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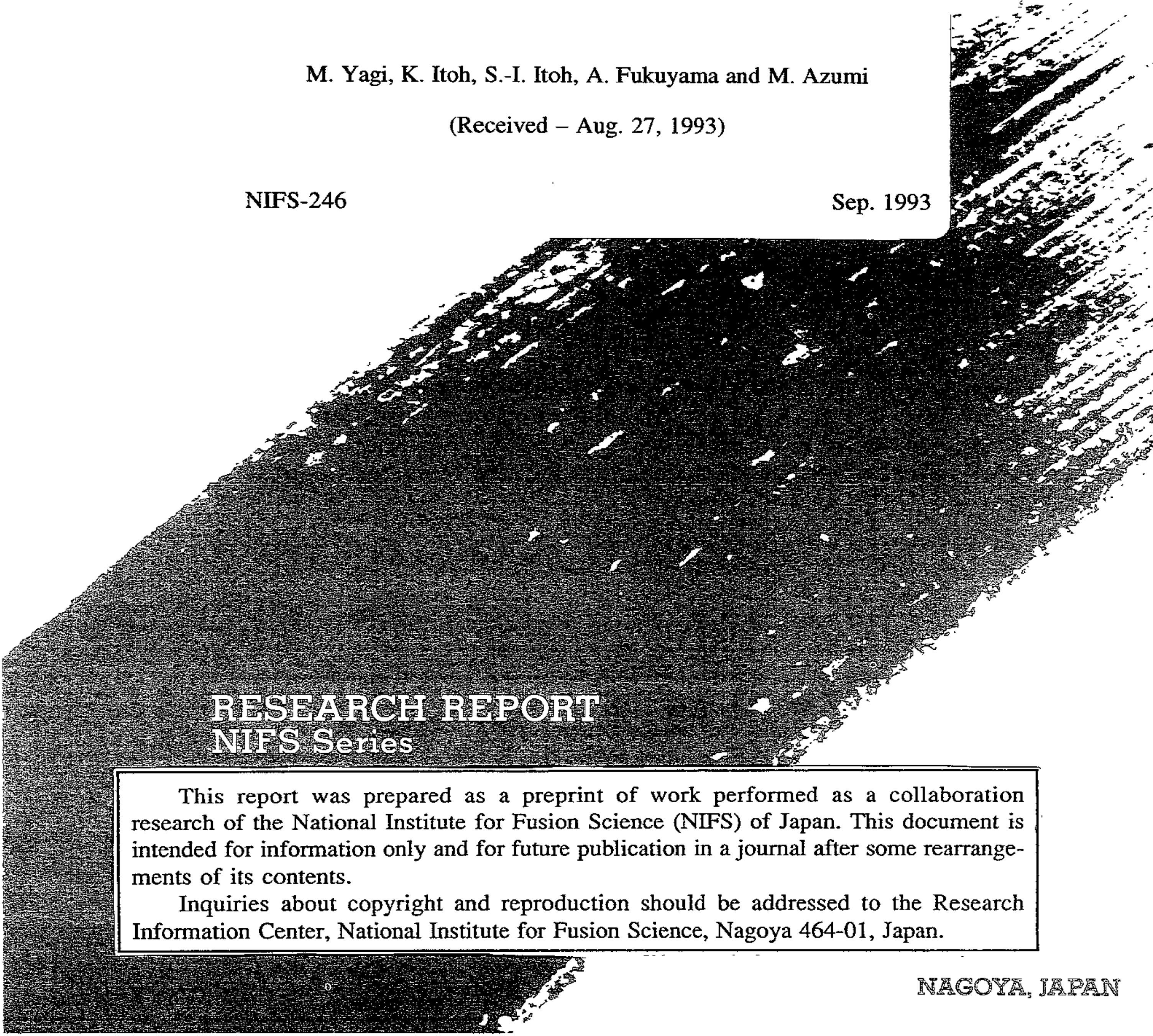

RESEARCH REPORT
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Current Diffusive Ballooning Mode in Second Stability Region of Tokamaks

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Abstract

Stability of the current diffusive ballooning mode in tokamaks with high toroidal mode number is analyzed in the region of second-stability against the ideal magnetohydrodynamic mode. It is found that the growth rate of the current diffusive ballooning mode is decreased by the reduction of the geodesic curvature driving force. The reduction of thermal conductivity in the limit of very weak shear or negative shear in comparision with standard shear is also shown.

