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# Trapped Electron Instabilities due to Electron Temperature Gradient and Anomalous Transport

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#### Abstract

The electrostatic trapped electron instabilities have been investigated by numerically solving the local single energy integral form dispersion relation. With the electron temperature gradient, the electron collision effect destabilizes the dissipative trapped electron mode (DTEM). For collisionless electrons, the energy dependent curvature drift effect strongly destabilizes the collisionless trapped electron mode. The growth rate is, however, reduced by a small collision effect. The anomaluous transport due to these trapped electron modes is discussed.

**Keywords:** Electrostatic local dispersion relation for trapped electrons, single energy integral form, numerical calculation, dissipative trapped electron mode, curvature drift, electron temperature gradient, collisionless trapped electron mode, anomalous electron transport.

#### §1. Introduction

Trapped particle instabilities have been studied by many authors in connection with the anomalous plasma transport  $^{1)-6}$ . It is well known that the electron collision effect excites so called dissipative trapped electron mode (DTEM). In the simple model of the constant collision frequency, the collision stabilizes by the collision damping effect. The DTEM is destabilized, combined with the electron temperature gradient, when the energy dependence of the collision frequency is taken into account. This situation is similar to the  $\eta_1$ -mode which is excited by taking into account the energy dependence of the curvature drift frequency  $\omega_D^{7}$ 

In addition to the collision effect, if we take into account the energy dependent electron curvature drift effect, the trapped electron mode may be more destabilized. Due to this energy dependence, the analytical solution to the dispersion relation is at best limited to perticular asymptotic cases. For arbitrary parameter ranges, the dispersion relation may only be solved by numerical calculations.

The purpose of this report is to examine the effects of electron temperature gradient energy dependent electron collision, and curvature drift frequency for trapped electron modes by numerically solving the electrostatic local dispersion relation, and compare with usual analytical results.

We assume a simple model for ion dynamics neglecting the effects of trapped ions, finite Larmor radius, curvature drift and transit frequency. By solving the complex integral equation by a comformal mapping method, the growth rate  $\gamma$  is found to be much larger than expected. Due to the combined effects of the electron temperature gradient and curvature drift,  $\gamma$  can even be larger than the electron diamagnetic drift frequency  $\omega^*_{\mathbf{e}}$ .

In actual situations, the growth rate may be smaller by various stabilizing effects such as the finite ion Larmor radius effect, transit particle effect and finite  $\beta$ -effect. Although our model may

be too simple to be realistic, we can see, at least, what is the most important source for the trapped electron instabilities.

### §2. Electrostatic Dispersion Relation

We start with the gyrokinetic solution for the perturbed ion distribution

$$\widetilde{f}_{\underline{i}} = -\frac{e\widetilde{\varphi}}{T_{\underline{i}}} \left\{ 1 - \frac{\overline{\omega} - \omega * \underline{\pi}_{\underline{i}}}{\overline{\omega} - \omega_{D,i}}, J_0(\alpha) \right\} F_{\underline{M}\underline{i}}$$
(1)

The perturbed distribution for trapped eletrons is given by

$$\tilde{\mathbf{f}}_{e} = -\frac{e\tilde{\varphi}}{T_{e}} \left\{ 1 - \frac{\bar{\omega} - \omega_{\star Te}}{\bar{\omega} - \omega_{De} + i v_{eff}} \right\} \mathbf{F}_{Me} \tag{2}$$

where all notations are standard  $^{7)}:\omega_{D}=\omega_{D}(v_{\!\!\!L}^{2}/2+v_{\!\!\!H}^{2})$ ,  $\omega_{D}=2\epsilon_{n}\omega^*/\tau$ ,  $\tau=T_{e}/T_{i}$ ,  $\epsilon_{n}=L_{n}/L_{B}$ ,  $1/L_{n}=-d\ln n/d\tau$ ,  $1/L_{B}=-d\ln B/d\tau$   $\omega^*_{T}=\omega^*_{e}(1+\eta(v_{\!\!L}^{2}+v_{\!\!H}^{2}-3/2))/\tau$ ,  $\eta=d\ln T/d\ln n$ ,  $\omega^*_{e}=k_{y}cT_{e}/eBL_{n}$ ,  $F_{M}=(\pi v_{th}^{2})^{-3/2}\exp(-E/T)$ , and  $v_{eff}=v_{e}/\epsilon$  with  $\epsilon=r/R$  for tokamak, and  $\epsilon=\epsilon_{h}$  (helical inhomogeniety factor) for the helically symmetric system. We neglect the passing electron distribution, because it is of the order  $\omega_*/\omega_t$  which must be much smaller than that of trapped electrons, where  $\omega_t$  is the electron transit frequency  $\omega_t=k_Hv_{\!\!H}$ .

