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(Received – Jan. 17, 1994)

NIFS-271

Feb. 1994

### RESEARCH REPORT NIFS Series

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Large amplitude Langmuir  
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Large amplitude Langmuir and ion-acoustic waves in a relativistic two-fluid plasma are analysed by the pseudo-potential method. The existence conditions for relativistic nonlinear Langmuir waves depend on the relativistic effect, the particular energy and the ion mass to electron mass ratio. The allowable range of the normalized potential depends on the relativistic effect. It is shown that the Mach number has the significant effect for the formation of relativistic nonlinear ion-acoustic waves rather than the ratio of the ion-acoustic velocity to the velocity of light. The allowable range of the normalized potential depends on the Mach number. The present investigation predicts new findings of relativistic nonlinear Langmuir and ion-acoustic waves in plasmas in which high-speed electrons and ions coexist.

Key Words: Langmuir Wave, Ion Acoustic Wave, Relativistic Two-Fluid Plasma,  
Pseudo-Potential Method

## 1. Introduction

In the recent space observations, it has been investigated that the high-speed streaming ions as well as the high-speed electrons play a major role in the physical mechanism for the nonlinear wave structures. When we assume that the ion and electron energies depend only on the kinetic energy, velocities of plasma particles in the solar atmosphere and the magnetosphere have to attain to relativistic speeds<sup>1-2</sup>. Thus, by considering such relativistic effects as the ion and electron velocities are about  $0.01c-0.1c$  ( $c$  is the velocity of light), we can take the relativistic motion of such particles into consideration in the study of nonlinear plasma waves. When the velocity of the particles approaches that of light, the nonlinear waves which occur in the space exhibit a peculiar feature due to the effect of the high-speed ions<sup>3-4</sup>.

Although relativistic Langmuir waves have been studied as the subjects of laser-plasma interaction and laboratory experiments, the studies on relativistic ion acoustic waves are also rapidly developing for recent several years in the space plasma<sup>5-7</sup>. In space, the low frequency solitary waves are frequently observed and the modulational stable and unstable waves in the finite wavenumber region are also detected<sup>8-9</sup>. We have presented the theories of relativistic ion acoustic waves to explain these phenomena<sup>4-8</sup>. In the actual situations, there exist not only high-speed streaming ions but also high-speed streaming electrons, and they cause the excitation of various kinds of nonlinear waves such as shock and solitary waves in the interplanetary space and the Earth's magnetosphere. The conventional investigations which treat the relativistic effect of only electrons or that of only ions have not been completed in considering the energetic particle phenomena of space plasmas. No one can truly understand these

phenomena without investigating both effects of relativistic electrons and ions. Hence, in this article, we consider a relativistic two-fluid plasma composed of the high-speed relativistic electrons and ions.

The purpose of this paper is to derive the pseudo(Sagdeev)-potential for a relativistic two-fluid plasma, to exhibit the possibility of the existence of novel solutions of relativistic nonlinear Langmuir and ion-acoustic waves. We show that, by considering both relativistic electrons and ions, the stationary nonlinear potential solutions in the plasma can be formulated in terms of an integral equation which expresses the same form as the equation governing the motion of particles in a potential well. In this paper, it is predicted that large amplitude relativistic nonlinear Langmuir waves coexist with relativistic ion-acoustic waves.

The layout of this paper is as follows. In section 2, we present the basic equations for a relativistic, two-fluid plasma and derive an energy equation with the effective potential. In section 3, we discuss the existence condition of large amplitude relativistic nonlinear Langmuir waves on the basis of the energy equation. The dependency of the pseudo-potential on the normalized potential, the relativistic effect and the particular energy is presented. In section 4, we study large amplitude relativistic nonlinear ion-acoustic waves. The effective potential structure is analyzed in Mach number and potential space. We show the existence regions of large amplitude relativistic ion-acoustic waves in relation to the Mach number, the normalized potential and the relativistic effect. The last section is devoted to the concluding discussions.

## 2. Basic equations

We consider an unmagnetized, collisionless relativistic two-fluid plasma consisting of the hot and isothermal electrons and cold ions. We do not take into account kinetic effects such as the deviation from the Maxwell distribution, Landau damping, etc, and assume the electron and ion flow velocities are relativistic, and thereby there exist high-speed streaming electrons and ions in an equilibrium state.

The equations of continuity, the equations of motion and Poisson's equation of a fully relativistic two-fluid plasma in a one-direction are described<sup>5, 10</sup> as:

$$\frac{\partial}{\partial t} \left[ \gamma_e n_e \right] + \frac{\partial}{\partial x} \left[ \gamma_e n_e v_e \right] = 0 \quad (1.a)$$

$$\frac{\partial}{\partial t} \left[ \gamma_i n_i \right] + \frac{\partial}{\partial x} \left[ \gamma_i n_i v_i \right] = 0 \quad (1.b)$$

$$m_e \left[ \frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x} \right] (\gamma_e v_e) = e \frac{\partial \phi}{\partial x} - \frac{1}{n_e} \frac{\partial P_e}{\partial x} \quad (1.c)$$

$$m_i \left[ \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x} \right] (\gamma_i v_i) = -e \frac{\partial \phi}{\partial x} - \frac{1}{n_i} \frac{\partial P_i}{\partial x} \quad (1.d)$$

and

$$\epsilon_0 \frac{d^2 \phi}{dx^2} = e (n_e - n_i) \quad (1.e)$$

where

$$\gamma_{e,i} = \frac{1}{\left[ 1 - \left( \frac{v_{e,i}}{c} \right)^2 \right]^{1/2}}$$

$$\approx 1 + \frac{1}{2} \left( \frac{v_{e,i}}{c} \right)^2.$$

The subscript e and i denote electrons and ions, respectively. The electron and ion pressures are determined by  $P_e = \kappa T_e n_e$  and  $P_i = \kappa T_i n_i$ , respectively, where  $T_e(T_i)$  and  $n_e(n_i)$  denote the electron(ion) temperature and electron(ion) density.

In the stationary state, the conservation of electron and ion fluxes are obtained as

$$n_e v_e \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right] = C_1 \quad (2.a)$$

$$n_i v_i \left[ 1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2 \right] = C_2, \quad (2.b)$$

from eqs.(1.a) and (1.b). The equations for conservation of electron and ion energy are described as:

$$\frac{m_e v_e^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_e}{c} \right)^2 \right] - e\phi + \kappa T_e \ln n_e = C_3 \quad (2.c)$$

$$\frac{m_i v_i^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_i}{c} \right)^2 \right] + e\phi + \kappa T_i \ln n_i = C_4 \quad (2.d)$$

which are obtained by integrating eqs.(1.c) and (1.d), in the stationary state.  $C_1 \sim C_4$  are integration constants.  $T_e$  and  $T_i$  are assumed to be constant.

Adding eq.(1.c) to (1.d), using eq.(1.e) and integrating, the conservation law of total pressure is given as

$$m_e n_e v_e^2 \left[ 1 + \frac{5}{6} \left( \frac{v_e}{c} \right)^2 \right] + m_i n_i v_i^2 \left[ 1 + \frac{5}{6} \left( \frac{v_i}{c} \right)^2 \right] + \kappa T_e n_e + \kappa T_i n_i - \varepsilon_0 \frac{1}{2} \left( \frac{d\phi}{dx} \right)^2 = -C_5. \quad (3)$$

The derivation of eq.(3) is discussed in Appendix.