Keywords: Ballooning instability, Thermal conductivity, Current diffusivity, Ion viscosity, Anomalous transport, Magnetic shear, Magnetic well Collisionless skin depth

Recently a new theoretical approach has been proposed to understand the anomalous transport phenomena in tokamaks. In this new approach, effects of the anomalous transport process on the mode stability itself are self-consistently treated taking into account of nonlinear interaction[1-5]. The destabilization of plasma due to the fluctuation-driven dissipation can overcome the stabilization effect of magnetic shear[1-8]. It is found that, below the beta-limit of the ideal magnetohydrodynamic (MHD) mode, the microscopic ballooning mode can be destabilized by the transport coefficient such as the current-diffusivity λ, and stabilized by other transport coefficients, such as the thermal diffusivity χ, and the ion viscosity, μ[2-5]. Renormalizing the $\mathbf{E} \times \mathbf{B}$ nonlinearity in a form of transport coefficient, the anomalous transport coefficient and characteristics of fluctuations (level, typical correlation length and time) are determined simultaneously by the marginal stability condition for the least stable mode. This result on the self-consistent treatment of the anomalous transport was confirmed by the scale invariance method [9]. The theoretical results was compared with experimental data[4,10], and a good agreement was seen for the L-mode plasma[11]. Extension of the theory to the case of the H-mode plasma has been performed[12].

Since the origin of the fluctuations comes from the plasma pressure coupled with the bad curvature, which is the cause of MHD ballooning modes, the improved

stability and transport is expected in the second stability region against ideal MHD ballooning mode. The transport coefficient in this region was partly discussed in previous articles[4,5], and seems to explain [10] the improved confinement phenomena in high-β_p plasma[13], PEP H-mode [14] and so on. In this note, we present the result of the stability analysis on the current diffusive ballooning mode in the region of very weak or negative shear region.

We consider large aspect ratio tokamaks with circular cross section in toroidal coordinates (r, θ, ζ). Starting from the reduced set of equations [15], the eigenmode equation is derived in the ballooning space where the valiable $\phi(r, \theta)$ is transformed to $d(\eta)$ (ϕ is the static potential). Details of the derivation is explained in Ref. [4,5]. By this procedure, the eigenmode equation is reduced to the ordinary differential equation as

$$\frac{d}{d\eta}\frac{F}{\hat{\gamma}+\Xi F+\Lambda F^2}\frac{d\phi}{d\eta}+\frac{\alpha}{\hat{\gamma}+XF}[\kappa+cos\eta+(s\eta-\alpha sin\eta)sin\eta]\phi$$
$$-(\hat{\gamma}+MF)F\phi=0 \qquad (1)$$

In Eq.(1), we use the normalization: $r/a \longrightarrow \hat{r}$, $t/\tau_{Ap} \longrightarrow \hat{t}$, $\chi\tau_{Ap}/a^2 \longrightarrow \hat{\chi}$,

$\mu\tau_{Ap}/a^2 \rightarrow \hat{\mu}$, $\tau_{Ap}/\mu_0\sigma a^2 \rightarrow 1/\hat{\sigma}$, $\lambda\tau_{Ap}/\mu_0 a^4 \rightarrow \hat{\lambda}$, $\gamma\tau_{Ap} \rightarrow \hat{\gamma}$. Here the notation is: $\tau_{Ap} \equiv a\sqrt{\mu}_0 m_i n_i/B_p$, $\Xi = n^2q^2/\hat{\sigma}$, $\Lambda = \hat{\lambda}n^4q^4$, $X = \hat{\chi}n^2q^2$, $M = \hat{\mu}n^2q^2$. Other notations are standard: m_i is the ion mass, n_i, the ion density, B, the main magnetic field, $B_p = Br/qR$, the poloidal magnetic field, $\epsilon = r/R$, inverse aspect ratio, a, the minor radius, R, the major radius, n, the toroidal mode number, β, the plasma pressure divided by the magnetic pressure ($\beta = \mu_0 n(T_e + T_i)/B^2$), γ, the growth rate, s, the shear parameter defined by $s = r(dq/dr)/q$, q, the safety factor, $F = 1 + (s\eta - \alpha sin\eta)^2$, the normalized perpendicular wave number, $\kappa \equiv -(r/R)(1 - 1/q^2)$, the average well, $\alpha = -q^2\beta'/\epsilon$, the normalized pressure gradient, and $\beta' \equiv d\beta/d\hat{r}$. The electric conductivity σ is given by the classical theory. Transport coefficients, i.e., the current-diffusivity λ, the thermal diffusivity χ and the ion viscosity μ, are given by renormalizing turbulence [2].