If we neglect the curvature drift frequency and also finite Larmor radius effect for ion in eq.(1), the perturbed ion density is approximated by $^{8}$ :

$$\widetilde{n}_{\underline{i}} = \frac{\omega_{\star e}}{\bar{\omega}} = \frac{e\widetilde{\varphi}}{T_{e}} N \tag{3}$$

where  $\bar{\omega}=\omega+\omega_{\rm E}$  with  $\omega_{\rm E}=v_{\rm E}k_{\theta}$ ,  $v_{\rm E}=cE_{\rm F}/B$  is the poloidal rotation velocity due to the radial electric field  $E_{\rm F}$ . Integrating eq.(2) over the velocity, we have the pertubed electron density

$$\widetilde{n}_{e} = \frac{e\widetilde{\phi}}{T_{e}} N \left( 1 - \int_{T} \frac{\widetilde{\omega} - \omega_{\star Te}}{\widetilde{\omega} - \omega_{De} + i v_{eff}} F_{Me} d^{3}v \right)$$
(4)

From the quasi-neutrality condition, we have the local electrostatic dispersion relation:

$$D_{\text{es}} = 1 - \frac{\omega_{\star}_{\text{e}}}{\bar{\omega}} - \int_{\mathbb{T}} \frac{\bar{\omega} - \omega_{\star}_{\text{Te}}}{\bar{\omega} - \omega_{\text{De}} + i\nu_{\text{eff}}} F_{\text{Me}} d^3 v = 0$$
 (5)

The velocty integral in eq.(5) can be rewritten in the form of double integral:

$$D_{es} = 1 - \frac{1}{\omega} - \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} dv_{\perp} v_{\perp} e^{-v_{\perp}^{2}} \int_{v_{\parallel} \le \sqrt{\epsilon v_{\perp}}} \frac{\left(\omega - 1 - \eta_{e} \left(v_{\perp}^{2} + v_{\parallel}^{2} - \frac{3}{2}\right)\right) e^{-v_{\parallel}^{2}}}{\omega - 2\epsilon_{n} \left(v_{\perp}^{2} / 2 + v_{\parallel}^{2}\right) + i v_{effo} \left(v_{\perp}^{2} + v_{\parallel}^{2}\right)^{-3/2}}$$
(6)

Equation (6) may only be solved by numerical calcultions. The double integration is, however, time consuming particulary near the resonance condition. We approximate 7) the curvature drift frequency by  $\omega_D=2\varepsilon_n\,(v_1^2/2+v_{ii}^2)\stackrel{\sim}{=}2\varepsilon_n\,(v_1^2+v_{ii}^2)=2\varepsilon_n E$ . Equation (6) in this case can be rewritten in the form of single integral with respect to the normalized energy E:

$$D_{\text{es}} = 1 - \frac{1}{\omega} - \frac{2}{\sqrt{\pi}} \sqrt{\varepsilon_{\text{T}}} \int_{0}^{\infty} dE \sqrt{E} e^{-E} \frac{\omega - 1 - \eta_{\text{e}} \left(E - \frac{3}{2}\right)}{\omega - 2\varepsilon_{\text{n}} E + i \nu_{\text{effo}} E^{-3/2}} = 0$$
 (7)

For the sake of simplicity, we express the normalized frequency  $\bar{\omega}/\omega_{\pm e}$  as  $\omega$ . To our trapped electron mode, the electron temperature gradient is essential. The importance of  $\eta_e$  can be seen by setting  $\eta_e=0$  in eq.(6) or (7). In this case, these equations yield an exact solution  $\omega=1$  and anothor damping branch, i.e., without electron temperature gradient all branches are stabilized.

