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Yoshiomi Kondoh

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Attractors of Dissipative Structure in Three Dissipative Fluids

Yoshiomi Kondoh

Department of Electronic Engineering, Gunma University

Kiryu, Gunma 376, Japan

(Received:)

A general theory with use of auto-correlations for distributions is presented to derive that realization of coherent structures in general dissipative dynamic systems is equivalent to that of self-organized states with the minimum dissipation rate for instantaneously contained energy. Attractors of dissipative structure are shown to be given by eigenfunctions for dissipative dynamic operators of the dynamic system and to constitute the self-organized and self-similar decay phase. Three typical examples applied to incompressible viscous fluids, to incompressible viscous and resistive magnetohydrodynamic (MHD) fluids and to compressible resistive MHD plasmas are presented to lead to attractors in the three dissipative fluids and to describe a common physical picture of self-organization and bifurcation of the dissipative structure.

Keywords: self-organization, eigenfunction of dissipative dynamic operator, attractor of dissipative structure, incompressible vicous fluids, incompressible viscous and resistive MHD fluids, resistive MHD plasma

I. INTRODUCTION

"Dissipative structures" realized in dissipating nonlinear dynamical systems have attracted much attention in many research fields. They include various self-organized structures in thermodynamic systems [1, 2], the force-free fields of cosmic magnetism [3], the self-organized relaxed state of the magnetized fusion plasmas such as in the reversed field pinch (RFP) experiment [4–6], in the spheromak experiment [7, 8] and in the simple toroidal Z pinch experiment [9], and further the flow structures in incompressible viscous fluids such as the two dimensional (2-D) flow patterns after grid turbulence [10] and the helical flow patterns which follow turbulent puffs [11]. We can see some common mathematical structure among the self-organized relaxed states of the dissipative structure and also among the proposed theories themselves to explain those dissipative structures [3, 12–18]. The study of the common universal mathematical structures embedded in dissipative nonlinear dynamic systems and leading to those dissipative structures is an area of deep interest. Using a thought analysis for the method of science [19–21], the present author has proposed recently a novel general theory which clarifies that attractors of the dissipative structure are given by eigenfunctions for dissipative dynamic operators in dynamic systems of interest [22]. Here, the word "thought analysis" means that we investigate logical structures, ideas or theories used in the objects being studied, and try to find some key elements for improvement and/or some other new theories which involve generality, by using such as a kind of thought experiments and mathematically reversible processes [18–21].

In this paper, we refine more the previous general theory [22] in Section II, by clarifying that realization of coherent structures in time evolution is equivalent to that of self-organized states with the minimum dissipation rate for instantaneously contained energy. We present three typical examples applied to incompressible viscous fluids in Section III, to incompressible viscous and resistive magnetohydrodynamic (MHD) fluids such as liquid metals in Section IV and to compressible resistive MHD plasmas in Section V in oreder to lead to attractors of the dissipative structure in these dissipative fluids and to describe a common physical picture of self-organization and bifurcation of the dissipative structure.

II. GENERAL THEORY OF SELF – ORGANIZATION AND DISSIPATIVE STRUCTURE

We present here the more refined general theory for the self-organization and the dissipative structure than the previous report [22]. Quantities with n elements in dynamic systems of interest shall be expressed as $\mathbf{q}(t, \mathbf{x}) = \{q_1(t, \mathbf{x}),\ q_2(t, \mathbf{x}),\ \cdots,\ q_n(t, \mathbf{x})\}$. Here, t is time, $\mathbf{x}$ denotes m-dimensional space variables, and $\mathbf{q}$ represents a set of physical quantities having n elements, some of which are vectors such as the velocity $\mathbf{u}$, the magnetic field $\mathbf{B}$, the current density $\mathbf{j}$, $\cdots$, and others are scalors such as the mass density, the energy density, the specific entropy and so on. We consider a general dissipative nonlinear dynamic system which may be described by

$$\frac{\partial q_i}{\partial t} = L_i^N[\mathbf{q}] + L_i^D[\mathbf{q}], \tag{1}$$

where $L_i^N[\mathbf{q}]$ and $L_i^D[\mathbf{q}]$ denote respectively nondissipative and dissipative, linear or nonlinear dynamic operators, such as $q_i = \mathbf{u}$, $L_i^N[\mathbf{q}] = -\nabla p/\rho - \nabla u^2/2 + \mathbf{u} \times \omega$, and $L_i^D[\mathbf{q}] = (\nu/\rho)\nabla^2\mathbf{u}$ in the Navier-Stokes equation for incompressible viscous fluid dynamics with the coefficient of viscosity ν. If this dynamic system has no dissipative term of $L_i^D[\mathbf{q}]$ and also has no external input, global auto-correlations $W_{ii}(t) = \int q_i(t, \mathbf{x}) q_i(t, \mathbf{x}) \mathrm{d}V = \int [q_i(t, \mathbf{x})]^2 \mathrm{d}V$ across the space volume of the system, which usually represent the system's total energy, are conserved because there is no dissipation by the nondissipative operator $L_i^N[\mathbf{q}]$. In this case, we accordingly obtain

$\partial W_{ii}/\partial t = 2\int q_i(\partial q_i/\partial t)\mathrm{d}V = 2\int q_i L_i^N[\mathrm{q}]\,\mathrm{d}V = 0$, and therefore the definition of the nondissipative operator $L_i^N[\mathrm{q}]$ is written by:

$$\int q_i L_i^N[\mathrm{q}]\,\mathrm{d}V \;=\; 0\,. \tag{2}$$

Using eq.(2), the dissipation rate of $W_{ii}(t)$ in the dissipative dynamic system of eq.(1) is written as follows:

$$\frac{\partial W_{ii}(t)}{\partial t} \;=\; 2\int q_i(t,\mathrm{x})\frac{\partial q_i(t,\mathrm{x})}{\partial t}\,\mathrm{d}V \;=\; 2\int q_i L_i^D[\mathrm{q}]\,\mathrm{d}V\,. \tag{3}$$

When the dynamic system has some unstable regions, the nondissipative dynamic operator $L_i^N[\mathrm{q}]$ may become dominant and lead to the rapid growth of perturbations there and further to turbulent phases. (In some cases, a nonlinearity of $L_i^N[\mathrm{q}]$ may lead to nonlinear saturation of perturbations.) This may yield spectrum transfers or spectrum spreadings toward the higher or wider wavenumber region in q_i distributions, as in the normal energy cascade and the inverse cascade shown by 3-D MHD simulations in [23–25] or in the turbulent region of the turbulent puffs in incompressible viscous fluids shown in Fig.4 of [11]. When the higher wavenumber becomes a large fraction of the spectrum, the dissipative dynamic operator $L_i^D[\mathrm{q}]$ may become dominant to yield higher dissipations for the higher wavenumber components of W_{ii}. In this rapid dissipation phase, which is far from equilibrium, the unstable regions in the dynamic system are considered to vanish to produce a stable configuration again. Since this newly self-organized relaxed state is identified by realization of its coherent structure, we notice and find the following definition of the configuration of the self-organized relaxed state, by using auto-correlations, $q_i(t_R,\mathrm{x})q_i(t_R+\Delta t,\mathrm{x})$, between the time of relaxed state, t_R, and slightly later time, $t_R+\Delta t$, with a small Δt:

$$min\ \left|\ \frac{\int q_i(t_R,\mathrm{x})q_i(t_R+\Delta t,\mathrm{x})\mathrm{d}V}{\int q_i(t_R,\mathrm{x})q_i(t_R,\mathrm{x})\mathrm{d}V} \;-\; 1\ \right|\ \text{state}\,. \tag{4}$$

Substituting the Taylor expansion of $q_i(t_R + \Delta t, \mathrm{x}) = q_i(t_R, \mathrm{x}) + [\partial q_i(t_R, \mathrm{x})/\partial t]\Delta t + (1/2)[\partial^2 q_i(t_R, \mathrm{x})/\partial t^2](\Delta t)^2 + \cdots$ into eq.(4), and taking account of the arbitrary smallness of Δt, we obtain the following equivalent definition of the configuration of the self-organized relaxed state from the first order of Δt in eq.(4):

$$min \ \left| \frac{\int q_i(t_R, \mathrm{x})[\partial q_i(t_R, \mathrm{x})/\partial t]\mathrm{d}V}{\int q_i(t_R, \mathrm{x}) q_i(t_R, \mathrm{x})\mathrm{d}V} \right| \ \text{state} . \tag{5}$$