The equation (1) is the generalization of the previous ballooning equations [16]. If we neglect $\hat{\lambda}, \hat{\chi}, \hat{\mu}$, and $1/\hat{\sigma}$, the ideal MHD mode equation is recovered as

$$\frac{d}{d\eta}F\frac{d\phi}{d\eta} + \alpha[\kappa + cos\eta + (s\eta - \alpha sin\eta)sin\eta]\phi - \hat{\gamma}^2F\phi = 0 \tag{2}$$

From Eq.(1), it is seen that the mode becomes more stable when the shear parameter becomes very small or negative. The potential $[\kappa + cos\eta + (s\eta - \alpha sin\eta)sin\eta]$ is the driving source of the high-n ballooning mode with dissipation and ideal MHD ballooning mode. For analytic insight, we expand this potential in the small η region for the case of weak shear, $1/2 + \alpha > s$. It is approximated as $\{1 + \kappa - (1/2 + \alpha - s)\eta^2\}$. When the geodesic curvature is small or negative, the coefficient $(1/2 + \alpha - s)$ becomes larger, so that the eigenmode is more strongly localized near the origin, $\eta = 0$.

This lead to the better stability.

Equation (1) is solved by shooting method. Figure 1 illustrates the eigenfunction for the marginal stability condition ($\gamma = 0$ in the case of $s = 0.5$ (a) and $s = -0.5$ (b). It is seen that the mode is more strongly localized in the case of negative shear.

Figure 2 shows the stability boundary on the $s - \alpha$ diagram for various values of the toroidal mode number, n, for the fixed value of the transport coefficient. For a fixed value of n, the mode is more easily stabilized in the strong shear limit and the weak shear limit. The envelope of the stability boundary is also drawn by the thick solid line in Fig.2. Left side of this envelope line is stable to all mode. The region of stability is found to expand to higher pressure region as s becomes very small or negative.

Based on the stability analysis, we derive the formula for the anomalous transport coefficient. The anomalous transport coefficient was given [2-4] in the dimensionless form as

$$\hat{\chi} = \frac{1}{f(s,\alpha)}\alpha^{3/2}\left(\frac{\delta}{a}\right)^2, \tag{3}$$

or in an explicit form of

$$\chi = \frac{q^2}{f(s,\alpha)}\left(\frac{R}{r}\frac{d\beta}{d\hat{r}}\right)^{3/2}\delta^2\frac{v_A}{R}. \tag{4}$$

Here we investigate the effect of geometrical factor, $f(s,\alpha)$, on the transport in negative shear region. The geometrical factor denotes the effect of the magnetic shear. It is not possible to express the factor f by using a single power of s. Figure 3 illustrates the contour of the anomalous transport coefficient in the $s-\alpha$ diagram. The unstable region for ideal MHD ballooning mode is also shown by the dashed line. It is demonstrated that the transport coefficient is reduced in accordance with the appearance of the second stability region.

In this note, we report the stability analysis on the current-diffusive ballooning mode in tokamaks, putting the emphasis on the negative shear region.

It is shown that the change of the sing in the geodesic curvature is effective in improving the stability. By this reduction in the driving source of the ballooning mode turbulence, the anomalous transport coefficient is also reduced in the region of negative shear.

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Figure Captions

Fig.1 Eigenmode structure at the marginal stability condition for the positive shear case (a) and negative shear case (b). $s = 0.5$, $\alpha = 0.145$, $n = 29$ in (a) and $s = -0.5$, $\alpha = 0.638$, $n = 45$ in (b). Other parameters are : $\hat{\chi} = \hat{\mu} = 10^{-5}$, $\hat{\lambda} = 10^{-9}$, $q = 3$ and $\epsilon = 1/8$.

Fig.2 Stability boundary on the $s - \alpha$ diagram for various values of toroidal node numbers. Transport coefficients are fixed. Bold line shows the envelope of the stability boundaries of various modes. Parameters are $\hat{\chi} = \hat{\mu} = 10^{-7}$, $\hat{\lambda}/\hat{\chi} = 10^{-4}$, $q = 3$ and $\epsilon = 1/8$.