We will solve eq.(7) numerically by making use of a conformal mapping method, i.e., by mapping certain orthognal curves in the complex  $\omega$ -plane onto the complex Des-plane and seek the point in the complex  $\omega$ -plane until Des tends to the origin in the complex Des-plane.

### §3. Dissipative Trapped Electron Mode

First we consider the trapped electron instability induced by the electron collision effect. If we assume the curvature drift and collision frequencies,  $\omega_{\rm D}$  and  $v_{\rm eff}$ , are respectively constants. In this case, eq.(7) can be written by

$$1 - \frac{1}{\omega} - \frac{\sqrt{\varepsilon_{\rm T}}}{\omega - 2\varepsilon_{\rm n} + i\nu_{\rm eff}} \left\{ \left( \omega - 1 + \frac{3}{2} \eta_{\rm e} \right) \right\} = 0$$
 (8)

where the moment integral I; has been defined by

$$I_{j} = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} dx \sqrt{x} e^{-x} x^{j}$$
 (9)

Since  $I_0=1$  and  $I_1=3/2$ ,  $\eta_e$  term is exactly cancelled out and eq.(8) yields the solution

$$\omega = 1 \quad \text{or } \omega = \frac{2\varepsilon_{\text{n}} - i v_{\text{eff}}}{1 - \sqrt{\varepsilon_{\text{T}}}}$$
 (10)

Both eigenvalues indicate the stable branches. This means that as long as  $\omega_{\rm D}$  and  $\nu_{\rm eff}$  are constants, we have no instabilty.

If we introduce the energy dependence of these frequencies, the trapped particle instabilities are excited. Let us solve eq.(7) in the collisional regime:  $v_{\rm eff} >> \omega_{\star}$ . Expanding the integrand in eq.(7) in powers of  $\omega/v_{\rm eff}$ , we have

$$D_{\text{es}} = 1 - \frac{1}{\omega} + 2i \frac{\sqrt{\epsilon_{\text{T}}}}{v_{\text{effo}}} \left[ 2(\omega - 1) - 3\eta_{\text{e}} + \frac{i}{v_{\text{effo}}} \left\{ (\omega - 1 + \frac{3}{2}\eta_{\text{e}}) (\omega \xi - 2\epsilon_{\text{n}}\xi) - \eta_{\text{e}} (\omega \xi - 2\epsilon_{\text{n}}\xi) \right\} \right] = 0 \text{ (11)}$$

To the first order of  $\omega/v_{\mbox{eff}}$ , the solution to eq.(11) is obtained by

$$\omega = 1 + i \sqrt{\epsilon_{\rm T}} \frac{6\eta_{\rm e}}{\sqrt{\pi \nu_{\rm effo}}}$$
 (12)

As compared to the usual growth rate  $v=\epsilon^{3/2}\omega_{\star}^2\eta_{\rm e}/v_{\rm e}^{1/2})$  for the DTEM, the growth rate given by eq.(12) is larger by a numerical factor  $6/\pi^{1/2}$ . The effect of curvature drift frequency  $\omega_{\rm D}$  is the second order to  $\omega/v_{\rm eff}$ , and may be negiligible in the collisional

regime. In the limit,  $v_e \rightarrow \infty$ , the eigenvalue  $\omega$  tends to the stable point 1. In the opposite collisionless limit,  $v_e \rightarrow 0$ , the eigenvalue  $\omega$  also tends to the same stable point 1, which can be seen from eq.(10).

For arbitrary collision frequencies, eq.(7) is numerically solved by the conformal mapping method. Variation of the discrete eigenvalue for the case of  $\omega_D$ =0 is presented in Fig.1 for various values of  $v_e$  and  $\eta_e$ . Since the eigenvalue  $\omega$  tends to the same limit 1 for  $v_e \rightarrow 0$  and  $v_e \rightarrow \infty$ , the eigenvalue trajectory for each  $\eta_e$  forms a closed contour. As  $\eta_e$  is reduced, the contour shrinks and finally tends to a stable point  $\omega$ =1, i.e., the electron temperature gradient is essential to the DTEM as mentioned in the above.