Using the term of $W_{ii}(t)$, this equivalent definition is rewritten as follows:

$$min \ \left| \frac{\partial W_{ii}(t_R)/\partial t}{W_{ii}(t_R)} \right| \ \text{state} . \tag{6}$$

This definition leads to the following two equivalent definitions for the configuration of the self-organized relaxed state :

$$min \ \left| \frac{\partial W_{ii}}{\partial t} \right| \ \text{state for a given value of } W_{ii} \text{ at } t = t_R . \tag{7}$$

$$max \ W_{ii} \ \text{state for a given value of } \left| \frac{\partial W_{ii}}{\partial t} \right| \ \text{at } t = t_R . \tag{8}$$

These two equivalent definitions belong to typical problems of variational calculus with respect to the spatial variable x to find the spatial profiles of $q_i(t_R, \mathrm{x})$, and they are known to be equivalent to each other by the reciprocity of the variational calculus.

We use the notation $\mathbf{q}^*(W_{ii}, \mathrm{x})$ or simply q_i^* for the profiles of q_i that satisfy eq.(7). Since $\partial W_{ii}/\partial t$ usually has a negative value, the mathematical expressions for eq.(7) are written as follows, defining a functional F with use of a Lagrange multiplier α:

$$F \equiv -\frac{\partial W_{ii}}{\partial t} - \alpha W_{ii} , \tag{9}$$

$$\delta F = 0 , \tag{10}$$

$$\delta^2 F > 0, \tag{11}$$

where δF and $\delta^2 F$ are the first and the second variations of F with respect to the variation $\delta \mathrm{q}(\mathbf{x})$ only for the spatial variable $\mathbf{x}$. Substituting eq.(3) into eqs.(9) - (11), we obtain:

$$\delta F \;=\; -\,2\int\{\; \delta q_i (L_i^D[\mathrm{q}] + \alpha q_i) + q_i \delta L_i^D[\mathrm{q}] \;\}\, \mathrm{d}V \;=\; 0 \,. \tag{12}$$

$$\delta^2 F \;=\; -\,2\int\; \delta q_i (\delta L_i^D[\mathrm{q}] \;+\; \frac{\alpha}{2}\, \delta q_i)\, \mathrm{d}V \;>\; 0 \,. \tag{13}$$

We now impose the following self-adjoint property upon the operators $L_i^D[\mathrm{q}]$:

$$\int q_i\, \delta L_i^D[\mathrm{q}]\mathrm{d}V = \int \delta q_i L_i^D[\mathrm{q}]\mathrm{d}V + \oint \mathbf{P}\cdot \mathrm{d}\mathbf{S}\,, \tag{14}$$

where $\oint \mathbf{P}\cdot \mathrm{d}\mathbf{S}$ denotes the surface integral term which comes out as from the Gauss theorem. The self-adjoint property of eq.(14) is satisfied by dissipative dynamic operators such as $(\nu/\rho)\nabla^2\mathbf{u}$ in the Navier-Stokes equation, and the Ohm loss term of $-\nabla \times (\eta \mathbf{j})$ in the magnetic field equation of resistive MHD plasmas with resistivity η. The surface integral term of $\oint \mathbf{P}\cdot \mathrm{d}\mathbf{S}$ sometimes vanishes because of the boundary condition, as in the case of the ideally conducting wall. Using the self-adjoint property of eq.(14), we obtain the following from eq.(12):

$$\delta F \;=\; -\,2\int \delta q_i (2L_i^D[\mathrm{q}] + \alpha q_i)\, \mathrm{d}V \;-\; 2\oint \mathbf{P}\cdot \mathrm{d}\mathbf{S} \;=\; 0 \,. \tag{15}$$

We then obtain the Euler-Lagrange equations from the volume integral term in eq.(15) for arbitrary variations of δq_i as follows:

$$L_i^D[\mathrm{q}^*] \;=\; -\,\frac{\alpha}{2}\, q_i^* \,. \tag{16}$$

We find from eq.(16) that the profiles of q_i^* are given by the eigenfunctions for the dissipative dynamic operators $L_i^D[\mathrm{q}^*]$, and therefore have a feature uniquely determined by the operators $L_i^D[\mathrm{q}^*]$ themselves. As a boundary value problem, we may assume that eq.(16) can be solved for given boundary values of q_i. The value of the Lagrange multiplier α is determined by using the given value of W_{ii} for the global constraint, as is common practice in the variational calculus. Since we cannot, a priori, predict the value of W_{ii} at the state with the minimum dissipation rate for every dissipative dynamic system, we have to measure the value of W_{ii} at such a state in order to determine the value of α. However, we can predict the type of the profile q_i^* for every dissipative dynamic system by using eq.(16), if the operator $L_i^D[\mathrm{q}^*]$ is given.

Substituting the eigenfunctions q_i^* into eq.(3), and using eq.(16), we obtain the following:

$$\frac{\partial W_{ii}{}^*}{\partial t} = -\alpha \int (q_i{}^*)^2 \, \mathrm{d}V = -\alpha \, W_{ii}{}^* , \tag{17}$$

$$W_{ii}^* = \mathrm{e}^{-\alpha t} W_{iiR}^* = \mathrm{e}^{-\alpha t} \int [q_{iR}{}^*(\mathrm{x})]^2 \, \mathrm{d}V = \int [q_{iR}{}^*(\mathrm{x}) \, \mathrm{e}^{-\frac{\alpha}{2}t}]^2 \, \mathrm{d}V , \tag{18}$$

$$q_i{}^* = q_{iR}{}^*(\mathrm{x}) \, \mathrm{e}^{-\frac{\alpha}{2}t} , \tag{19}$$

$$\frac{\partial q_i{}^*}{\partial t} = -\frac{\alpha}{2} \, q_i{}^* = L_i^D[\mathrm{q}^*] , \tag{20}$$

where $q_{iR}{}^*(\mathrm{x})$ denotes the eigensolution for eq.(16) which is supposed to be realized at the state with the minimum dissipation rate during the time evolution of the dynamical system of interest. Substituting the eigenfunctions q_i^* into eq.(1), and comparing with eq.(20), we obtain the following equilibrium equations at $t = t_R$:

$$\textit{equilibrium equations} \qquad L_i^N[\mathrm{q}^*] = 0 . \tag{21}$$

We find from eqs.(19) - (21) that the eigenfunctions q_i^* for the dissipative dynamic operators $L_i^D[\mathrm{q}^*]$ constitute the self-organized and self-similar decay phase with the minimum dissipation rate and with the equilibrium state of eq.(21) in the time evolution of the present dynamic system. We see from eq.(17) that the factor α of eq.(16), the Lagrange multiplier, is equal to the decay constant of W_{ii} at the self-organized and self-similar decay phase. Since the present dynamic system evolves basically by eq.(1), the dissipation by $L_i^D[\mathrm{q}^*]$ of eq.(16) during the self-similar decay couples with $L_i^N[\mathrm{q}]$ and the boundary wall conditions to cause gradual deviation from self-similar decay. This gradual deviation may yield some new unstable region in the dynamic system. When some external input is applied in order to recover the dissipation of W_{ii}, the present dynamic system is considered to return repeatedly close to the self-organized and self-similar decay phase. Observation of the time evolution of the system of interest for long periods reveals a physical picture in which the system appears to be repeatedly attracted towards and trapped in the self-organized and self-similar decay phase of eq.(19). The system stays in this phase for the longest time during each cycle of the time evolution because this is where it has the minimum dissipation rate. In this sense, the eigenfunctions q_i^* of eq.(16) for the dissipative dynamic operators $L_i^D[\mathrm{q}^*]$ are "the attractors of the dissipative structure" introduced by Prigogine [1, 2].