From eq. (3), one obtain

$$\frac{1}{2} \left[ \frac{d\phi}{dx} \right]^2 + V = 0 \quad (4)$$

where the pseudo-potential is given by

$$V = U - W \quad (5)$$

and

$$U = -\frac{1}{\epsilon_0} \left\{ \kappa T_e n_e + \kappa T_i n_i + m_e n_e v_e^2 \left[ 1 + \frac{5}{6} \left[ \frac{v_e}{c} \right]^2 \right] + m_i n_i v_i^2 \left[ 1 + \frac{5}{6} \left[ \frac{v_i}{c} \right]^2 \right] \right\} \quad (6)$$

where

$$W = \frac{C_s}{\epsilon_0} \quad (7)$$

Equation (4) represents an energy equation for a classical particle moving with the velocity  $d\phi/dx$  in a potential  $V(\phi)$ .

We study the nonlinear potential structures for Langmuir wave and ion-acoustic wave on the basis of eqs. (4) and (5) in the following sections.

### 3. Relativistic nonlinear Langmuir waves

We consider large amplitude relativistic Langmuir waves in the

case that the electron inertia is important. We assume that the electron flux and the ion flux are equal, that is,

$$n_e v_e = n_i v_i = \varphi_0 . \quad (8)$$

In the case of  $v_e = v_i = v_0$  at  $\phi = 0$  and  $T_e = T_i = 0$ , from eqs. (2.c) and (2.d), the integration constants  $C_3$  and  $C_4$  are given as

$$C_3 = \frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right] ,$$

$$C_4 = \frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right] ,$$

where  $v_0$  is the constant velocity. In this case, eqs. (2.c) and (2.d) reduce to

$$\frac{m_e v_e^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_e}{c} \right)^2 \right] - e\phi = \frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right] , \quad (9.a)$$

and

$$\frac{m_i v_i^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_i}{c} \right)^2 \right] + e\phi = \frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right] . \quad (9.b)$$

Using eqs. (9.a) and (9.b), we get, from eqs. (2.c) and (2.d),

$$\frac{v_e}{v_0} \approx \left[ 1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} , \quad (10.a)$$

$$\frac{v_i}{v_0} \approx \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} , \quad (10.b)$$



where we approximated  $[1+(3/4)(v_0/c)^2] \approx [1+(3/4)(v_{e,1}/c)^2]$ .  
Then the potential  $U(\phi)$  reduces to

$$\begin{aligned}
U(\phi) = & -\omega_e^2 v_0^2 \left[ \frac{m_e}{e} \right]^2 \left\{ \left[ 1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} \right. \\
& \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left[ 1 + \frac{e\phi}{\frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right] \right] \\
& + \frac{m_i}{m_e} \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} \\
& \left. \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right] \right] \right\} \quad (11)
\end{aligned}$$

by using eqs.(10.a) and (10.b), where we define  $\omega_e = (n_e e^2 / \epsilon m_e)^{1/2}$  and this is the electron plasma frequency.

The oscillatory solution of the nonlinear Langmuir wave exist when the following two conditions are satisfied:

(i)The potential  $U(\phi)$  has a minimum value  $W_{min}$  at  $\phi=0$ .

The minimum energy  $W_{min}$  is

$$\begin{aligned}
W_{min} &= U(\phi=0) \\
&= -\omega_e^2 v_0^2 \left[ \frac{m_e}{e} \right]^2 \left[ 1 + \frac{m_i}{m_e} \right] \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \right]. \quad (12)
\end{aligned}$$

Nonlinear Langmuir waves exist provided that the constant energy  $W$  in eq.(7) exceeds a minimum value  $W_{min}$ .

(ii) The maximum potential  $W_{\max}$  satisfies when  $W_{\max} = U(\phi_c)$ , where  $\phi_c = -(m_e v_0^2 / 2e) [1 + (3/4)(v_0/c)^2]$ . Here  $W_{\max}$  is given as

$$\begin{aligned}
 W_{\max} &= U(\phi_c) \\
 &= -\omega_e^2 v_0^2 \left( \frac{m_e}{e} \right)^2 \frac{m_i}{m_e} \left( 1 + \frac{m_e}{m_i} \right)^{1/2} \\
 &\quad \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left( 1 + \frac{m_e}{m_i} \right) \right]. \quad (13)
 \end{aligned}$$

Oscillatory Langmuir soliton solutions exist if  $W$  is less than the maximum value  $W_{\max}$ . From eqs.(12) and (13), we obtain the relativistic nonlinear Langmuir wave by an arbitrary choice of  $W$  in the range

$$\begin{aligned}
 -\omega_e^2 v_0^2 \left( \frac{m_e}{e} \right)^2 \left( 1 + \frac{m_i}{m_e} \right) \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \right] &< W \\
 < -\omega_e^2 v_0^2 \left( \frac{m_e}{e} \right)^2 \frac{m_i}{m_e} \left( 1 + \frac{m_e}{m_i} \right)^{1/2} \\
 &\quad \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left( 1 + \frac{m_e}{m_i} \right) \right]. \quad (14)
 \end{aligned}$$

Large amplitude nonlinear Langmuir waves can propagate when  $W$  satisfies eq.(14). The energy  $W$  depends on  $m_i/m_e$  and the relativistic effect  $v_0/c$ .

It should be noted that  $U(\phi)$  is real, if

$$-1 < \frac{e\phi}{\frac{m_e v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} < \frac{m_i}{m_e}.$$

We show a Bird's eye view of  $V(\phi)/(\omega_e^2 v_o^2 (m_e/e)^2)$  when  $W=1849.3$ , in Fig.1. Figure 1 illustrates the dependence on the normalized potential  $e\phi/(m_e v_o^2/2)$  and the effect of the streaming velocity  $v_o/c$  of the normalized pseudo-potential  $V(\phi)/(\omega_e^2 v_o^2 (m_e/e)^2)$ . From Figs.1-3, we can understand the following:

- 1) In the range of  $v_o/c < 0.144$ ,  $V(\phi)/(\omega_e^2 v_o^2 (m_e/e)^2)$  is always positive. In this case, the potential well does not exist.
- 2) If  $0.144 < v_o/c < 0.156$ ,  $V(\phi)/(\omega_e^2 v_o^2 (m_e/e)^2)$  forms the potential well. In the potential well, large amplitude relativistic nonlinear Langmuir waves can exist. For the calculation, we assume that the ion is proton, that is  $m_i/m_e \approx 1836$ . As an example of this case, we illustrate the pseudo-potential in Fig.2 when  $v_o/c = 0.150$  and  $W = 1870.5$ .
- 3) In the range of  $0.156 < v_o/c$ ,  $V(\phi)/(\omega_e^2 v_o^2 (m_e/e)^2)$  is always negative. We show a typical example of this case in Fig.3, where  $v_o/c = 0.20$  and  $W = 1898.4$ .

We illustrate the existence region of large amplitude relativistic nonlinear Langmuir waves depending on the energy  $W$  and the relativistic effect  $v_o/c$  in Fig.4. Langmuir waves propagate in the shaded region. Figure 5 shows the existence region of nonlinear Langmuir waves as the relation between the normalized potential  $e\phi/(m_e v_o^2/2)$  and the relativistic effect, where  $W = 1837.5$ . Large amplitude Langmuir waves exist in the region A but not exist in the region B.

We now understand that large amplitude relativistic nonlinear Langmuir waves can propagate under proper conditions, mentioned above.

#### 4. Relativistic ion-acoustic waves

We consider large amplitude relativistic ion-acoustic waves. Since the motion of massive ions will be involved, these will be low-frequency oscillations. The electrons move so fast relative to the ions and the electrons therefore are isothermal.