The normalized frequency and growth rate are also plotted versus the normalized collision frequency  $v_{\rm e}$  for different values of  $\eta_{\rm e}$  in Fig.2. The asymptotic solution (12) presented by the broken curve is also compared in Fig.2. As seen in Figs. 1 and 2, the growth rate  $\gamma$  sharply increases in the weakly collisional regime,  $v_{\rm e}<1$ . As  $v_{\rm e}$  increases, however,  $\gamma$  suffers collision damping and decreases. When the curvature drift effect is taken into account, the trapped electron mode is strongly destabilized as seen by the case of  $\varepsilon_{\rm n}=0.1$  in Fig.2. This branch seems to be strongly destabilized particularly in the collisionless limit.

### §4. Collisionless Trapped Electron Mode

We now consider the effect of curvature drift on the trapped electron mode in the collisionless limit. In this case, eq.(7) can be rewritten in the form

$$D_{es} = 1 - \frac{1}{\omega} - \sqrt{\varepsilon_{T}} \frac{\eta_{e}}{2\varepsilon_{n}} - 2\sqrt{\frac{\varepsilon_{T}}{\pi}} \left\{ 1 - \frac{3}{2}\eta_{e} - (1 - \frac{\eta_{e}}{2\varepsilon_{n}})\omega \right\}_{0}^{\infty} \sqrt{\frac{Ee^{-E}dE}{2\varepsilon_{n}E - \omega}}$$
(13)

If we change the variable by  $E=x^2$ , the integral in eq.(13) can be expressed in terms of the usual plasma dispersion function Z:

$$\frac{1}{\sqrt{\pi}} \int_{0}^{\infty} \frac{\sqrt{Ee^{-E}dE}}{2\varepsilon_{n}E^{-\omega}} = \frac{1}{2\varepsilon_{n}} \left\{ 1 + \sqrt{\frac{\omega}{2\varepsilon_{n}}} Z \left( \sqrt{\frac{\omega}{2\varepsilon_{n}}} \right) \right\}$$
(14)

Introducing eq.(14) into eq.(13), we have

$$D_{es} = \left(1 - \sqrt{\varepsilon_{T}} \frac{\eta_{e}}{2\varepsilon_{n}}\right) \omega - 1 - \frac{\sqrt{\varepsilon_{T}\omega}}{\varepsilon_{n}} \left\{1 - \frac{3}{2}\eta_{e} - \left(1 - \frac{\eta_{e}}{2\varepsilon_{n}}\right)\omega\right\} \left\{1 + \sqrt{\frac{\omega}{2\varepsilon_{n}}} Z \left(\sqrt{\frac{\omega}{2\varepsilon_{n}}}\right)\right\}$$
(15)

The complex function  $(\omega/\omega_D)^{1/2}$  has the branch points at  $\omega=0$  and at infinity. The function Z is discontinuous across the branch cut  $[0,\infty]$  on the positive real axis. This branch cut corresponds to the continuous eigenvalue of the original gyrokinetic equation in the collisionless case.

When  $\omega/\omega_D>>1$ , applying the asymptotic formula  $Z(\zeta)=(1+1/2\zeta^2+3/4\zeta^4)/\zeta$ , we find the eigenvalue  $\omega=1$  in the limit  $\omega_D\to 0$ . Since the quantity  $(\omega/\omega_D)^{1/2}$  is approximatly unity, and also the real and imaginary parts of  $\omega$  are nearly the same, Re $\omega$ \*Im $\omega$ , in general, we have no available asymptotic formula. Equation (15) is, therefore, not so useful to derive approximate analytical solution for the eigenvalue. We solve directly eq.(7) numerically by the conformal mapping method.

Variation of the discrete eigenvalue, the solution of eq.(7), is presented in Fig.3 for various values of  $\eta_e$  and  $\epsilon_n$ . As seen in Fig.3, the normalized growth rate  $\gamma$  increases as  $\eta_e$  increases. For each  $\eta_e$ , as  $\epsilon_n$  increases, the trajectory of the eigenvalue tends to the continuum, i.e., the positive real axis at which the mode becomes marginal. Figures 1 and 3 also indicate that the trapped electron modes are essentially the resonant mode at the electron diamagnetic drift frequency,  $\omega=\omega_{*e}$ .