Using eq.(13), we next discuss the mode transition point or bifurcation point of the self-organized dissipative structure. We consider the following associated eigenvalue problem for critical perturbations δq_i that make $\delta^2 F$ in eq.(13) vanish:

$$(\delta L_i^D[\mathrm{q}])_k + \frac{\alpha_k}{2} \delta q_{ik} = 0 , \tag{22}$$

with boundary conditions given for δq_i, for example $\delta q_i = 0$ at the boundary wall. Here, α_k is the eigenvalue, and $(\delta L_i^D[\mathrm{q}])_k$ and δq_{ik} denote the eigensolution. Substituting the eigensolution δq_{ik} into eq.(13) and using eq.(22), we obtain the following:

$$\delta^2 F = (\alpha_k - \alpha) \int \delta q_{ik}{}^2 \, \mathrm{d}V > 0 . \tag{23}$$

Since eq.(23) is required for all eigenvalues, we obtain the following condition for the state with the minimum dissipation rate:

$$0 < \alpha < \alpha_1 , \tag{24}$$

where α_1 is the smallest positive eigenvalue, and α is assumed to be positive. When the value of α goes beyond the condition of eq.(24), as when $\alpha_1 < \alpha$, then the mixed mode, which consists of the basic mode by the solution of eq.(16) where $\alpha = \alpha_1$ and the lowest eigenmode of eq.(22), becomes the self-organized dissipative structure with the minimum dissipation rate. The bifurcation point of the dissipative structure is given by $\alpha = \alpha_1$.

III. ATTRACTORS IN INCOMPRESSIBLE VISCOUS FLUIDS

We apply here the general theory in the previous section to incompressible viscous fluids described by the Navier-Stokes equation:

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla p + \nu \nabla^2 \mathbf{u}, \tag{25}$$

where ρ, $\mathbf{u}$, and p are the fluid mass density, the fluid velocity, and the pressure, respectively, and $\nabla \cdot \mathbf{u} = 0$. For simplicity, we assume ν to be spatially uniform. Using $\nabla \cdot \mathbf{u} = 0$ and the two vector formulas of $\nabla u^2 = 2\mathbf{u} \times (\nabla \times \mathbf{u}) + 2(\mathbf{u} \cdot \nabla)\mathbf{u}$ and $\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times \nabla \times \mathbf{u}$, eq.(25) is rewritten as:

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\nabla p}{\rho} - \frac{1}{2}\nabla u^2 + \mathbf{u} \times \omega - \frac{\nu}{\rho}\nabla \times \nabla \times \mathbf{u}\, , \tag{26}$$

where $\omega = \nabla \times \mathbf{u}$ is the vorticity. We find from eq.(26) that $L_i^N[\mathbf{q}] = -\nabla p/\rho - \nabla u^2/2 + \mathbf{u} \times \omega$ and $L_i^D[\mathbf{q}] = -(\nu/\rho)\nabla \times \nabla \times \mathbf{u}$, where $q_i \equiv \mathbf{u}$. Using the vector formula $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, $\partial W_{ii}/\partial t$ is known to be rewritten by volume integrals of $\nu\omega^2/\rho$. Substituting these two operators of $L_i^N[\mathbf{q}]$ and $L_i^D[\mathbf{q}]$ into eqs.(1) - (15), and using $\delta\omega = \nabla \times \delta\mathbf{u}$, $\omega = \nabla \times \mathbf{u}$, $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot \nabla \times \mathbf{a} - \mathbf{a} \cdot \nabla \times \mathbf{b}$, and the Gauss theorem, we obtain the following:

$$\delta F = 4\int \delta\mathbf{u} \cdot \left(\frac{\nu}{\rho}\nabla \times \nabla \times \mathbf{u} - \frac{\alpha}{2}\mathbf{u}\right)\mathrm{d}V$$
$$+ \frac{2\nu}{\rho}\oint [\, \delta\mathbf{u} \times (\nabla \times \mathbf{u}) + (\nabla \times \delta\mathbf{u}) \times \mathbf{u}\,] \cdot \mathrm{d}\mathbf{S} = 0\, , \tag{27}$$

$$\delta^2 F \;=\; 2\int \delta\mathbf{u}\cdot(\frac{\nu}{\rho}\nabla\times\nabla\times\delta\mathbf{u} - \frac{\alpha}{2}\delta\mathbf{u})\mathrm{d}V \;>\; 0\,. \qquad (28)$$

Here, we notice that the present dissipative operator $L_i^D[\mathrm{q}]$ satisfies the self-adjoint property of eq.(14) as follows:

$$\int \mathbf{u}\cdot(\frac{\nu}{\rho}\nabla\times\nabla\times\delta\mathbf{u})\mathrm{d}V \;=\; \int \delta\mathbf{u}\cdot(\frac{\nu}{\rho}\nabla\times\nabla\times\mathbf{u})\mathrm{d}V$$
$$+\,\frac{\nu}{\rho}\oint[\,\delta\mathbf{u}\times(\nabla\times\mathbf{u}) \;+\; (\nabla\times\delta\mathbf{u})\times\mathbf{u}\,]\cdot\mathrm{d}\mathbf{S}, \qquad (29)$$

where the vector formula of $\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot\nabla\times\mathbf{a} - \mathbf{a}\cdot\nabla\times\mathbf{b}$ is used twice. We obtain the Euler-Lagrange equation from the volume integral term in eq.(27) for the arbitrary variation $\delta\mathbf{u}$, corresponding to eq.(16), as follows:

$$\nabla\times\nabla\times\mathbf{u}^* \;=\; \frac{\alpha\rho}{2\nu}\mathbf{u}^*\,. \qquad (30)$$

The eigenfunction of this eq.(30) can be obtained for a given boundary value of $\mathbf{u}$, as a boundary value problem.

Using the eigenfunction of eq.(30) and refering eqs.(17) - (20), we obtain the following:

$$\frac{\partial W_{ii}{}^*}{\partial t} \;=\; -\,\alpha\int(\mathbf{u}^*)^2\,\mathrm{d}V \;=\; -\,\alpha\,W_{ii}{}^*\,, \qquad (31)$$

$$W_{ii}^* \;=\; \mathrm{e}^{-\alpha t}W_{iiR}{}^* \;=\; \mathrm{e}^{-\alpha t}\int[\mathbf{u}_R{}^*(\mathbf{x})]^2\,\mathrm{d}V \;=\; \int[\mathbf{u}_R{}^*(\mathbf{x})\mathrm{e}^{-\frac{\alpha}{2}t}]^2\,\mathrm{d}V\,, \qquad (32)$$

$$\mathbf{u}^* \;=\; \mathbf{u}_R{}^*(\mathbf{x})\,\mathrm{e}^{-\frac{\alpha}{2}t}\,, \qquad (33)$$

$$\frac{\partial\mathbf{u}^*}{\partial t} \;=\; -\,\frac{\alpha}{2}\,\mathbf{u}^* \;=\; -\,\frac{\nu}{\rho}\nabla\times\nabla\times\mathbf{u}^*\,, \qquad (34)$$

where $\mathbf{u}_R{}^*(\mathbf{x})$ denotes the eigensolution for eq.(30) for the given boundary value of $\mathbf{u}$, which is supposed to be realized at the state with the minimum dissipation rate during the time evolution of the dynamical system of interest. Substituting the eigenfunction $\mathbf{u}^*$ into eq.(26), and using eq.(34), we obtain the equilibrium equation at $t = t_R$:

$$\nabla p^* + \frac{\rho}{2}\nabla(u^*)^2 = \rho(\mathbf{u}^* \times \omega^*) . \tag{35}$$

We find from eq.(33) that the eigenfunction $\mathbf{u}^*$ for the present dissipative dynamic operator $-(\nu/\rho)\nabla \times \nabla \times \mathbf{u}$ constitutes the self-organized and self-similar decay phase during the time evolution of the present dynamic system. We see from eq.(31) that the factor α of eq.(30), which is the Lagrange multiplier, is equal to the decay constant of flow energy W_{ii} at the self-organized and self-similar decay phase.