In order to obtain the pseudo-potential for the relativistic ion-acoustic wave, we focus our attention again on eq.(3). In the following discussions, we assume that the electron flux is equal to the ion flux, that is,

$$n_e v_e = n_i v_i = \varphi_0 . \quad (15)$$

We consider the case that  $m_e=0$ ,  $T_i=0$  and  $n_e=n_0$  at  $\phi=0$ . In this case, we can obtain the electron density distribution from eq.(2.c) as

$$n_e = n_0 \exp \left[ \frac{e\phi}{k T_e} \right] . \quad (16)$$

Since we consider the case,  $v_e = v_i = v_0$  at  $\phi=0$ , we can rewrite eq.(2.d) to

$$\frac{m_i v_i^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_i}{c} \right)^2 \right] + e\phi = C_4 = \frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right] , \quad (17)$$

which yields

$$\frac{v_i}{v_0} \approx \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} , \quad (18)$$

in the same manner as the derivation of eq.(10.b), where we used that  $[1+(v_0/c)^2] \approx [1+(v_i/c)^2]$ . Using eq.(18), the term  $n_i m_i v_i^2 [1+(5/6)(v_i/c)^2]$  in eq.(6) reduces to

$$\begin{aligned}
 & n_i m_i v_i^2 \left[ 1 + \frac{5}{6} \left( \frac{v_i}{c} \right)^2 \right] \\
 &= m_i \varphi_0 v_0 \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} \\
 & \quad \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right] \right] \quad . \quad (19)
 \end{aligned}$$

Substituting eqs.(16) and (19) into eq.(6), we obtain the pseudo-potential as

$$\begin{aligned}
 U(\phi) = & -\frac{1}{\varepsilon_0} \left\{ \kappa T_e n_0 \exp \left( \frac{e\phi}{\kappa T_e} \right) \right. \\
 & + m_i \varphi_0 v_0 \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right]^{1/2} \\
 & \quad \times \left. \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left[ 1 - \frac{e\phi}{\frac{m_i v_0^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right]} \right] \right] \right\} \quad . \quad (20)
 \end{aligned}$$

Equation (20) implies that the solution exists if

$$\frac{m_1 v_0^2}{2} \left[ 1 + \frac{3}{4} \left[ \frac{v_0}{c} \right]^2 \right] > e\phi ,$$

holds. The particular solution of the ion-acoustic wave is obtained by imposing the boundary conditions:

$$V(\phi=0) = V(\phi=\phi_0) = 0 ,$$

$$\left. \frac{dV}{d\phi} \right|_{\phi=0} = 0 .$$

The constant potential  $W$  is determined by the conditions that  $V(\phi) = d\phi/dx = 0$  and  $V(\phi) = U(\phi) - W = 0$  at  $\phi = 0$ , that is,

$$W = - \frac{1}{\varepsilon_0} \left\{ \kappa T_e n_0 + m_1 \varphi_0 v_0 \left[ 1 + \frac{5}{6} \left[ \frac{v_0}{c} \right]^2 \right] \right\} . \quad (21)$$

We here introduce the dimensionless variables as

$$\Phi = \frac{e\phi}{\kappa T_e} , \quad (22.a)$$

$$\mathcal{M}^2 = \frac{v_0^2}{\kappa T_e / m_1} = \frac{v_0^2}{c_s^2} , \quad (22.b)$$

where  $c_s$  and  $\mathcal{M}$  denote the ion-acoustic speed and the Mach number, respectively. In this case, the pseudo-potential is given by

$$\begin{aligned}
V(\Phi) = & -\omega_i^2 c_s^2 \left( \frac{m_i}{e} \right)^2 \left\{ \exp(\Phi) - 1 - \mathcal{M}^2 \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \right. \right. \\
& - \left. \left. \left[ 1 - \frac{\Phi}{\frac{\mathcal{M}^2}{2} \left( 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right)} \right]^{1/2} \right. \right. \\
& \left. \left. \left. \times \left[ 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \left[ 1 - \frac{\Phi}{\frac{\mathcal{M}^2}{2} \left( 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right)} \right] \right] \right] \right\} \right\}
\end{aligned}
\tag{23}$$

where the ion plasma frequency is defined by  $\omega_i = (n_0 e^2 / \epsilon_0 m_i)^{1/2}$ .

It should be noted that  $(v_0/c)^2 = \mathcal{M}^2 (c_s/c)^2$ , where  $c_s$  is the ion-acoustic velocity. Since the value of  $V(\Phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  is invariable even if we change  $c_s/c$  in the range of  $0.01 < c_s/c < 0.05$ , we show a Bird's eye view of the pseudo-potential in Fig.6 in the range of  $0.1 < \Phi < 0.8$  and  $1.2 < \mathcal{M} < 1.58$ . Figure 7 shows a Bird's eye view in the range of  $0.5 < \Phi < 1.28$  and  $1.50 < \mathcal{M} < 1.58$ . Here, we use  $c_s/c = 0.05$ . From Figs.6 and 7, we obtain the following facts:

1) In the range of  $1.00 < \mathcal{M} < 1.58$ , the pseudo-potential

$V(\Phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  varies from negative to positive as  $\Phi$  increases. That is,  $V(\Phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  forms a potential well. As an example, when  $\mathcal{M} \sim 1.25$ , the critical potential where  $V(\Phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  vanishes is  $\Phi \sim 0.65$ .

As two typical examples of this case, we illustrate pseudo-potential curves for  $\mathcal{M} = 1.20$  and  $1.50$  in Figs.8 and 9, respectively. The allowable range of relativistic ion-acoustic waves where  $V(\Phi)/(\omega_i^2 c_s^2 (m_i/e)^2) < 0$  is narrow when  $\mathcal{M}$  is small, but

it becomes wider as  $\mathcal{M}$  increases.

2) The potential well becomes deep as  $\mathcal{M}$  increases and  $\Phi$  increases. The bottom of the curves in Fig.6 and 7 shifts from small  $\Phi$  to large  $\Phi$  as  $\mathcal{M}$  increases.

Since the square root of eq.(23) is positive,  $\Phi < (\mathcal{M}^2/2) \times (1 + (3/4)(v_0/c)^2)$  is hold. Hence, figure 10 shows the allowable region of nonlinear ion-acoustic waves depending on the normalized potential  $\Phi$  and the Mach number  $\mathcal{M}$ . Large amplitude nonlinear ion-acoustic waves propagate in the region A but not propagate in the region B.

Next, we obtain the maximum potential

$$\Phi_{\text{max}} = \frac{\mathcal{M}^2}{2} \left[ 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right],$$

which is determined by the constraint that the square root of eq.(23) is positive.

The maximum Mach number  $\mathcal{M}_{\text{max}}$  is determined from the condition that the potential becomes well. That is,  $\mathcal{M}_{\text{max}}$  has to satisfy the condition

$$\exp \left[ \frac{\mathcal{M}^2}{2} \left( 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right) \right] < 1 + \mathcal{M}^2 \left( 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \right).$$

The maximum Mach number holds when

$$\exp \left[ \frac{\mathcal{M}_{\text{max}}^2}{2} \left( 1 + \frac{3}{4} \left( \frac{v_0}{c} \right)^2 \right) \right] = 1 + \mathcal{M}_{\text{max}}^2 \left( 1 + \frac{5}{6} \left( \frac{v_0}{c} \right)^2 \right).$$

The maximum Mach number  $\mathcal{M}_{\text{max}}$  lies in the vicinity of 1.60.