Let us analytically evaluate the critical value of  $\eta_{\rm e}$  above which the trapped particle mode becomes unstable. At the marginal state, y=0, applying the formula:  $(x+i0)^{-1}=Px^{-1}-i\pi\delta(x)$  to the integrand of eq.(13), from the real and imaginary parts, we have two equations

$$\left(1 - \sqrt{\varepsilon_{\mathrm{T}}} \frac{\eta_{\mathrm{e}}}{2\varepsilon_{\mathrm{n}}}\right) \omega - 1 - \sqrt{\varepsilon_{\mathrm{T}}} \omega \left\{1 - \frac{3}{2} \eta_{\mathrm{e}} - \left(1 - \frac{\eta_{\mathrm{e}}}{2\varepsilon_{\mathrm{n}}}\right) \omega\right\} \frac{2}{\sqrt{\pi}} P \int_{0}^{\infty} \frac{\sqrt{\mathrm{E}\mathrm{e}^{-\mathrm{E}}} dE}{2\varepsilon_{\mathrm{n}} E - \omega} = 0 \tag{16}$$

$$\left\{ i - \frac{3}{2} \eta_{e} - \left( 1 - \frac{\eta_{e}}{2\varepsilon_{n}} \right) \omega \right\} \frac{1}{2\varepsilon_{n}} \sqrt{\frac{\omega}{2\varepsilon_{n}}} \exp \left( -\sqrt{\frac{\omega}{2\varepsilon_{n}}} \right) = 0$$
 (17)

From eq.(17), we have  $\omega=(1-3\eta_{e}/2)/(1-\eta_{e}/\omega_{D})$ . Introducing this relation into eq.(16), we have  $\omega=\omega_{D}/(\omega_{D}-\epsilon^{1/2}\eta_{e})$  and the relation between  $\eta_{e}$  and  $\epsilon_{n}$  in the form

$$\varepsilon_{\rm n} = \frac{1}{3} \left( 1 - \sqrt{\varepsilon_{\rm T}} + \frac{3}{2} \sqrt{\varepsilon_{\rm T}} \eta_{\rm e} \right) \tag{18}$$

In the derivation of eq.(18), we assumes no approximation, i.e., the critical condition (18) is exact. The boundary in the  $(\epsilon_n,\eta_e)$ -plane given by eq.(18) is shown in Fig.4. The evaluation of the critical value by numerical method has the difficulty due to the singularity. The numerical integration for the singular function must be very careful. The boundary values obtained by numerical calculations are close to the curve shown in Fig.4. For the collisionless case, the unstable region below the line given by Eq.(18) is limited in the region  $\epsilon_n > 0$ .

The critical boudary curve is also calculated numerically for the case with the collision effect. As seen in Fig.4, when the collision effect  $(\hat{\nu}_e=1)$  is introduced, the stable region in the  $(\eta_e,\epsilon_n)$ -plane is enlarged. How the collision effect stabilizes the trapped electron mode can also be seen by the eigenvalue shown by the dotted curve in Fig. 3. As compared with the eigenvalue of  $\eta_e=1$ , the dotted trajectory with the collision effect,  $\hat{\nu}_e=1$ , is much reduced.

The weak collision effect exites the DTEM. On the other hand, the same weak collision effect strongly stabilizes the collisionless trapped electron mode induced by the curvature drift. This situation can be seen in Fig.5, where the normalized growth rates for  $\epsilon_{\rm n}=0$  (DTEM) and  $\epsilon_{\rm n}=0.2$  are plotted versus  $\hat{\nu}_{\rm e}$  for comparison.