Refering to eqs.(13) and (22) - (24), we next discuss the mode transition point or bifurcation point of the self-organized dissipative structure due to the present dissipative dynamic operator $L_i^D[\mathbf{q}] = -(\nu/\rho)\nabla \times \nabla \times \mathbf{u}$. We obtain the associated eigenvalue problem from eq.(28) for critical perturbations $\delta\mathbf{u}$ that make $\delta^2 F$ vanish, and the condition for the state with the minimum dissipation rate that corresponds to eq.(24), as follows:

$$\nabla \times \nabla \times \delta\mathbf{u}_k - \frac{\alpha_k \rho}{2\nu}\delta\mathbf{u}_k = 0 , \tag{36}$$

$$0 < \alpha < \alpha_1 . \tag{37}$$

Here, α_k is the eigenvalue, $\delta\mathbf{u}_k$ denotes the eigensolution, α_1 is the smallest positive eigenvalue, the boundary conditions are $\delta\mathbf{u}_w \cdot d\mathbf{S} = 0$ and $[\ \delta\mathbf{u}_w \times (\nabla \times \delta\mathbf{u}_w)\ ] \cdot d\mathbf{S} = 0$, and the subscript w denotes the value at the boundary wall. Since the present dissipative dynamic operators $L_i^D[\mathbf{q}]$ satisfy the the self-adjoint property of eq.(29) and the boundary conditions are $\delta\mathbf{u}_w \cdot d\mathbf{S} = 0$ and $[\ \delta\mathbf{u}_w \times (\nabla \times \delta\mathbf{u}_w)\ ] \cdot d\mathbf{S} = 0$ for eq.(36), the eigenfunctions, $\mathbf{a}_k$, for the associated eigenvalue problem of eq.(36) form a complete orthogonal set and the appropriate normalization is written as:

$$\begin{aligned}\int \mathbf{a}_k \cdot (\nabla \times \nabla \times \mathbf{a}_j)\, dV &= \int \mathbf{a}_j \cdot (\nabla \times \nabla \times \mathbf{a}_k)\, dV \\ &= \frac{\alpha_k \rho}{2\nu} \int \mathbf{a}_j \cdot \mathbf{a}_k\, dV \\ &= \frac{\alpha_k \rho}{2\nu}\,\delta_{jk} ,\end{aligned} \tag{38}$$

where $\nabla \times \nabla \times \mathbf{a}_k - (\alpha_k \rho / 2\nu)\ \mathbf{a}_k = 0$ is used. When the flow-dynamics system has some unstable regions, the nondissipative dynamic operator $L_i^N[\mathbf{q}] = -\nabla p/\rho - (1/2)\nabla u^2 + \mathbf{u} \times \omega$ may become dominant, leading to the rapid growth of perturbations and finally to turbulent phases. This process may yield spectrum transfers or spectrum spreadings toward the higher or wider mode number region in the flow $\mathbf{u}$ distribution. The amplitudes of perturbations are considered to grow to nonlinear saturation, and not infinitely. We next investigate the change of flow $\mathbf{u}$ distribution for a short time around or after the saturation of perturbation growth. In this phase, operator $L_i^N[\mathbf{q}]$ has become less dominant and $L_i^D[\mathbf{q}]$ becomes more so. The flow $\mathbf{u}$ distribution can be written by using the eigensolution $\mathbf{u}^*$ for the boundary value problem of eq.(30) for the given boundary value and also by using orthogonal eigenfunctions $\mathbf{a}_k$ for the eigenvalue problem of eq.(36) with the boundary conditions of $\mathbf{a}_k \cdot d\mathbf{S} = 0$ and $[\ \mathbf{a}_k \times (\nabla \times \mathbf{a}_k)\ ] \cdot d\mathbf{S} = 0$ at the boundary, as follows:

$$\mathbf{u} = \mathbf{u}^* + \sum_{k=1}^{\infty} c_k\, \mathbf{a}_k\ . \tag{39}$$

Substituting eq.(39) into eq.(26) and using eq.(30) and eq.(36), we obtain the following:

$$\frac{\partial \mathbf{u}^*}{\partial t} + \sum_{k=1}^{\infty} \frac{\partial (c_k \mathbf{a}_k)}{\partial t} = L_i^N[\mathbf{q}] - \frac{\alpha}{2}\mathbf{u}^* - \sum_{k=1}^{\infty} \frac{\alpha_k}{2} c_k \mathbf{a}_k\ , \tag{40}$$

where $L_i^N[\mathbf{q}] = -\nabla p/\rho - \nabla u^2/2 + \mathbf{u} \times \omega$ acts now as a less dominant operator, the eigenvalues α_k are positive, and α_1 is the smallest positive eigenvalue. We find from eq.(40) that the flow components of $\mathbf{u}^*$ and $c_k \mathbf{a}_k$ decay approximately by the decay constants of $\alpha/2$ and $\alpha_k/2$, respectively, in the present short time interval, in the same way as in eqs.(33) and (34). Since the components with the larger eigenvalue α_k decay faster, we see that this decay process yields the selective dissipation for the higher mode number components. We understand from eq.(40) that if $\alpha < \alpha_1$, the basic component $\mathbf{u}^*$ remains last and the flow distribution of $\mathbf{u}$ at the minimum dissipation rate phase is represented approximately by $\mathbf{u}^*$. If the value of α becomes greater than α_1, then the basic component $\mathbf{u}^*$ decays faster than the eigenmode $\mathbf{a}_1$. This faster decay of the basic component $\mathbf{u}^*$ continues to yield further decrease of W_{ii}, resulting in the decrease of α itself, until α becomes equal to α_1, i.e. the same decay rate state with the lowest eigenmode $\mathbf{a}_1$. Consequently, the mixed mode which consists of both $\mathbf{u}^*$, having $\alpha = \alpha_1$, and the lowest eigenmode $\mathbf{a}_1$, remains last and the flow distribution of $\mathbf{u}$ at the minimum dissipation rate phase is represented approximately by this mixed mode. The flow energy of this mixed mode decays as $W_{ii}^* \cong e^{-\alpha_1 t} \int (\mathbf{u}_R{}^* + c_1\mathbf{a}_1)^2 \, dV$. This argument gives us a detailed physical picture of the self-organization of the dissipative nonlinear dynamic system approaching the basic mode $\mathbf{u}^*$ and also of the bifurcation of the self-organized dissipative structure from the basic mode $\mathbf{u}^*$ to the mixed mode with $\mathbf{u}^*$ and $\mathbf{a}_1$ which takes place at $\alpha = \alpha_1$.

If $\mathbf{g}(\mathbf{x})$ is a sulution of eq.(30), it is easy to show that $\mathbf{h}(\mathbf{x}) \equiv \nabla \times \mathbf{g}(\mathbf{x})$ satisfies again eq.(30) and has the same decay constant α with that of the component $\mathbf{g}(\mathbf{x})$, by taking rotation of eq.(30). Linear combinations of $\mathbf{u}^* = e_1\mathbf{g}(\mathbf{x}) + e_2\mathbf{h}(\mathbf{x})$ also satisfy eq.(30) and have the same decay constant α. In a special case with $e_2 = \sqrt{2\nu/\alpha\rho}\, e_1$, the linear combinations of $\mathbf{u}^*$ can be shown straightway to satisfy the following:

$$\nabla \times \mathbf{u}^* = \kappa \mathbf{u}^* \quad \left(|\, \kappa \,| \equiv \sqrt{\frac{\alpha\rho}{2\nu}} \right) . \tag{41}$$

In this spacial case, $\mathbf{u}^* \times \omega^* = 0$, and then the equilibrium equation, eq.(35), becomes:

$$\nabla p^* + \frac{\rho}{2}\nabla(u^*)^2 = 0 . \tag{42}$$

In more general case with $e_2 \neq \sqrt{2\nu/\alpha\rho}\; e_1$, $\mathbf{u}^*$ contains other component so that $\mathbf{u}^* \times \omega^* \neq 0$.