Fig.3 Contour of the transport coefficient on the $s - \alpha$ diagram. Parameters are $\hat{\lambda}/\hat{\chi} = 10^{-4}$, $q = 3$ and $\epsilon = 1/8$.

Fig.1

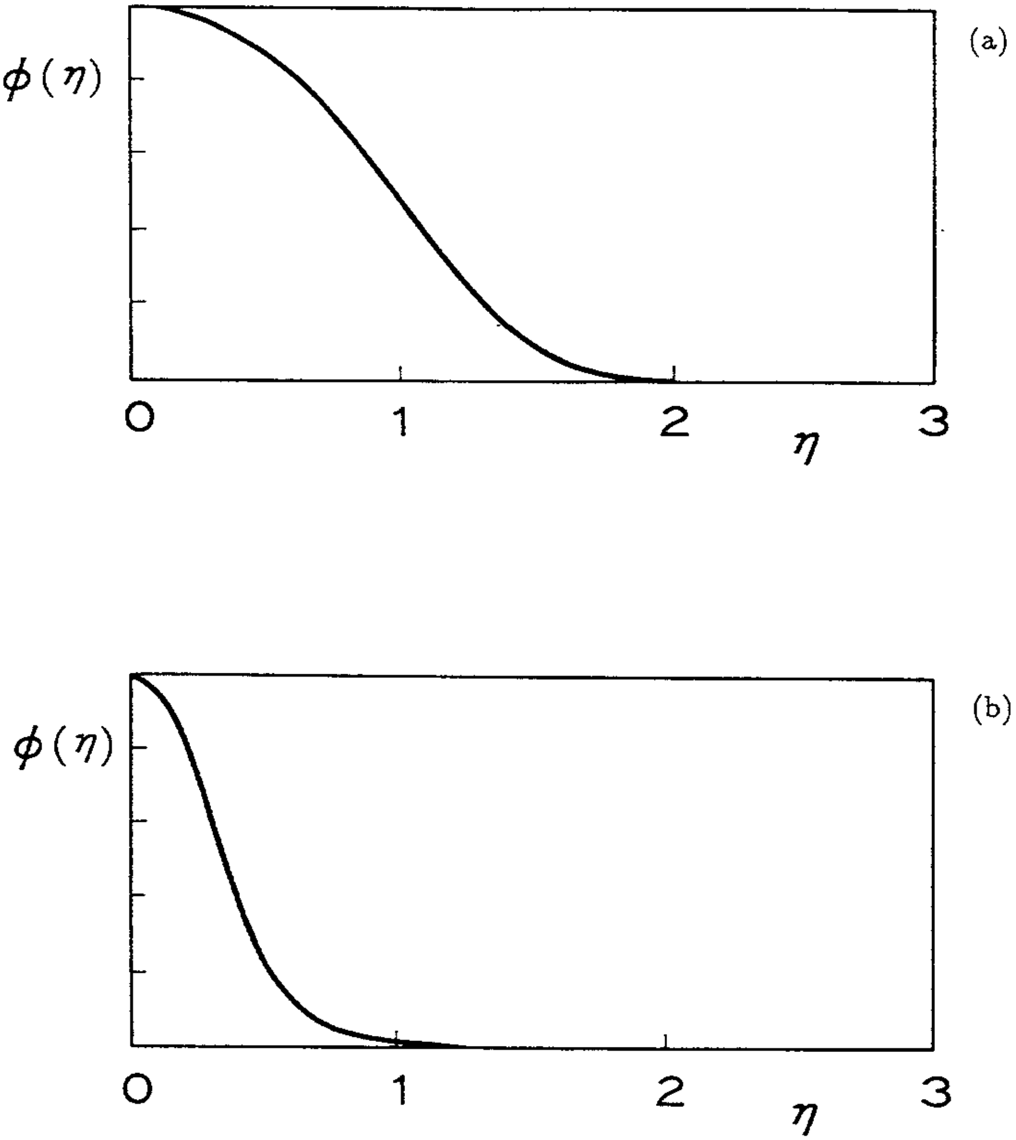


Fig.2

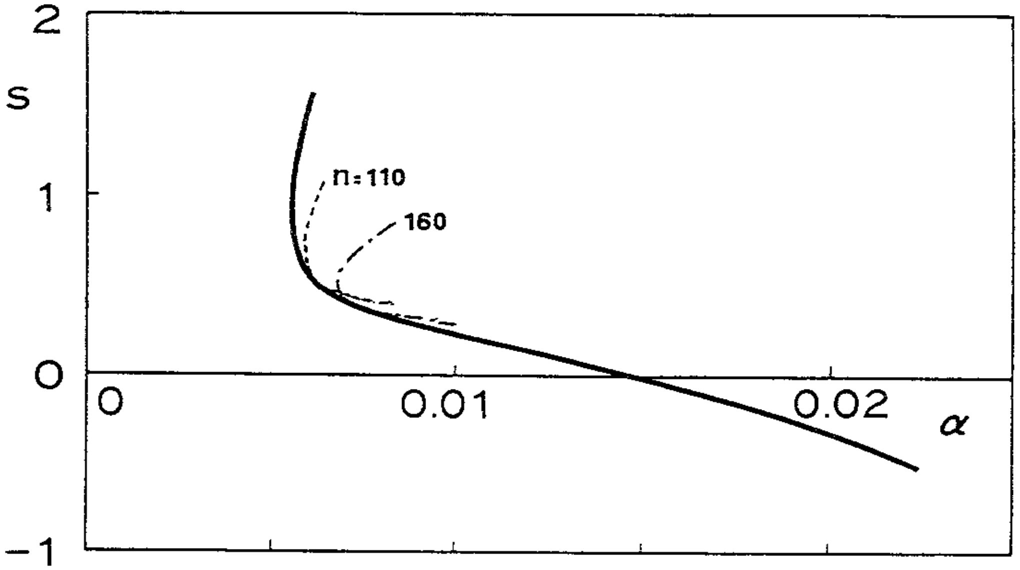
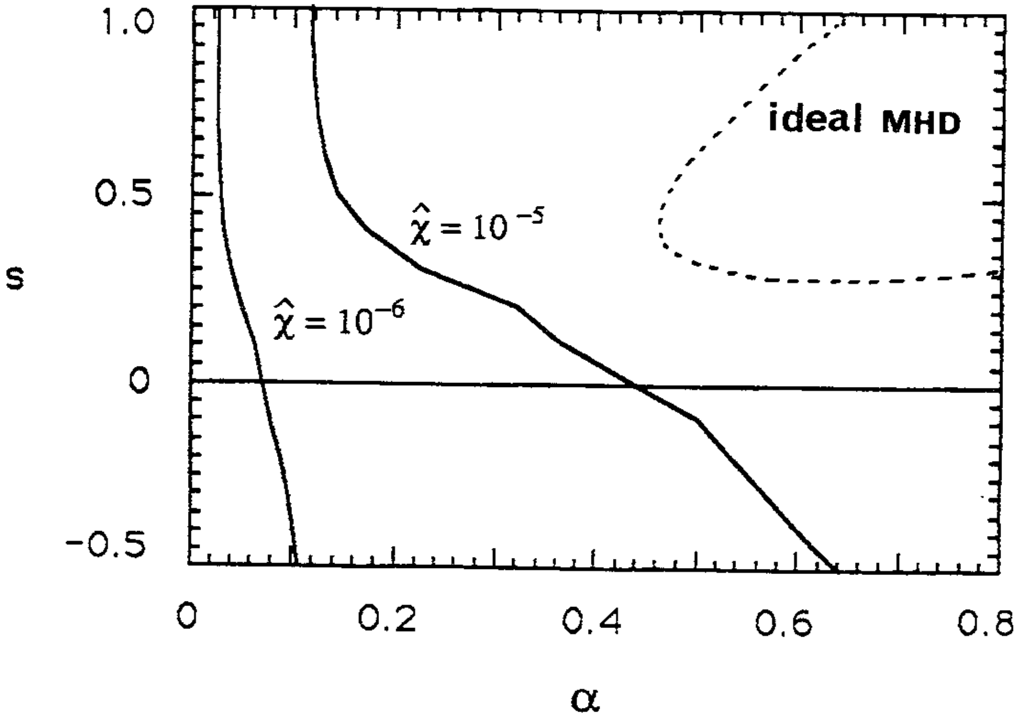


Fig.3



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