#### §5. Anomalous Electron Diffusion

We proceed to the derivation of electron diffusion coefficient induced by the trapped electron modes. Let us consider the cross field quasi-linear electron flux

$$\Gamma_{e} = \int d^{3}v \langle v_{x}f_{e} \rangle \tag{19}$$

where the angular brackets means the ensemble average. Introducing Fourier representation of  $\overset{\sim}{v}_{x}=ick_{\theta}\overset{\sim}{\phi}/B$  and eq.(2) into eq.(19), we have

$$\Gamma_{e} = \sum \left| \frac{e\phi}{T_{e}} \right|_{k_{0}}^{2} \frac{cT_{e}}{eB} k_{\theta} Im \left( \int d^{3}v \frac{\tilde{\omega} - \omega_{+Te}}{\tilde{\omega} - \omega_{De} + iv_{eff}} F_{Me} \right)$$
 (20)

The flux due to the discrete trapped electron modes must be evaluated by introducing the discrete eigenvalue  $\omega=\omega_{\mathbf{r}}+i\gamma$  given by the dispersion relation (5). From eq.(5), the imaginary part of the integral in eq.(20) can be expressed by the discrete eigenvalue in the form<sup>9</sup>)

$$\operatorname{Im} \left( \int d^{3} v \frac{\bar{\omega} - \omega_{+ \mathrm{Te}}}{\bar{\omega} - \omega_{\mathrm{De}} + i v_{\mathrm{eff}}} F_{\mathrm{Me}} \right) = \frac{\omega_{+ \mathrm{e}} \gamma}{\bar{\omega}_{r}^{2} + \gamma^{2}}$$
 (21)

which is exact for arbitrary  $\omega_D$  and  $v_{\mbox{eff}}$ .

If we apply the mixing length assumption:  $|e\tilde{\phi}T_e|^2 = (k_{\perp}L_n)^{-2}$  in eq.(20), and bearing in mind the relation:  $\Gamma = -D_{\perp} dn/dr$ , we have the electron diffusion coefficient from eqs.(20) and (21)

$$D_{\perp} = \frac{\omega \star_{e}}{k_{\perp}^{2}} \frac{\omega \star_{e} \gamma}{\bar{\omega}_{\perp}^{2} + \gamma^{2}}$$
 (22)

When  $\omega_{\mathbf{r}} = \omega_{\star}$  and  $\gamma << \omega_{\star}$ , eq.(22) reduces to the usual result

$$D_{\perp} = \frac{\gamma}{k_{\perp}^2} \tag{23}$$

As we have seen in Fig.3, this assumption does not hold in general, i.e., the usual formula (23) is valid only in the limited parameter region. As seen in Fig.3, when  $\eta_e>1$ , and  $\epsilon_n=0.1$ , the magnitude of the growth rate  $\gamma$  becomes nearly the same as that of the real frequency, and the assumption  $\gamma<<\omega_{\star}$  is broken down.

As long as the test particle diffusion model and the mixing length ansatz are applied, the diffusion coefficeient due to the continuum

contribution has been obtained in the similar form<sup>9)</sup>.

For the DTEM, applying the growth rate given by eq.(12) to eq.(23), we have

$$D_{\perp} \simeq \frac{6 \varepsilon^{\frac{3}{2}} \omega \star_{e}^{2} \eta_{e}}{\sqrt{\pi} v_{e} k_{\perp}^{2}}$$
 (24)

If we introduce the gryro-Bohm coefficient by

$$D_{GB} = \frac{\rho_{\underline{i}}^2 v_{\underline{i}}}{L_{B}} \tag{25}$$

for  $k = k_0$ , eq.(24) can be rewitten in the form

$$D_{\underline{i}} = D_{GB} \frac{6\sqrt{\epsilon \tau^2 V_{\underline{i}}}}{\sqrt{\pi \epsilon_n L_T V_e}}$$
 (26)

The diffusion coefficient (24) has the scaling: D  $\propto$  T<sup>7/2</sup> $\epsilon^{3/2}$ B<sup>-2</sup>n<sup>-1</sup>/L<sub>n</sub>L<sub>T</sub>, i.e., D has a strong temperature dependence.

If we assume k  $\rho_1$ \*constant as experimentally observed<sup>10)</sup>, eq.(22) can also by expressed in terms of  $D_{GB}$ :

$$D_{\perp} = D_{GB} \frac{\omega \star e^{\gamma}}{\tilde{\omega}_{r}^{2} + \gamma^{2}} \frac{\tau}{\varepsilon_{r_{1}}}$$
(27)

For the collisionless plasma, the diffusion coefficient  $D_{\perp}$  normalized by  $D_{GB}$  is plotted versus  $\epsilon_n$  in Fig.6. As seen in Fig.6, the quantity  $D_{\perp}/D_{GB}$  increses as  $\eta_e$  increases, and is sharply peaked at  $\epsilon_n$  = 0.05 which is due to the behavior of the growth rate  $\gamma$  versus  $\epsilon_n$ .