When self-organized relaxed states of interest have some kind of symmetry along one coordinate x_s in $\mathbf{x}$ (for examples, translational, axial, toroidal, or helical symmetry), or depend on only two dimensional variables perpendicular to x_s, i.e. $\partial/\partial x_s = 0$ (two dimensional flow systems are also included in this case), then eq.(30) can be separated into two mutually independent equations, by using two components of $\mathbf{u}_s^*$ along x_s and $\mathbf{u}_{s\perp}^*$ perpendicular to x_s, as follows:

$$\nabla \times \nabla \times \mathbf{u}_s^* = \frac{\alpha\rho}{2\nu}\mathbf{u}_s^* , \tag{43}$$

$$\nabla \times \nabla \times \mathbf{u}_{s\perp}^* = \frac{\alpha\rho}{2\nu}\mathbf{u}_{s\perp}^* . \tag{44}$$

Time evolution of self-organized and coherent surface flow structures after grid turbulence shown in Figs.1 and 4 in [10] are considered to be represented by eq.(40) with use of eq.(44). In three dimensional flow systems, when self-organized states have a feature of $\sqrt{\alpha\rho/2\nu}\,\mathbf{u}_s^* = \nabla \times \mathbf{u}_{s\perp}^*$, then the total flow of $\mathbf{u}^* = \mathbf{u}_s^* + \mathbf{u}_{s\perp}^*$ can be shown straightforward to constitute solutions of the helical flow of eq.(41), by using eq.(44). This type of helical flow solution for eq.(41) is considered to represent approximately the helical flow pattern after the turbulent puffs shown in Fig.4 of [11] with use of the NMR imaging observation.

IV. ATTRACTORS IN INCOMPRESSIBLE VISCOUS AND RESISTIVE MHD FLUIDS

We show here another application of the general theory in Section II to incompressible viscous and resistive MHD fluids such as liquid metals which are described by the following extended Navier-Stokes equation and the equation for the magnetic field,

$$\rho\frac{\partial \mathbf{u}}{\partial t} = \mathbf{j} \times \mathbf{B} - \nabla p - \frac{\rho}{2}\nabla u^2 + \rho\mathbf{u} \times \omega - \nu\nabla \times \nabla \times \mathbf{u} , \tag{45}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\, \mathbf{u} \times \mathbf{B}\,) - \nabla \times (\eta \mathbf{j}), \tag{46}$$

where Ohm's law is used and $\mathrm{d}\mathbf{u}/\mathrm{d}t$ is rewritten by $\partial \mathbf{u}/\partial t$ in eq.(45) in the same way used at eq.(25) for eq.(26). In this system, the flow energy $\rho u^2/2$ and the magnetic energy $B^2/2\mu_0$ interchange with each other through the terms of $\mathbf{j}\times\mathbf{B}$ and $\nabla\times(\,\mathbf{u}\times\mathbf{B}\,)$ in eqs.(45) and (46). The global auto-correlation W_{ii}, corresponding to the total energy, and its dissipation rate $\partial W_{ii}/\partial t$ are written respectively as $W_{ii} = 2\int[(\rho u^2/2)+(B^2/2\mu_0)]\mathrm{d}V$ and $\partial W_{ii}/\partial t = -2\int[\,\nu\mathbf{u}\cdot\nabla\times\nabla\times\mathbf{u} + \mathbf{B}\cdot\nabla\times(\eta\mathbf{j})/\mu_0\,]\mathrm{d}V$. Using the vector formula $\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot\nabla\times\mathbf{a} - \mathbf{a}\cdot\nabla\times\mathbf{b}$, and the Gauss theorem, $\partial W_{ii}/\partial t$ is known to be rewritten by volume integrals of $(\nu\,\omega^2 + \eta\, j^2)$. We assume here, for simplicity, that the resistivity η at the relaxed state has a fixed spatial dependence like as $\eta(\mathbf{x})$. In the same way as was used at eqs.(27) and (28), substituting those of W_{ii} and $\partial W_{ii}/\partial t$ into eqs.(6) - (11), we obtain the followings:

$$\begin{aligned}\delta F = {} & 4\int\{\,\delta\mathbf{u}\cdot(\nu\nabla\times\nabla\times\mathbf{u} - \frac{\alpha}{2}\rho\mathbf{u}) + \frac{1}{\mu_o}\delta\mathbf{B}\cdot[\,\nabla\times(\eta\mathbf{j}) - \frac{\alpha}{2}\mathbf{B}]\,\}\mathrm{d}V \\ & + 2\oint[\,\nu(\,\delta\mathbf{u}\times\omega + \delta\omega\times\mathbf{u}) + \frac{\eta}{\mu_0}(\delta\mathbf{B}\times\mathbf{j} + \delta\mathbf{j}\times\mathbf{B})\,]\cdot\mathrm{d}\mathbf{S} = 0,\end{aligned} \tag{47}$$

$$\begin{aligned}\delta^2 F = {} & 2\int\{\,\delta\mathbf{u}\cdot(\nu\nabla\times\nabla\times\delta\mathbf{u} - \frac{\alpha}{2}\rho\delta\mathbf{u}) \\ & + \frac{1}{\mu_o}\delta\mathbf{B}\cdot[\,\nabla\times(\eta\delta\mathbf{j}) - \frac{\alpha}{2}\delta\mathbf{B}]\,\}\mathrm{d}V > 0,\end{aligned} \tag{48}$$

where $\mu_o\delta\mathbf{j} = \nabla\times\delta\mathbf{B}$ is used. Here, we notice again that the dissipative operator $-\nabla\times(\eta\mathbf{j})$ [i.e. $-\nabla\times(\eta\nabla\times\mathbf{B}/\mu_o)$] satisfies the self-adjoint property of eq.(14) as follows:

$$\begin{aligned}\int \mathbf{b}_k\cdot[\nabla\times(\eta\nabla\times\mathbf{b}_j)]\,\mathrm{d}V = {} & \int \mathbf{b}_j\cdot[\nabla\times(\eta\nabla\times\mathbf{b}_k)]\,\mathrm{d}V \\ & + \oint[\eta(\nabla\times\mathbf{b}_j)\times\mathbf{b}_k - \eta(\nabla\times\mathbf{b}_k)\times\mathbf{b}_j]\cdot\mathrm{d}\mathbf{S}.\end{aligned} \tag{49}$$

We then obtain the Euler-Lagrange equations for arbitrary variations of $\delta\mathbf{u}$ and $\delta\mathbf{B}$ from the volume integral terms of eq.(47), as follows:

$$\nabla \times \nabla \times \mathbf{u}^* = \frac{\alpha\rho}{2\nu}\mathbf{u}^* , \tag{50}$$

$$\nabla \times (\eta \mathbf{j}^*) = \frac{\alpha}{2}\mathbf{B}^*, \tag{51}$$

$$\nabla \times \nabla \times \mathbf{B}^* = \frac{\alpha\mu_o}{2\eta}\mathbf{B}^* \quad \text{for } \eta = \text{const.} , \tag{52}$$

where $\mu_0 \mathbf{j} = \nabla \times \mathbf{B}$ is used. Using the eigenfunctions of eqs.(50) and (51), and refering to eqs.(17) - (20), we obtain the following:

$$\frac{\partial W_{ii}{}^*}{\partial t} = -\,\alpha \int [\rho(\mathbf{u}^*)^2 + \frac{(\mathbf{B}^*)^2}{\mu_0}]\, \mathrm{d}V = -\,\alpha\, W_{ii}{}^* , \tag{53}$$

$$W_{ii}^* = \mathrm{e}^{-\alpha t} W_{iiR}{}^* = \int \{\rho[\mathbf{u}_R{}^*(\mathbf{x})\mathrm{e}^{-\frac{\alpha}{2}t}]^2 + \frac{[\mathbf{B}_R{}^*(\mathbf{x})\mathrm{e}^{-\frac{\alpha}{2}t}]^2}{\mu_0}\}\mathrm{d}V , \tag{54}$$

$$\mathbf{u}^* = \mathbf{u}_R{}^*(\mathbf{x})\, \mathrm{e}^{-\frac{\alpha}{2}t} , \tag{55}$$

$$\mathbf{B}^* = \mathbf{B}_R{}^*(\mathbf{x})\, \mathrm{e}^{-\frac{\alpha}{2}t} , \tag{56}$$

$$\rho\frac{\partial \mathbf{u}^*}{\partial t} = -\,\frac{\alpha}{2}\,\rho\mathbf{u}^* = -\,\nu\nabla \times \nabla \times \mathbf{u}^* , \tag{57}$$