The minimum Mach number is determined by expanding the right-hand



side of eq.(23) in the small amplitude limit, such that  $\Phi \ll 1$ . We obtain the condition

$$\mathcal{M}^2 > \frac{1 + \frac{5}{6} \left[ \frac{v_0}{c} \right]^2}{\left[ 1 + \frac{3}{4} \left[ \frac{v_0}{c} \right]^2 \right]^2} ,$$

under the approximation that  $(5/6)(v_0/c)^2 \approx (3/4)(v_0/c)^2 \ll 1$ . The minimum Mach number holds when

$$\mathcal{M}_{\min}^2 = \frac{1 + \frac{5}{6} \left[ \frac{v_0}{c} \right]^2}{\left[ 1 + \frac{3}{4} \left[ \frac{v_0}{c} \right]^2 \right]^2} .$$

The minimum Mach number  $\mathcal{M}_{\min}$  lies in the vicinity of range of 1.0 .

Thus,  $\mathcal{M}_{\max}$  and  $\mathcal{M}_{\min}$  are almost invariable even if  $v_0/c$  varies.

## 5. Concluding Discussion

The nonlinear wave structures of large amplitude relativistic nonlinear Langmuir and ion-acoustic waves are studied in a relativistic two-fluid plasma.

First, we investigated the existence conditions for the stationary wave solutions of large amplitude relativistic Langmuir waves, by analysing the structure of pseudo-potential, which is illustrated in Figs.1-5. The results are briefly summarized as

follows:

- 1) The existence conditions for large amplitude relativistic nonlinear Langmuir waves strongly depend on the relativistic factor  $v_0/c$ , the energy  $W$ , the normalized potential  $e\phi/(m_e v_0^2/2)$  and the mass ratio  $m_i/m_e$ .
- 2) Since  $m_i/m_e$  is constant, the range of the energy  $W$  sensitively depends on the relativistic effect.
- 3) Large amplitude relativistic Langmuir waves can propagate if the relativistic effect lies in the range  $0.144 < v_0/c < 0.156$  and the energy  $W$  lies in the range  $-1837 < W < -1899$ .
- 4) The allowable range of the normalized potential where Langmuir waves exist depends on the relativistic effect.

Second, we studied the proper existence conditions of large amplitude nonlinear ion-acoustic waves. The pseudo-potential shown in Figs.6-10 confirms the existence of large amplitude nonlinear ion-acoustic waves. We obtained the existence conditions for large amplitude nonlinear ion-acoustic waves in detail by the calculation. The results are summarized as:

- 1) The existence conditions for large amplitude nonlinear ion-acoustic waves sensitively depend on the Mach number rather than the relativistic effect.
- 2) Large amplitude nonlinear ion-acoustic waves can exist if the Mach number lies in the proper range.
- 3) Large amplitude nonlinear ion-acoustic waves can propagate even if the high-speed streaming velocity  $v_0 \sim 0$ .
- 4) The pseudo-potential is sensitively dependent on the Mach number but independent of the change of the ratio of the ion-acoustic velocity to the velocity of light  $c_s/c$ .
- 5) The allowable range of the normalized potential where nonlinear ion-acoustic waves exist depends on the Mach number.

5) If the relativistic effect is neglected, eq.(4) with eq.(23) coincides with the result of Chen (eq.8-29 of Ref.11). In this case, the allowable range of the Mach number of large amplitude relativistic ion-acoustic waves lies in the range  $1.00 < \mathcal{M} < 1.58$ .

This investigation predicts new findings on large amplitude relativistic Langmuir and ion-acoustic waves such as shock and solitary waves in plasmas composed of relativistic high-speed streaming electrons and ions. Although we have no direct observational data of high-speed energetic events, the present theory is applicable to analyze large amplitude energetic shock and solitary waves associated with relativistic Langmuir and ion-acoustic waves which occur in space.

#### Acknowledgement

This work was a joint research effort with the National Institute for Fusion Science.

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Appendix

Detailed derivation of eq. (3)

In the stationary state, eqs. (1.c) and (1.d) become

$$m_e v_e \frac{\partial}{\partial x} \gamma_e v_e - e \frac{d\phi}{dx} + \kappa T_e \frac{1}{n_e} \frac{\partial n_e}{\partial x} = 0 \quad (\text{A.a})$$

and

$$m_i v_i \frac{\partial}{\partial x} \gamma_i v_i + e \frac{d\phi}{dx} + \kappa T_i \frac{1}{n_i} \frac{\partial n_i}{\partial x} = 0. \quad (\text{A.b})$$

It is obvious that the integration of (A.a) is eq. (2.c) and the integration of (A.b) is (2.d).

Multiplying (A.a) by  $n_e$  and (A.b) by  $n_i$ , we have

$$n_e m_e v_e \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right] v_e - n_e e \frac{d\phi}{dx} + \kappa T_e \frac{\partial n_e}{\partial x} = 0, \quad (\text{B.a})$$

and

$$n_i m_i v_i \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2 \right] v_i + n_i e \frac{d\phi}{dx} + \kappa T_i \frac{\partial n_i}{\partial x} = 0. \quad (\text{B.b})$$

Adding eq. (B.a) to eq. (B.b) and using Poisson's equation, we obtain

$$\begin{aligned} & n_e m_e v_e \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right] v_e + n_i m_i v_i \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2 \right] v_i \\ &= \epsilon_0 \frac{d\phi}{dx} \frac{d^2 \phi}{dx^2} - \kappa T_e \frac{\partial n_e}{\partial x} - \kappa T_i \frac{\partial n_i}{\partial x}. \end{aligned} \quad (\text{C})$$

In order to obtain eq.(3), we integrate eq.(C), and define its left-hand side and right-hand side as

$$L \equiv L_1 + L_2,$$

$$L_1 \equiv \int \left( n_e m_e v_e \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right] v_e \right) dx ,$$

$$L_2 \equiv \int \left( n_i m_i v_i \frac{\partial}{\partial x} \left[ 1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2 \right] v_i \right) dx ,$$

for the left-hand side and

$$R \equiv \int \left( \varepsilon_0 \frac{d\phi}{dx} \frac{d^2\phi}{dx^2} - \kappa T_e \frac{\partial n_e}{\partial x} - \kappa T_i \frac{\partial n_i}{\partial x} \right) dx$$

for the right-hand side, respectively.