As shown in Refs.(1) and (2), and eq.(24), the asymptotic formula for the growth rate for the DTEM is proportional to the factor  $\eta_{\rm e}.$  The importance of the electron temperature gradient effect  $\eta_{\rm e}$  has also been seen in the numerical results in Figs.1 and 3, i.e., without  $\eta_{\rm e}$  the trapped electron instability disappears. Recently the diffusion coefficient induced by the DTEM without the  $\eta_{\rm e}$ -effect has been employed, among many other models, for interpretations of experimental observations  $^{11})^{12})$ . Since the electron temperature gradient is an essential source for the trapped electron instability as pointed out in the above, the effect of  $\eta_{\rm e}$  should be taken into account.

## §6. Conclusion

Neglecting the finite Larmor radius effect and transit frequency for ions, the trapped electron instabilities induced by the electron temperature gradient have been investigated by numerically calculating the electrostatic local dispersion relation. In our model, the negative electron temperature gradient is essential to excites the dissipative and collisionless trapped electron modes. When  $\eta_e{=}0$ , all these instabilities are stabilized. Even when  $\eta_e \neq 0$ , if the collision frequency and curvature drift frequency are independent of velocity, the trapped electron modes are stable, as in the case of  $\eta_1$ -mode. The DTEM is, therefore, becomes unstable when  $\eta_e \neq 0$  and  $v_{eff}$  is velocity dependent. In the highly collisional regime,  $v_e >> \omega_\star$ , the asymptotic formula for the growth rate agrees with the numerical results calculated by a conformal mapping method.

In the collisionless regime, the combination of electron temperature gradient and curvature drift effects make the trapped electron mode strongly unstable. The frequency  $\omega_{\Gamma}$  is positive, i.e., in the electron diamagnetic drift direction, and  $\omega_{\Gamma}^{s}\omega_{\star}$ . The growth rate  $\gamma$  also becomes comparable to or even larger than the electron drift frequency  $\omega_{\star}$ , i.e.,  $\gamma$  is even larger than that of the  $\eta_{1}$ -mode. In this regime,  $\omega_{\Gamma}^{s}\gamma^{s}\omega_{\star}$ , the asymptotic expansion formula for the plasma dispersion function is not available, and we have no analytical expression for the dispersion relation. In the limit,  $\gamma$ ->0, of the marginal state, however, the exact analytical expression as given by eq.(18) for the stability boundary has been obtained in the collisionless case. The collision effect destabilizes the TEM in the small  $\eta_{\Theta}$  regime, while it stabilizes the TEM in the larger  $\eta_{\Theta}$  regime as seen in Fig.4.

The cross field electron flux which is induced by the discrete TEM has been calculated consistently with the dispersion relation, and the diffusion coefficient has been evaluated in term of the gyro-Bohm coefficient as given by eq.(24). The electron diffusion coefficient normalized by  $D_{\rm GB}$  increases as  $\eta_{\rm e}$  increases, and is sharply peaked at  $\epsilon_{\rm n} \approx 0.1$  as  $\epsilon_{\rm n}$  varies. At the peak value  $D_{\perp}$  is much

larger than DGB as shown in Fig.6.

In this study a simple model has been employed to see what is the important source of the trapped electron instabilty. If we introduce more detail of ion dynamics such as the finite Larmor radius effect, the growth rate of the TEM may be reduced<sup>13</sup>.