$$\frac{\partial \mathbf{B}^*}{\partial t} = -\,\frac{\alpha}{2}\,\mathbf{B}^* = -\,\nabla \times (\eta\mathbf{j}^*) , \tag{58}$$

where $\mathbf{u}_R{}^*(\mathbf{x})$ and $\mathbf{B}_R{}^*(\mathbf{x})$ denote again the eigensolutions for eqs.(50) and (51) for given boundary values of $\mathbf{u}$ and $\mathbf{B}$, which are supposed to be realized at the state with the minimum dissipation rate during the time evolution of the dynamical system of interest. Substituting the eigenfunctions $\mathbf{u}^*$ and $\mathbf{B}^*$ into eqs.(45) and (46), and using eqs.(57) and (58), we obtain the equilibrium equation at $t = t_R$:

$$\nabla p^* + \frac{\rho}{2}\nabla u^{*2} = \mathbf{j}^* \times \mathbf{B}^* + \rho(\mathbf{u}^* \times \omega^*) . \tag{59}$$

$$\nabla \times (\, \mathbf{u}^* \times \mathbf{B}^* \,) = 0 . \tag{60}$$

We find from eqs.(55) and (56) that the eigenfunctions $\mathbf{u}^*$ and $\mathbf{B}^*$ for the present two dissipative dynamic operators, $-\ \nu\nabla \times \nabla \times \mathbf{u}$ and $-\ \nabla \times (\eta\mathbf{j})$, constitute the self-organized and self-similar decay phase with the minimum dissipation rate and with equilibrium equations of eqs.(59) and (60) during the time evolution of the present dynamic system. We see from eq.(53) that the factor α of eqs.(50) and (51), which is the Lagrange multiplier, is equal to the decay constant of energy W_{ii} at the self-organized and self-similar decay phase.

Refering to eqs.(13) and (22) - (24) for discussion of the bifurcation point of dissipative structure, we obtain two associated eigenvalue problems from eq.(48) for critical perturbations $\delta\mathbf{u}$ and $\delta\mathbf{B}$ that make $\delta^2 F$ vanish, and the condition for the state with the minimum dissipation rate that corresponds to eq.(24), as follows:

$$\nabla \times \nabla \times \delta\mathbf{u}_k \ - \ \frac{\alpha_k \rho}{2\nu}\,\delta\mathbf{u}_k \ = \ 0\,, \tag{61}$$

$$\nabla \times (\eta\nabla \times \delta\mathbf{B}_k) \ - \ \frac{\mu_0\beta_k}{2}\delta\mathbf{B}_k \ = \ 0, \tag{62}$$

$$0 \ < \ \alpha \ < \ \alpha_1 \ \text{and} \ \beta_1\,. \tag{63}$$

Here, α_k and β_k are eigenvalues, $\delta\mathbf{u}_k$ and $\delta\mathbf{B}_k$ denote the eigensolutions, α_1 and β_1 are the smallest positive eigenvalue of α_k and β_k, respectively, the boundary conditions are $\delta\mathbf{u}_w \cdot d\mathbf{S} = 0$, $[\ \delta\mathbf{u}_w \times (\nabla \times \delta\mathbf{u}_w)\ ] \cdot d\mathbf{S} = 0$, $\delta\mathbf{B}_w \cdot d\mathbf{S} = 0$ and $[\eta(\nabla \times \delta\mathbf{B}_w) \times \delta\mathbf{B}_w] \cdot d\mathbf{S}$ $= 0$. Since the dissipative operator $-\ \nabla \times (\eta\mathbf{j})$ satisfies again the the self-adjoint property of eq.(49), the eigenfunctions, $\mathbf{b}_k$, for the associated eigenvalue problem of eq.(62) for the magnetic field with the boundary conditions of $\mathbf{b}_{kw} \cdot d\mathbf{S} = 0$ and $[\eta(\nabla \times \mathbf{b}_{kw}) \times \mathbf{b}_{kw}] \cdot d\mathbf{S} = 0$ form a complete orthogonal set and the appropriate normalization is written as

$$\int \mathbf{b}_k \cdot [\nabla \times (\eta\nabla \times \mathbf{b}_j)]\, dV \ = \ \int \mathbf{b}_j \cdot [\nabla \times (\eta\nabla \times \mathbf{b}_k)]\, dV$$

$$= \frac{\mu_0 \beta_k}{2} \int \mathbf{b}_j \cdot \mathbf{b}_k \, dV$$

$$= \frac{\mu_0 \beta_k}{2} \, \delta_{jk} \, , \tag{64}$$

where $\nabla \times (\eta \nabla \times \mathbf{b}_k) - (\mu_0 \beta_k / 2) \, \mathbf{b}_k = 0$ is used. After spectrum transfers or spectrum spreadings toward the higher or wider mode number region by instabilities and field reconnections which are possibly followed by nonlinear saturation of perturbation growth, it is considered that the nondissipative operator becomes less dominant and the dissipative operator becomes more so. (Field reconnections have features to induce spectrum transfers toward both the lower and the higher mode number regions.) In this phase, $\mathbf{u}$ and $\mathbf{B}$ can be written by using eigensolutions $\mathbf{u}^*$ and $\mathbf{B}^*$ for the boundary value problem and orthogonal eigenfunctions $\mathbf{a}_k$ and $\mathbf{b}_k$ for eigenvalue problems, as follows;

$$\mathbf{u} = \mathbf{u}^* + \sum_{k=1}^{\infty} c_k \, \mathbf{a}_k \, , \tag{65}$$

$$\mathbf{B} = \mathbf{B}^* + \sum_{k=1}^{\infty} C_k \, \mathbf{b}_k \, . \tag{66}$$

Substituting eqs.(65) and (66) into eqs.(45) and (46) and using eqs.(50), (51), (61) and (62) , we obtain the followings:

$$\frac{\partial \mathbf{u}^*}{\partial t} + \sum_{k=1}^{\infty} \frac{\partial (c_k \mathbf{a}_k)}{\partial t} = L_1^N[\mathbf{q}] - \frac{\alpha}{2} \mathbf{u}^* - \sum_{k=1}^{\infty} \frac{\alpha_k}{2} c_k \mathbf{a}_k \, , \tag{67}$$

$$\frac{\partial \mathbf{B}^*}{\partial t} + \sum_{k=1}^{\infty} \frac{\partial (C_k \mathbf{b}_k)}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \frac{\alpha}{2} \mathbf{B}^* - \sum_{k=1}^{\infty} \frac{\beta_k}{2} C_k \mathbf{b}_k \, , \tag{68}$$

where $L_1^N[\mathbf{q}]$ [$\equiv \mathbf{j} \times \mathbf{B} - \nabla p - (\rho/2) \nabla u^2 + \rho \mathbf{u} \times \omega$] and $\nabla \times (\mathbf{u} \times \mathbf{B})$ act now as less dominant operators, the eigenvalues α_k and β_k are positive, and α_1 and β_1 are the smallest positive eigenvalues. We find again from eqs.(67) and (68) that selective dissipations for the higher eigenmode components give us a detailed physical picture of the self-organization and the bifurcation of the dissipative structure, in the same way as was shown after eq.(38) in the previous section. If $\alpha <$ (α_1 and β_1), then through interchange between the two of the flow and the magnetic energies by the two terms of $\mathbf{j} \times \mathbf{B}$ and $\nabla \times (\mathbf{u} \times \mathbf{B})$ in eqs.(45) and (46), and after catching up slower decay component of the two energies by the other faster one, the basic components of $\mathbf{u}^*$ and $\mathbf{B}^*$ with the same value of α remain last. The bifurcation of the self-organized dissipative structure takes place when the value of α becomes equal to the lower one of α_1 and β_1, where the mixed mode with ($\mathbf{u}^*$ and $\mathbf{B}^*$) and the corresponding lowest eigenmode ($\mathbf{a}_1$ or $\mathbf{b}_1$) remains last.

