mathrm{d}}{\mathrm{d}u}u^2\frac{\mathrm{d}\Phi^{(0)}}{\mathrm{d}u}+\frac{P'}{S^2}g_0f_1\Phi^{(0)}=0, \tag{69}$$

in the limit of $q\to 0$. Therefore, we can obtain the stability condition for the ideal mode

$$\frac{S^2}{4}-P'g_0f_1\geq 0, \tag{70}$$

or, by substitution of eqs.(67)-(68),

$$\frac{S^2}{4}-P'\left\langle\frac{1}{G_{ss}}\right\rangle\left\{-\frac{V''}{\Psi_{\mathrm{p}}'}+P'\left\langle\frac{1}{\overline{B}^2}\right\rangle\right\}+P'S\left\langle\frac{\sigma_1}{G_{ss}}\right\rangle-\left\langle\frac{1}{G_{ss}}\right\rangle\left\langle\frac{\sigma^2}{G_{ss}}\right\rangle+\left\langle\frac{\sigma}{G_{ss}}\right\rangle^2\geq 0. \tag{71}$$

This is an extension of condition given in Ref.1. Comparing inequality (71) with that in Ref.5, the quantity $|\nabla\psi|^2/B^2$ in Ref.4 is replaced by G_{ss}.

As is shown in Appendix B, when Fourier transformed, the above set of equations are equivalent to those discussed in Ref.5. Hence, the stability condition for the resistive interchange mode can be written as

$$D_R\equiv\frac{P'}{S^2}g_0f_1-\frac{P'g_0h_1}{Sh_0}+\left(\frac{P'g_0h_1}{Sh_0}\right)^2\leq 0. \tag{72}$$

Substituting eqs.(66)-(68) into eq.(72) we obtain

$$\begin{aligned}D_R\equiv&\frac{P'}{S^2}\left\langle\frac{1}{G_{ss}}\right\rangle\left\{-\frac{V''}{\Psi_{\mathrm{p}}'}+P'\left\langle\frac{1}{\overline{B}^2}\right\rangle\right\}-\frac{P'}{S}\left\langle\frac{1}{G_{ss}}\right\rangle\frac{\langle\sigma_1\overline{B}^2\rangle}{\langle\overline{B}^2\rangle}\\&+\frac{P'^2}{S^2}\left\langle\frac{1}{G_{ss}}\right\rangle\left\{\left\langle\frac{\sigma_1^2}{G_{ss}}\right\rangle-2\left\langle\frac{\sigma_1}{G_{ss}}\right\rangle\frac{\langle\sigma_1\overline{B}^2\rangle}{\langle\overline{B}^2\rangle}+\left\langle\frac{1}{G_{ss}}\right\rangle\frac{\langle\sigma_1\overline{B}^2\rangle^2}{\langle\overline{B}^2\rangle^2}\right\}\leq 0.\end{aligned} \tag{73}$$

§6. Conclusion.

The averaged equations of motion are derived with the assumption of the single toroidal harmonics, and without other ordering. The single toroidal harmonics means that the toroidal mode number n does not resonate with the toroidal period N.

The averaged energy principle given in Ref.1 is correct if we assume the coordinates such that the Jacobian is independent of the toroidal angle. In the coordinates with the coordinates ϕ_1=0, it implies the assumption that

Y=0.

By using the Lagrangian formulation, the linearized resistive MHD equations consistent to the 3D equilibrium are derived. The stability criteria for the localized mode are also derived. These criteria are simple, because they are expressed only by the averaged quantities appearing in the averaged equilibrium equations.

The equations can be applied for the equilibrium obtained by the three dimensional equilibrium codes as well as the averaged equilibrium codes. The set of the averaged equations may be useful in practice, since the full 3D calculation is still very expensive in the present computer capabilities.

Appendix A.

We shall consider the construction of the coordinates which are used in this paper, when the some kind of magnetic coordinates are known.

We suppose that the magnetic coordinate system (ψ,θ,ζ) is given, the coordinate ψ being the surface function (say, the toroidal flux), and the magnetic field being straight on these coordinates; the equation of the field line is expressed as

$$\frac{d\theta}{d\zeta}=\iota(\psi). \tag{A.1}$$

The cylindrical coordinates (r,z,ϕ) are expanded into Fourier series

$$r=R(\psi,\theta,\zeta)=\sum_{m,n}R_{m,n}(\psi)\cos(m\theta-n\zeta),$$
$$z=Z(\psi,\theta,\zeta)=\sum_{m,n}Z_{m,n}(\psi)\sin(m\theta-n\zeta),$$
$$\phi=\Phi(\psi,\theta,\zeta)=\zeta+\sum_{m,n}\Phi_{m,n}(\psi)\sin(m\theta-n\zeta). \tag{A.2}$$

Here, we have assumed a symmetry for simplicity, so that the only one trigonometric function appears in each series. If we put

$$X=X(\psi,\theta)\equiv\sum_{m}R_{m,0}(\psi)\cos(m\theta),$$
$$Y=Y(\psi,\theta)\equiv\sum_{m}Z_{m,0}(\psi)\sin(m\theta), \tag{A.3}$$

then, we have

$$H^X=-\iota(\psi)\frac{\partial\psi}{\partial Y},\quad H^Y=\iota(\psi)\frac{\partial\psi}{\partial X},\quad H^\zeta=\frac{\partial(\psi,\theta)}{\partial(X,Y)}. \tag{A.4}$$