First, we consider  $L_1$ , that is,

$$\begin{aligned} L_1 &\equiv \int n_e m_e \frac{\partial}{\partial x} \left( \frac{v_e^2}{2} \right) dx + \frac{3}{2c^2} \int n_e m_e \frac{\partial}{\partial x} \left( \frac{v_e^4}{4} \right) dx \\ &= C_1 m_e \int \frac{1}{v_e \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]} \frac{d}{dx} \left( \frac{v_e^2}{2} \right) dx \\ &\quad + \frac{3}{2c^2} C_1 m_e \int \frac{1}{v_e \left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]} \frac{d}{dx} \left( \frac{v_e^4}{4} \right) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} C_1 m_e \left[ \frac{v_e}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} + \int \frac{1 + \frac{3}{2} \left( \frac{v_e}{c} \right)^2}{\left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]^2} dv_e \right] \\
&+ \frac{3}{8c^2} C_1 m_e \left[ \frac{v_e^3}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} + \int \frac{v_e^2 \left[ 1 + \frac{3}{2} \left( \frac{v_e}{c} \right)^2 \right]}{\left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]^2} dv_e \right]. \tag{D}
\end{aligned}$$

Here, we used eq.(2.a) and carried out the partial integration. The integrations in (D) can be solved in the following approximated form

$$\begin{aligned}
&\int \frac{1 + \frac{3}{2} \left( \frac{v_e}{c} \right)^2}{\left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]^2} dv_e = \frac{c}{2^{1/2}} \left[ 3 \int \frac{dX}{X(X-1)^{1/2}} - 2 \int \frac{dX}{X^2(X-1)^{1/2}} \right] \\
&= 2^{1/2} c \left[ 2 \arctan \left( \frac{v_e}{2^{1/2} c} \right) - \frac{\frac{v_e}{2^{1/2} c}}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} \right] \\
&= 2^{1/2} c \left[ 2 \left[ \frac{v_e}{2^{1/2} c} - \frac{1}{3} \left( \frac{v_e}{2^{1/2} c} \right)^3 + \frac{1}{5} \left( \frac{v_e}{2^{1/2} c} \right)^5 - \dots \right] \right. \\
&\quad \left. - \frac{\frac{v_e}{2^{1/2} c}}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} \right]. \tag{D.1}
\end{aligned}$$

where  $X$  is defined by  $X=1+(1/2)(v_e/c)^2$ . Here we expanded  $\arctan(v_e/2^{1/2}c)$  under the condition  $|v_e/2^{1/2}c| \ll 1$ . Performing the similar calculation, one obtain

$$\begin{aligned}
& \int \frac{v_e^2 \left[ 1 + \frac{3}{2} \left( \frac{v_e}{c} \right)^2 \right]}{\left[ 1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2 \right]^2} dv_e \\
&= (2^{1/2}c)^3 \left[ 3 \int \frac{dX}{(X-1)^{1/2}} - 5 \int \frac{dX}{X(X-1)^{1/2}} + 2 \int \frac{dX}{X^2(X-1)^{1/2}} \right] \\
&= (2^{1/2}c)^3 \left[ 3 \frac{v_e}{2^{1/2}c} + \frac{\frac{v_e}{2^{1/2}c}}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} - 4 \arctan \left( \frac{v_e}{2^{1/2}c} \right) \right] \\
&= (2^{1/2}c)^3 \left[ 3 \frac{v_e}{2^{1/2}c} + \frac{\frac{v_e}{2^{1/2}c}}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} \right. \\
&\quad \left. - 4 \left[ \frac{v_e}{2^{1/2}c} - \frac{1}{3} \left( \frac{v_e}{2^{1/2}c} \right)^3 + \frac{1}{5} \left( \frac{v_e}{2^{1/2}c} \right)^5 - \dots \right] \right].
\end{aligned} \tag{D.2}$$

The use of eqs.(D.1) and (D.2) yields

$$\begin{aligned}
L_1 &= C_1 m_e v_e \left[ 1 - \frac{1}{6} \left( \frac{v_e}{c} \right)^2 \right] \\
&+ \frac{3}{8c^2} C_1 m_e v_e \left[ \frac{v_e^2}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} + 2c^2 \left[ \frac{1}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} \right. \right. \\
&\quad \left. \left. - \left[ 1 - \frac{2}{3} \left( \frac{v_e}{c} \right)^2 \right] \right] \right],
\end{aligned}$$



(E.1)

where we approximated that  $(v_e/c)^4 \approx 0$ ,  $(v_e/c)^8 \approx 0$ .

The integration  $L_2$  can also be obtained similarly,

$$L_2 = C_2 m_i v_i \left[ 1 - \frac{1}{6} \left( \frac{v_i}{c} \right)^2 \right] + \frac{3}{8c^2} C_2 m_i v_i \left[ \frac{v_i^2}{1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2} + 2c^2 \left[ \frac{1}{1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2} - \left[ 1 - \frac{2}{3} \left( \frac{v_i}{c} \right)^2 \right] \right] \right].$$

(E.2)

Here we used eq. (2.b) and the approximation  $(v_e/c)^4 \approx 0$ ,  $(v_e/c)^8 \approx 0$ .

Using eqs. (E.1) and (E.2), we can reduce the integration  $L$  to

$$L = C_1 m_e v_e \left[ 1 - \frac{1}{6} \left( \frac{v_e}{c} \right)^2 + \frac{3}{8c^2} \left\{ \frac{v_e^2}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} + 2c^2 \left[ \frac{1}{1 + \frac{1}{2} \left( \frac{v_e}{c} \right)^2} - 1 + \frac{2}{3} \left( \frac{v_e}{c} \right)^2 \right] \right\} \right] + C_2 m_i v_i \left[ 1 - \frac{1}{6} \left( \frac{v_i}{c} \right)^2 \right]$$

$$\begin{aligned}
& + \frac{3}{8c^2} \left\{ \frac{v_i^2}{1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2} + 2c^2 \left[ \frac{1}{1 + \frac{1}{2} \left( \frac{v_i}{c} \right)^2} \right. \right. \\
& \qquad \qquad \qquad \left. \left. - 1 + \frac{2}{3} \left( \frac{v_i}{c} \right)^2 \right] \right\} . \tag{F}
\end{aligned}$$

Expansion of  $1/[1+(1/2)(v_{e,i}/c)^2]$  in (F) under the condition  $(v_{e,i}/c)^2 \ll 1$ . Then eq.(F) gives rise to

$$L = C_1 m_e v_e \left[ 1 + \frac{1}{3} \left( \frac{v_e}{c} \right)^2 \right] + C_2 m_i v_i \left[ 1 + \frac{1}{3} \left( \frac{v_i}{c} \right)^2 \right], \tag{G}$$

where we approximated that  $(v_{e,i}/c)^4 \approx 0$ ,  $(v_{e,i}/c)^6 \approx 0$ .

Next, integrating the right-hand side of eq.(C) and using eq.(G), we obtain, from  $L = R$ ,

$$\begin{aligned}
& C_1 m_e v_e \left[ 1 + \frac{1}{3} \left( \frac{v_e}{c} \right)^2 \right] + C_2 m_i v_i \left[ 1 + \frac{1}{3} \left( \frac{v_i}{c} \right)^2 \right] \\
& = \frac{1}{2} \varepsilon_0 \left[ \frac{d\phi}{dx} \right]^2 - \kappa T_e n_e - \kappa T_i n_i - C_5 .
\end{aligned}$$

Using eqs.(2.a) and (2.b) for  $C_1$  and  $C_2$ , and putting  $(v_{e,i}/c)^4 \approx 0$ , we finally obtain

$$m_e n_e v_e^2 \left[ 1 + \frac{5}{6} \left( \frac{v_e}{c} \right)^2 \right] + m_i n_i v_i^2 \left[ 1 + \frac{5}{6} \left( \frac{v_i}{c} \right)^2 \right]$$

$$+ \kappa T_e n_e + \kappa T_i n_i - \frac{1}{2} \varepsilon_0 \left( \frac{d\phi}{dx} \right)^2 = -C_5 , \quad (\text{H})$$

which is eq.(3).

## Captions of figures

Fig.1:

Bird's eye view of the pseudo-potential  $V(\phi)/(\omega_e^2 v_0^2 (m_e/e)^2)$  for relativistic Langmuir waves represented by eq.(11) under the conditions of  $0.150 < v_0/c < 0.170$  and  $0 < e\phi/(m_e v_0^2/2) < 1837$ .  $W = 1849.3$ .