In the same way as was used at eq.(41), two eqs.(50) and (52) can be shown to have the following helical solutions:

$$\nabla \times \mathbf{u}^* = \kappa \mathbf{u}^* \quad (\mid \kappa \mid \equiv \sqrt{\frac{\alpha \rho}{2\nu}}) . \tag{69}$$

$$\nabla \times \mathbf{B}^* = \lambda \mathbf{B}^* \quad (\mid \lambda \mid \equiv \sqrt{\frac{\alpha \mu_0}{2\eta}}) . \tag{70}$$

In this spacial case, $\mathbf{u}^* \times \omega^* = 0$ and $\mathbf{j}^* \times \mathbf{B}^* = 0$, and then the equilibrium equation, eq.(59), becomes:

$$\nabla p^* + \frac{\rho}{2} \nabla (u^*)^2 = 0 . \tag{71}$$

In more general cases, $\mathbf{u}^*$ and $\mathbf{B}^*$ contain other components so that $\mathbf{u}^* \times \omega^* \neq 0$ and $\mathbf{j}^* \times \mathbf{B}^* \neq 0$.

In the same way as was used at eqs.(43) and (44), when self-organized relaxed states of interest have some kind of symmetry along one coordinate x_s in $\mathbf{x}$ or depend on only two dimensional variables perpendicular to x_s, i.e. $\partial / \partial x_s = 0$ (two dimensional systems are included in this case), then two eqs.(50) and (51) can be separated into two mutually independent equations, by using two components of $\mathbf{u}_s^*$ and $\mathbf{B}_s^*$ along x_s, and $\mathbf{u}_{s\perp}^*$ and $\mathbf{B}_{s\perp}^*$ perpendicular to x_s, as follows:

$$\nabla \times \nabla \times \mathbf{u}_s^* = \frac{\alpha\rho}{2\nu}\mathbf{u}_s^* , \tag{72}$$

$$\nabla \times \nabla \times \mathbf{u}_{s\perp}^* = \frac{\alpha\rho}{2\nu}\mathbf{u}_{s\perp}^* . \tag{73}$$

$$\nabla \times (\eta\nabla \times \mathbf{B}_s^*) = \frac{\alpha\mu_0}{2}\mathbf{B}_s^* , \tag{74}$$

$$\nabla \times (\eta\nabla \times \mathbf{B}_{s\perp}^*) = \frac{\alpha\mu_0}{2}\mathbf{B}_{s\perp}^* , \tag{75}$$

where $\mu_0\mathbf{j} = \nabla \times \mathbf{B}$ is used. In three dimensional systems, when self-organized states with uniform η have a feature of $\sqrt{\alpha\mu_0/2\eta}\ \mathbf{B}_s^* = \nabla \times \mathbf{B}_{s\perp}^*$, then the total field of $\mathbf{B}^* = \mathbf{B}_s^* + \mathbf{B}_{s\perp}^*$ can be shown straightforward to constitute solutions of the helical force-free field of eq.(70), by using eq.(75), in the same way as was used for $\mathbf{u}^*$ after eq.(44).

V. ATTRACTORS IN COMPRESSIBLE RESISTIVE MHD PLASMAS

We show here the third application of the general theory in Section II to compressible resistive MHD plasmas described by the following simplified equations:

$$\rho\frac{d\mathbf{u}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p, \tag{76}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta\mathbf{j}), \tag{77}$$

where the viscosity is assumed to be negligibly small. In this system, W_{ii} and its dissipation rate $\partial W_{ii}/\partial t$ are written respectively as $W_{ii} = 2\int[B^2/2\mu_0 + \rho u^2/2]dV$ and $\partial W_{ii}/\partial t = -(2/\mu_0)\int[\mathbf{B} \cdot \nabla \times (\eta\mathbf{j})]dV$. We assume here, for simplicity, that the resistivity η at the relaxed state has a fixed spatial dependence like as $\eta(\mathbf{x})$, as is indeed the case in all experimental plasmas where η goes up to infinity near the boundary wall. Substituting those of W_{ii} and $\partial W_{ii}/\partial t$ into eqs.(6) - (11) in the same way as was used at eqs.(47) and (48), and taking account of compressible ρ, we obtain the followings:

$$\delta F = \frac{2}{\mu_0}\int\{2\delta\mathbf{B}\cdot[\nabla\times(\eta\mathbf{j}) - \frac{\alpha}{2}\mathbf{B}] - \alpha\mu_0(\delta\rho\frac{u^2}{2} + \rho\delta\mathbf{u}\cdot\mathbf{u})\}dV + \frac{2}{\mu_0}\oint(\eta\delta\mathbf{B}\times\mathbf{j} + \eta\delta\mathbf{j}\times\mathbf{B})\cdot d\mathbf{S} = 0, \tag{78}$$

$$\delta^2 F = \frac{2}{\mu_0}\int\{\delta\mathbf{B}\cdot[\nabla\times(\eta\delta\mathbf{j}) - \frac{\alpha}{2}\delta\mathbf{B}] - \alpha\mu_0(\delta\rho\delta\mathbf{u}\cdot\mathbf{u} + \rho\frac{\delta u^2}{2})\}dV > 0\ . \tag{79}$$

We obtain the Euler-Lagrange equation from the volume integral term in eq.(78) for arbitrary variations of $\delta\mathbf{B}$, $\delta\rho$ and $\delta\mathbf{u}$ as follows:

$$\nabla\times(\eta\mathbf{j}^*) = \frac{\alpha}{2}\mathbf{B}^*\ , \tag{80}$$

$$u^* = 0\ , \qquad \rho^*\mathbf{u}^* = \mathbf{0}\ , \tag{81}$$

where eq.(80) is the same with eq.(51) in Section IV. Using eqs.(80) and (81), and refering to eqs.(17) - (20), we obtain the following:

$$\frac{\partial W_{ii}{}^*}{\partial t} = -\alpha\int\frac{(\mathbf{B}^*)^2}{\mu_0}dV = -\alpha\, W_{ii}{}^*\ , \tag{82}$$

$$W_{ii}^* = e^{-\alpha t}W_{iiR}{}^* = \int\frac{[\mathbf{B}_R{}^*(\mathbf{x})e^{-\frac{\alpha}{2}t}]^2}{\mu_0}dV\ , \tag{83}$$

$$\mathbf{B}^* = \mathbf{B}_R{}^*(\mathbf{x})\, e^{-\frac{\alpha}{2}t}\ , \tag{84}$$

$$\frac{\partial\mathbf{B}^*}{\partial t} = -\frac{\alpha}{2}\mathbf{B}^* = -\nabla\times(\eta\mathbf{j}^*)\ , \tag{85}$$

where eqs.(84) and (85) are the same with eqs.(56) and (58) in Section IV. Substituting $\mathbf{u}^*$ and $\mathbf{B}^*$ into eqs.(76) and (77), and using eqs.(80), (81) and (85), we obtain the equilibrium equation at $t = t_R$:

$$\nabla p^* = \mathbf{j}^* \times \mathbf{B}^* , \tag{86}$$

$$\nabla \times (\mathbf{u}^* \times \mathbf{B}^*) = \mathbf{0} . \tag{87}$$

We find again from eq.(84) that the eigenfunction $\mathbf{B}^*$ for the present dissipative dynamic operator, $- \nabla \times (\eta \mathbf{j})$, constitutes the self-organized and self-similar decay phase with the minimum dissipation rate and with equilibrium equations of eqs.(86) and (87) during the time evolution of the present dynamic system. We also see from eq.(82) that the factor α of eq.(80), which is the Lagrange multiplier, is equal to the decay constant of energy W_{it} at the self-organized and self-similar decay phase, as was shown at eqs.(16) - (21) in the general self-organization theory.