The Jacobian of the coordinates (X,Y,ζ)

$$\sqrt{g} = \mathcal{J}\frac{\partial(\psi,\theta)}{\partial(X,Y)}, \tag{A.5}$$

may depend on ζ, because the Jacobian of the (ψ,θ,ζ) coordinates $\mathcal{J}$ may depend on ζ. In order to construct the coordinates on which the Jacobian is a function of X and Y only, the coordinates (ψ,θ,ζ) should be transformed beforehand to those such as $\mathcal{J}=\mathcal{J}(\psi,\theta)$. Such a transformation is given by the following two steps. At the first step, the coordinates are transformed to the Hamada coordinates[7] $(\psi,\theta^{\mathrm{H}},\zeta^{\mathrm{H}})$ on which the Jacobian $\mathcal{J}^{\mathrm{H}}$ is a function of ψ only. Since the ζ=constant plane of the Hamada coordinates is greatly slanted,[8] in order to satisfy the last condition in eqs.(3), the following transformation is carried out as the second step,

$$\zeta = \zeta^{\mathrm{H}} + \sum_m \Phi^{\mathrm{H}}_{m,0}(\psi)\sin(m\theta^{\mathrm{H}}), \qquad \theta = \theta^{\mathrm{H}} + \iota(\psi)\sum_m \Phi^{\mathrm{H}}_{m,0}(\psi)\sin(m\theta^{\mathrm{H}}), \tag{A.6}$$

where $\Phi^{\mathrm{H}}_{m,n}(\psi)$ is the Fourier components of ϕ in the Hamada coordinates.

Appendix B.

In Ref.5 the equations are written in terms of normalized variables

$$\frac{\mathrm{d}^2\Xi}{\mathrm{d}X^2} - H\frac{\mathrm{d}\Upsilon}{\mathrm{d}X} = Q(\Psi - X\Xi), \tag{B.1}$$

$$Q^2\frac{\mathrm{d}^2\Xi}{\mathrm{d}X^2} - QX^2\Xi + E\Upsilon + QX\Psi + \Gamma = 0, \tag{B.2}$$

$$Q\frac{\mathrm{d}^2\Upsilon}{\mathrm{d}X^2} - X^2\Upsilon - GQ^2\Upsilon + Q^2(G-KE)\Xi + X\Psi - Q^2K\Gamma = 0, \tag{B.3}$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}X} = H\frac{\mathrm{d}^2\Psi}{\mathrm{d}X^2} + F\frac{\mathrm{d}\Upsilon}{\mathrm{d}X}, \tag{B.4}$$

Fourier transforming these equations, and eliminating Γ, we obtain

$$Q\frac{\mathrm{d}\Xi}{\mathrm{d}X} + \mathrm{i}(Q+X^2)\Psi - HX\Upsilon = 0, \tag{B.5}$$

$$Q\frac{\mathrm{d}^2\Xi}{\mathrm{d}X^2} - Q^2X^2\Xi + (E+F)\Upsilon + \mathrm{i}Q\frac{\mathrm{d}\Psi}{\mathrm{d}X} + \mathrm{i}HX\Psi = 0, \tag{B.6}$$

$$\frac{\mathrm{d}^2\Upsilon}{\mathrm{d}X^2} - \left\{QX^2 + Q^2(G+KF)\right\}\Upsilon + Q^2(G-KE)\Xi + \mathrm{i}\frac{\mathrm{d}\Psi}{\mathrm{d}X} - \mathrm{i}Q^2KHX\Psi = 0. \tag{B.7}$$

Here, we have used the same symbols to its Fourier transform. From eq.(63) we eliminate μ_0, and differentiate eq.(65) with respect to u. Then we obtain the following equations.

$$\frac{\mathrm{d}^2\Phi_0}{\mathrm{d}u^2}-\frac{q\rho\eta}{S^2}(h_0h_2-h_1^{\,2})u^2\Phi_0-\frac{\eta h_0}{qS^2}(f_1-S\frac{h_1}{h_0})\Psi_0$$

$$-\frac{g_0}{S}\frac{\mathrm{d}}{\mathrm{d}u}\frac{\Theta_0}{u^2}-\frac{\eta P'g_0}{qS^2}h_1\frac{\Theta_0}{u}=0, \tag{B.8}$$

$$\frac{\mathrm{d}^2\Psi_0}{\mathrm{d}u^2}-\frac{q^2p}{S^2}\left\{h_0f_0+\frac{\eta}{q}u^2(h_0h_2-h_1^{\,2})\right\}\Psi_0+\frac{q^2p\,h_0}{S^2}(f_1-P'f_0-\frac{Sh_1}{h_0})\Phi_0$$

$$+\frac{P'g_0}{S}\frac{\mathrm{d}}{\mathrm{d}u}\left\{\frac{\Theta_0}{u^2}\right\}-\frac{q^2p\,g_0}{S^2}h_1\frac{\Theta_0}{u}=0. \tag{B.9}$$

The variables Φ_0, Ψ_0, Θ_0 , u, q in eqs.(61), (B.8), and (B.9) correspond to Ξ, Y, i$X^2\Psi$, X, Q in eqs.(B.5)-(B.7), respectively, except for some normalization factors. Comparison of two set of equations yields

$$E+F+H=\frac{P'}{S^2}g_0f_1, \qquad H=-\frac{P'g_0}{Sh_0}h_1. \tag{B.10}$$

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