Fig.2:

A typical example in the case where the pseudo-potential forms the potential well.  $W = 1870.5$  and  $v_0/c = 0.150$ .

Fig.3:

A typical example of the pseudo-potential that does not form potential well.  $W = 1898.4$  and  $v_0/c = 0.20$ .

Fig.4:

The existence region of large amplitude relativistic Langmuir waves depending on the relativistic effect  $v_0/c$  and the energy  $W$ .

Fig.5:

The existence region of relativistic Langmuir waves depending on the normalized potential  $e\phi/(m_e v_0^2/2)$  and the relativistic effect  $v_0/c$ .  $W = 1837.5$ .

Fig.6:

Bird's eye view of the pseudo-potential  $V(\phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  for relativistic ion-acoustic waves represented by eq.(23) under the conditions of  $0.10 < \Phi < 0.80$  and  $1.20 < \mathcal{M} < 1.58$ .

Fig.7:

Bird's eye view of the pseudo-potential  $V(\phi)/(\omega_i^2 c_s^2 (m_i/e)^2)$  for relativistic ion-acoustic waves under the conditions of  $0.50 < \Phi < 1.28$  and  $1.50 < \mathcal{M} < 1.58$ .

Fig.8:

A pseudo-potential curve of large amplitude ion-acoustic waves for  $\mathcal{M}=1.20$ .

Fig.9:

A pseudo-potential curve of large amplitude ion-acoustic waves for  $\mathcal{M}=1.50$ .

Fig.10:

The existence region of large amplitude relativistic ion-acoustic waves depending on the normalized potential  $\Phi$  and the Mach number  $\mathcal{M}$ . Large amplitude nonlinear ion-acoustic waves exist in the region A.

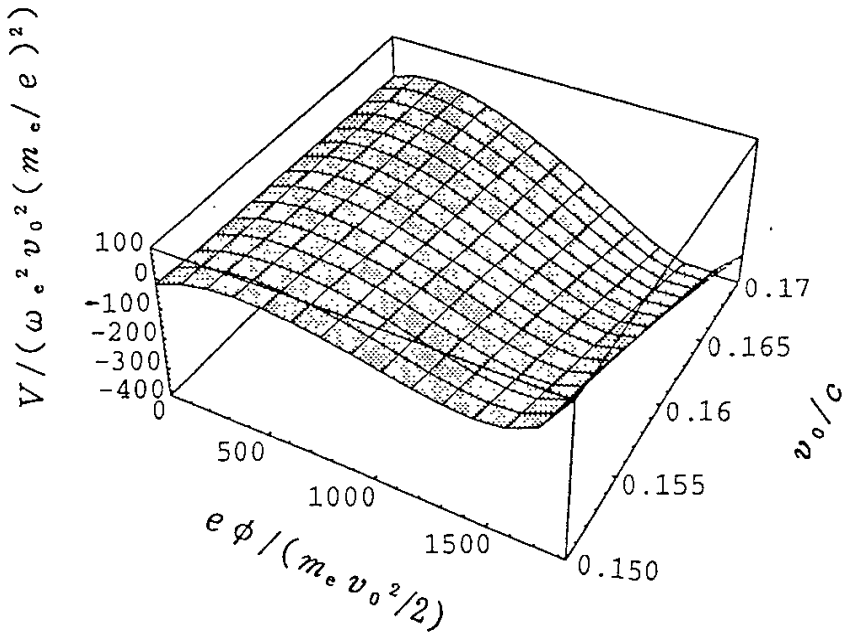


Figure 1

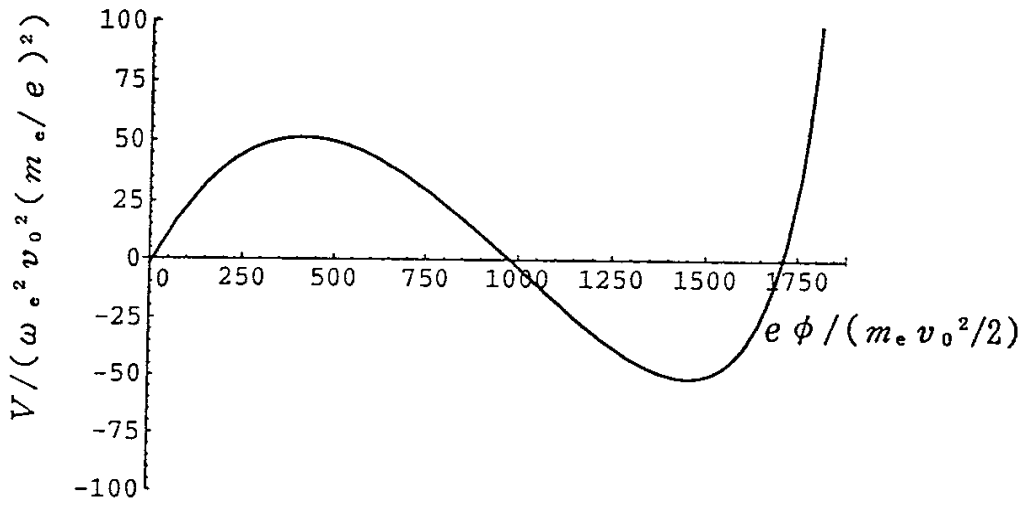


Figure 2

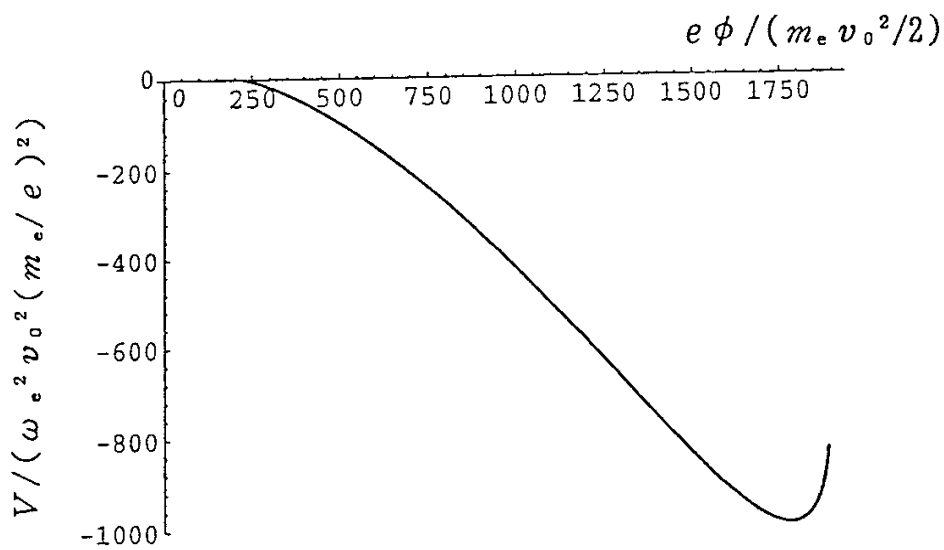
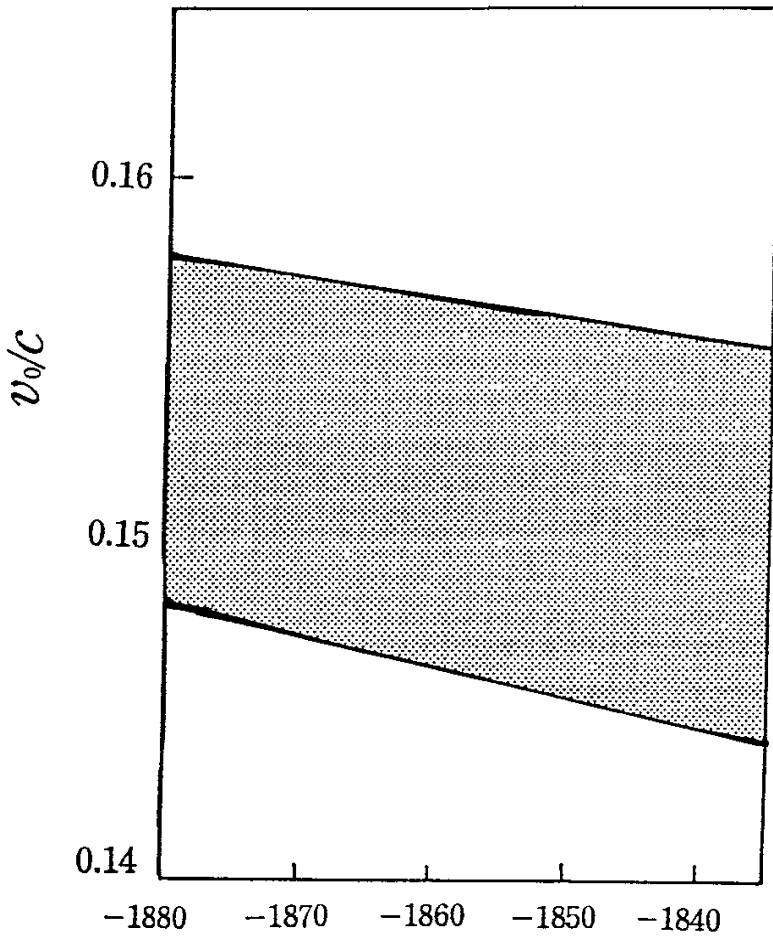