Refering to eqs.(13) and (22) - (24) for discussion of the bifurcation point of dissipative structure, we obtain again the associated eigenvalue problem from eq.(79) for critical perturbations $\delta\mathbf{B}$ that make $\delta^2 F$ vanish, and the condition for the state with the minimum dissipation rate that corresponds to eq.(24), as follows:

$$\nabla \times (\eta \nabla \times \delta\mathbf{B}_k) - \frac{\mu_0 \beta_k}{2} \delta\mathbf{B}_k = 0, \tag{88}$$

$$0 < \alpha < \beta_1 , \tag{89}$$

where eq.(88) is the same with eq.(62) in Section IV with the boundary conditions of $\delta\mathbf{B}_w \cdot d\mathbf{S} = 0$ and $[\eta(\nabla \times \delta\mathbf{B}_w) \times \delta\mathbf{B}_w] \cdot d\mathbf{S} = 0$. In the same way as was used at eqs.(64), (66) and (68), we obtain the same eigenmode expansion of $\mathbf{B}$ by the eigenslution $\mathbf{B}^*$ for the boundary value problem and the orthogonal eigenfunction $\mathbf{b}_k$ for eigenvalue problems, and also the same field equation, as follows;

$$\mathbf{B} = \mathbf{B}^* + \sum_{k=1}^{\infty} C_k \, \mathbf{b}_k , \tag{90}$$

$$\frac{\partial \mathbf{B}^*}{\partial t} + \sum_{k=1}^{\infty} \frac{\partial (C_k \mathbf{b}_k)}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \frac{\alpha}{2}\mathbf{B}^* - \sum_{k=1}^{\infty} \frac{\beta_k}{2} C_k \mathbf{b}_k . \tag{91}$$

We find again from eq.(91) that selective dissipations for the higher eigenmode components give us a detailed physical picture for the self-organization process and the bifurcation of the dissipative structure at $\alpha = \beta_1$, in the same way as was shown after eq.(68) in the previous section. The flow energy in the present system dissipates to vanish by the dissipation term of $- \nabla \times (\eta \mathbf{j})$ in eq.(77) through interchange between the two of the flow and the magnetic energies by the two terms of $\mathbf{j} \times \mathbf{B}$ and $\nabla \times (\mathbf{u} \times \mathbf{B})$ in eqs.(76) and (77).

In the same way as was used at eq.(41), eq.(80) can be shown to have the same force-free field solution with eq.(70) for the case with spatially uniform η :

$$\nabla \times \mathbf{B}^* = \lambda \mathbf{B}^* \quad (\, | \, \lambda \, | \equiv \sqrt{\frac{\alpha \mu_0}{2\eta}} \,) . \tag{92}$$

In this spacial case, $\mathbf{j}^* \times \mathbf{B}^* = 0$, and then the equilibrium equation, eq.(86), becomes:

$$\nabla p^* = 0 . \tag{93}$$

In more general cases, $\mathbf{B}^*$ contains other components so that $\mathbf{j}^* \times \mathbf{B}^* \neq 0$.

In the same way as was used at eqs.(43) and (44), when self-organized relaxed states of interest have some kind of symmetry along one coordinate x_s in $\mathbf{x}$ or depend on only two dimensional variables perpendicular to x_s, i.e. $\partial / \partial x_s = 0$ (two dimensional systems are included in this case), then eq.(80) can be separated again into the same two mutually independent equations with eqs.(74) and (75), as follows:

$$\nabla \times (\eta \nabla \times \mathbf{B}_s^*) = \frac{\alpha \mu_0}{2} \mathbf{B}_s^* , \tag{94}$$

$$\nabla \times (\eta \nabla \times \mathbf{B}_{s\perp}^*) = \frac{\alpha \mu_0}{2} \mathbf{B}_{s\perp}^* . \tag{95}$$

Conventional notations of $\mathbf{B}^*_s \equiv \mathbf{B}^*_t$ (toroidal component) and $\mathbf{B}^*_{s\perp} \equiv \mathbf{B}^*_p$ (poloidal component) are used for the case of toroidal symmetric relaxed states. The field reversal configuration (FRC) branch ($\mathbf{B}^*_t = 0$) of relaxed states of plasmas observed in merging experiments of two spheromaks shown in Fig.2 in [8] can be represented by eq.(95).

As was shown after eq.(75), when self-organized states with uniform η in three dimensional system have a feature of $\sqrt{\alpha\mu_0/2\eta}\,\mathbf{B}^*_s = \nabla \times \mathbf{B}^*_{s\perp}$, then the total field of $\mathbf{B}^* = \mathbf{B}^*_s + \mathbf{B}^*_{s\perp}$ can be shown straightforward to constitute solutions of the helical force-free field of eq.(92), by using eq.(95). This force-free field is realized approximately in experimental low β plasmas (i.e. no pressure gradient of $\nabla p^* = 0$) when spatially uniform resistivity η is assumed. In more general cases with nonumiform η, substituting $\mathbf{j}^* = \mathbf{j}^*_{\|} + \mathbf{j}^*_{\perp}$ and $\mu_0\mathbf{j}^*_{\|} = f(\mathbf{x})\mathbf{B}^*$ into $\nabla \times (\eta\mathbf{j}^*) = (\alpha/2)\mathbf{B}^*$ of eq.(80), using $\mu_0\mathbf{j} = \nabla \times \mathbf{B}$, and comparing the factor of $\mathbf{B}^*$, we obtain the following approximate solution for $\mathbf{j}^*_{\|}$ at the self-organized relaxed state:

$$\mu_0\mathbf{j}^*_{\|} \simeq \sqrt{\frac{\mu_0\alpha}{2\eta}}\,\mathbf{B}^* , \tag{96}$$

where the subscripts $\|$ and $\perp$ denote respectively the parallel and the perpendicular components to the field $\mathbf{B}^*$. As was reported in [26], comparison between this theoretical result of eq.(96) with 3-D MHD simulations with both "nonuniform η" and "uniform η" supports this dependence of $\mathbf{j}^*_{\|}$ on η profiles.

VI. SUMMARY

As one of universal mathematical structures embedded in dissipative dynamic systems, we have presented a more refined general theory on attractors of the dissipative structure in Section II, and have clarified that realization of coherent structures in time evolution, which is expressed by eq.(4) with use of auto-correlations, is equivalent to that of self-organized states with the minimum dissipation rate for instantaneously contained W_{ii}, expressed by eq.(7). It is seen from comparison between eqs.(5)- (7) and eqs.(1) - (3) that this coherent structure of the self-organized state with the minimum dissipation rate is determined essentially by the equations of the dynamic system themselves, which rule the time evolution of the system, and key terms are dissipative dynamic operators $L_i^D[\mathbf{q}]$ in the system. We find from the variational calculus of eqs.(9) - (16) and from eqs.(17) - (21) that attractors of the dissipative structure are given by eigenfunctions q_i^* of eq.(16) for dissipative operators $L_i^D[\mathbf{q}]$, they constitute the self-organized and self-similar decay phase with the minimum dissipation rate and with equilibrium states of eq.(21), and the Lagrange multiplier α becomes equal to the decay constant of W_{ii} in this phase. The bifurcation point of the dissipative structure is generally given by $\alpha = \alpha_1$ with use of the smallest positive eigenvalue α_1 of the associated eigenvalue problem of eq.(22).

We have presented three typical examples of applications of the present general theory to incompressible viscous fluids in Section III, to incompressible viscous and resistive MHD fluids such as liquid metals in Section IV and to compressible resistive MHD plasmas in Section V, and have derived attractors of the dissipative structure in these dissipative fluids. All of the attractors in the three dissipative fluids have been clarified to have the same features with those of attractors in the general theory mentioned above. Using eigensolutions of basic modes for boundary value problems and complete orthogonal sets by eignenfunctions for associated eigenvalue problems for the three dissipative fluids, we have presented detailed physical pictures of the self-organization of these dynamic systems approaching basic modes and also of the bifurcation of the dissipative structures from basic modes to mixed modes. Those physical pictures consist of two common fundamental processes; the first is spectrum transfers or spectrum spreadings toward both the higher and the lower eigenmode regions for dissipative dynamic operators, caused by such as instabilities and field reconnections, and the second is selective dissipations by higher eigenmodes associated with dissipative operators.

Corresponding to the Fourier spectrum analysis shown in [23–25], eqs.(40), (67), (68) and (91) with use of eigenmode expansions suggest us that an eigenfunction spectrum analysis associated with dissipative dynamical operators $L_i^D[\mathrm{q}]$ will be useful to understand self-organization processes. This type of eigenfunction spectrum analysis for our computer simulations of self-organization processes in resistive MHD plasmas [26] and in incompressible viscous fluids are under investigations, whose results will be reported elsewhere.

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