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## Figures Captions

- Fig.1: Variation of normalized discrete time eigenvalue for various values of  $\stackrel{\wedge}{v_e}=v_e/\omega_*$  and  $\eta_e$  in the complex  $\omega$ -plane.
- Fig.2: Variations of normalized frequency and growth rate for dissipative trapped electron mode vursus normalized collision frequency  $\hat{v}_e$  for  $\eta_e$ =1
- Fig.3: Variation of normalized discrete eigenvalue for collisionless trapped electron mode for various values of  $\epsilon_n$  and  $\eta_e$ . The discrete eigenvalue for DTEM is presented by the broken curve for comparison.
- Fig.4: Critical boundary curves for  $v_e=0$  and  $v_e=1$  in  $(\eta_e, \epsilon_n)$ -plane.
- Fig.5: Comparison of normalizes growth rates for  $\epsilon_n$ =0(DTEM) and  $\epsilon_n$ =0.2 as functions of normalized collision frequency  $\stackrel{\wedge}{\nu}_e$ .
- Fig.6: Variation of electron diffusion coefficient normalized by gyroBohm coefficient versus  $\epsilon_n$  for various values of  $\eta_e$  for collisionless plasma.

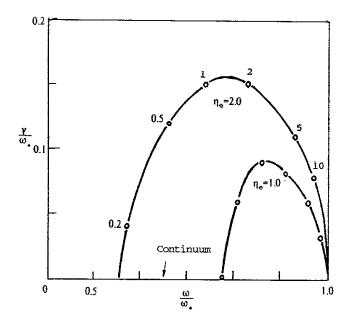


Fig.1

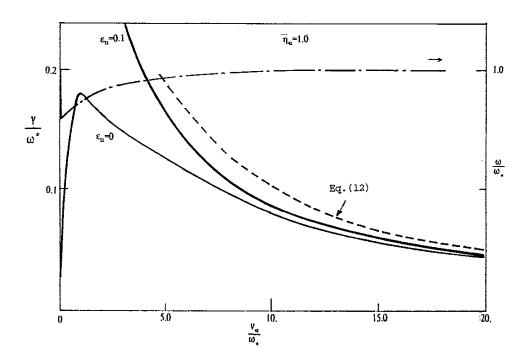


Fig.2

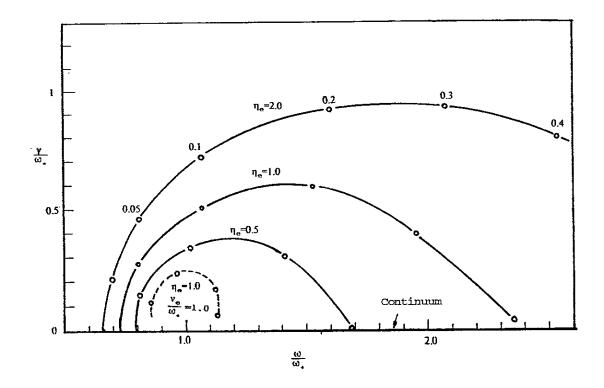


Fig. 3

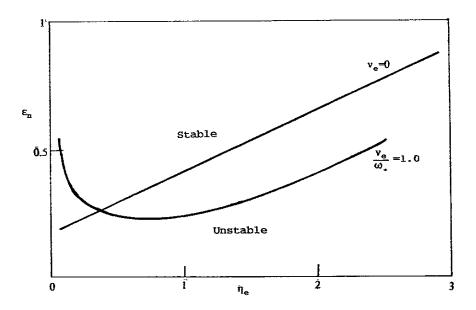


Fig. 4

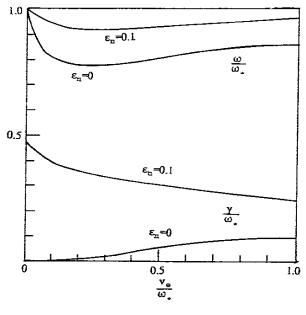


Fig.5

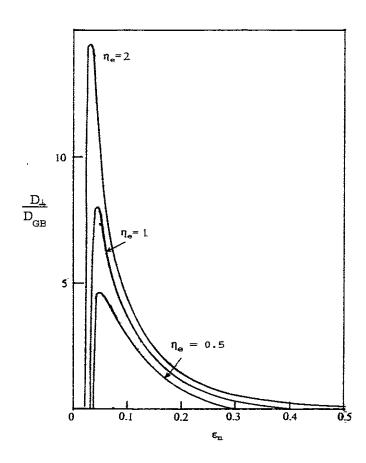


Fig.6

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