Figure 3





$W$

Figure 4

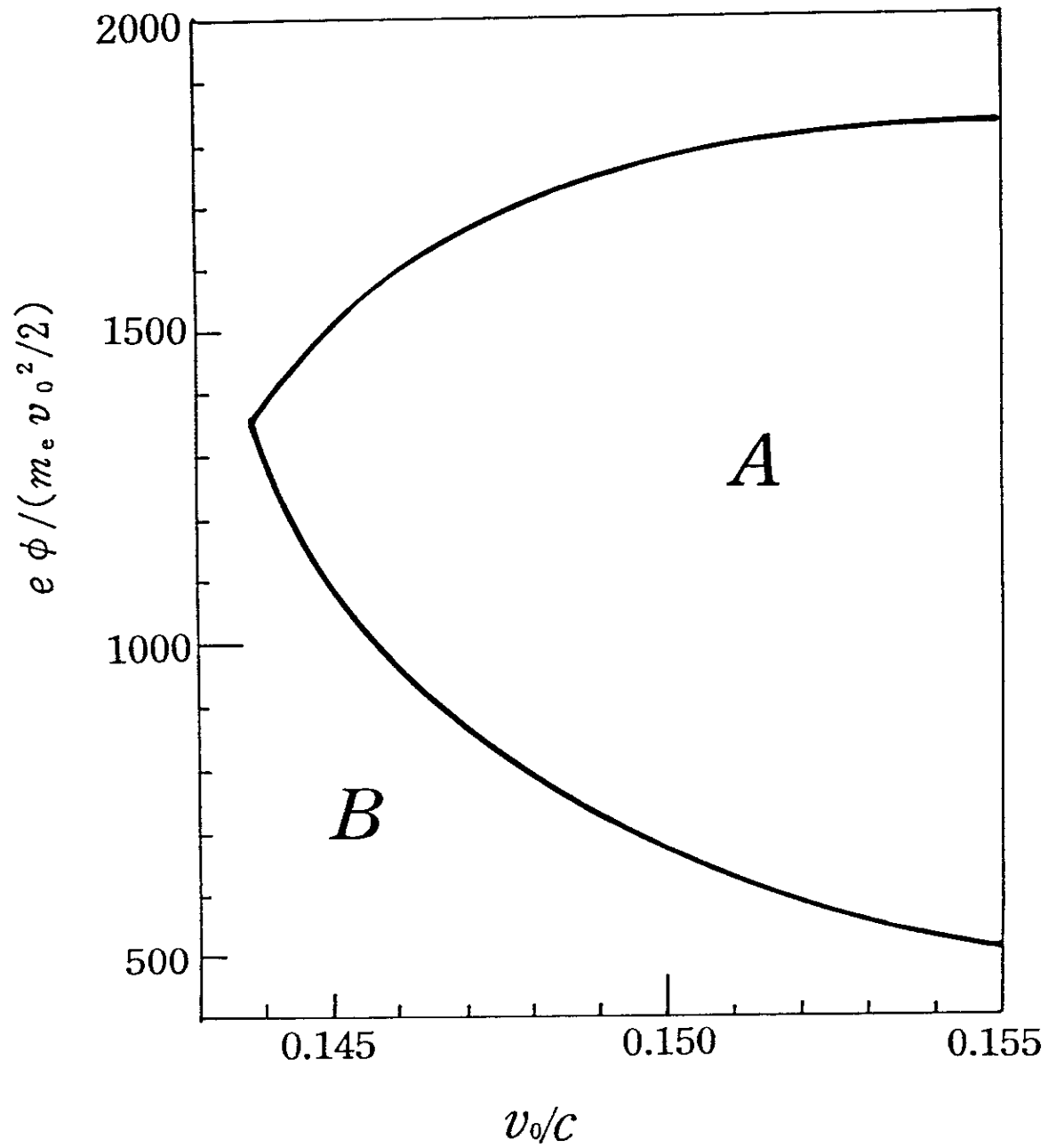


Figure 5

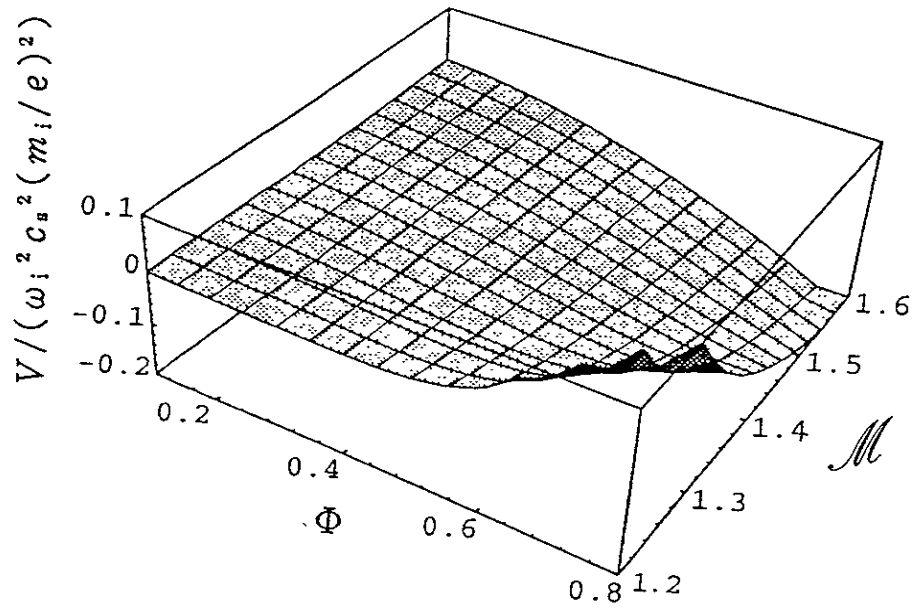


Figure 6

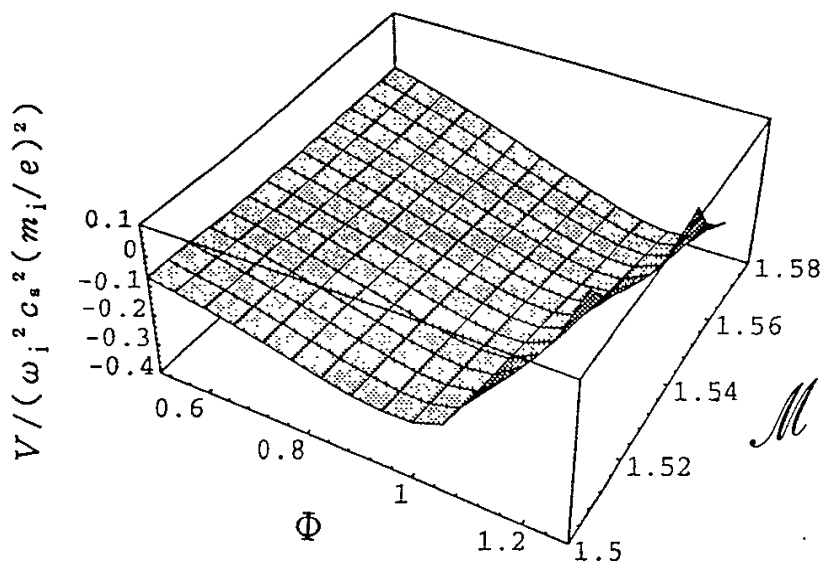


Fig.7

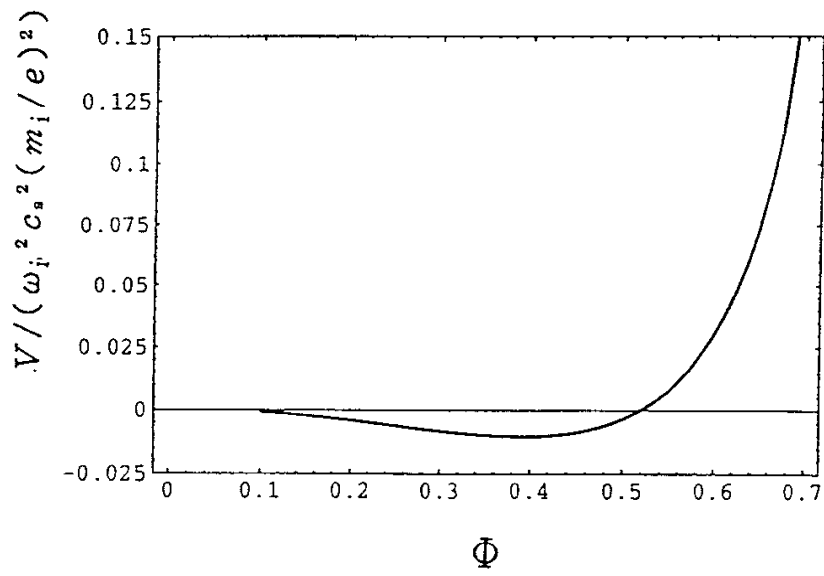


Fig.8

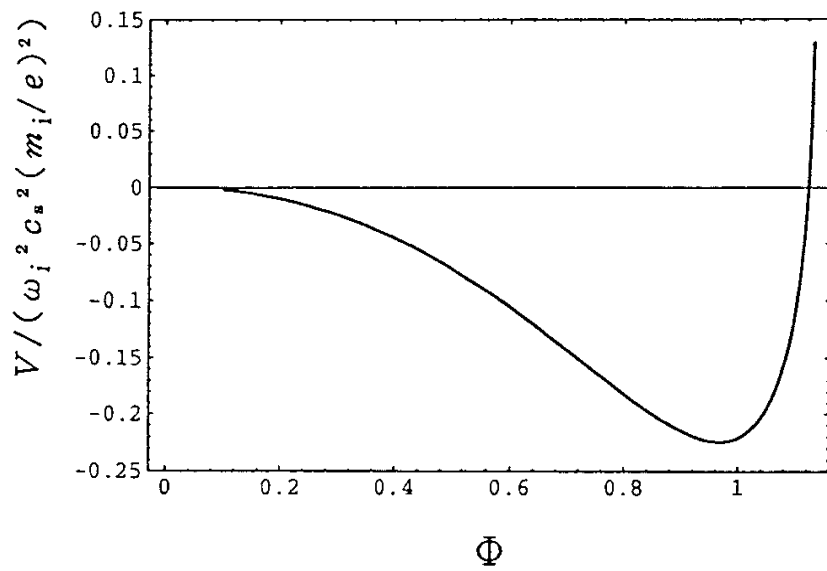


Fig.9

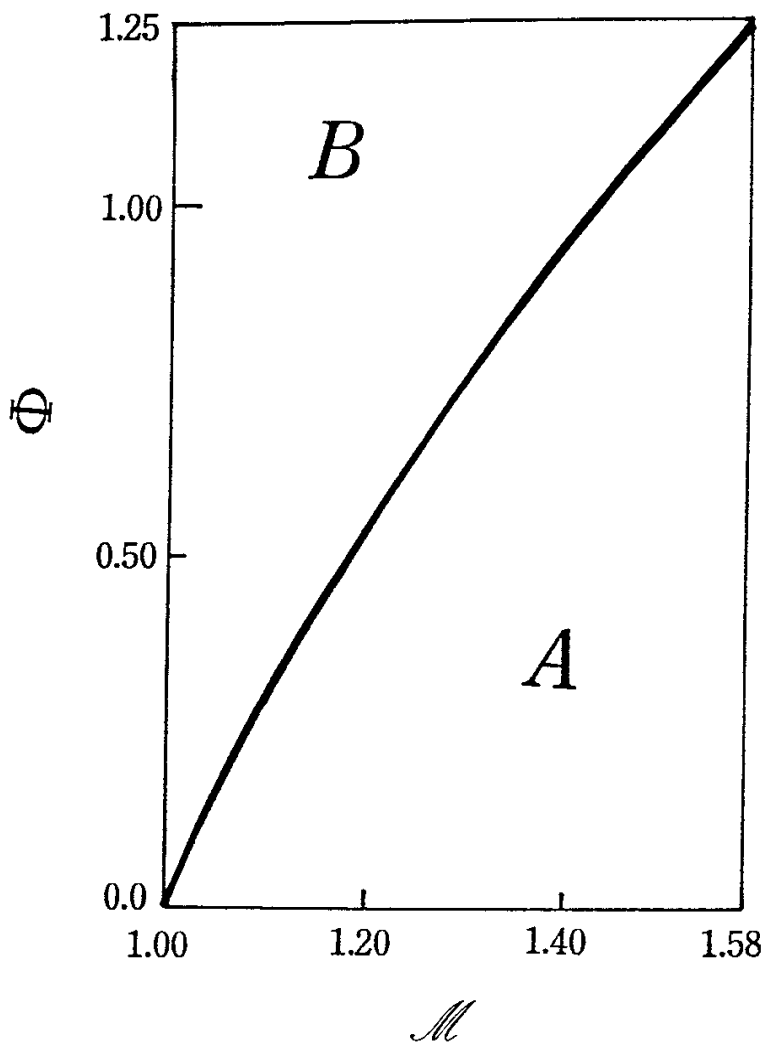


Fig.10

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