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A. Ida, H. Sanuki and J. Todoroki

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An Extended K-dV Equation for Nonlinear Magnetosonic Wave in a Multi-Ion Plasma

A. Ida
Department of Energy Engineering and Science,
Nagoya University, Nagoya 464-01 Japan

and

H. Sanuki and J. Todoroki
National Institute for Fusion Science, Nagoya 464-01,
Nagoya Japan

Abstract

Nonlinear magnetosonic waves propagating perpendiclarly to a magnetic field are studied in two-ion plasma. It is shown that high frequency magnetosonic wave under the influence of finite cut-off frequency is described by an extended K-dV equation, rather than conventional K-dV equation. Modulational stability of this mode is strongly affected by the finite cut-off frequency in two-ion plasma.

Keywords: Magnetosonic wave, Multi-ion species,
Finite cut-off, Extended K-dV

1. Introduction

Nonlinear magnetosonic waves have been actively studied because on the one hand they play an important role on particle acceleration and heating of plasmas, on the other hand its nonlinear behaviour is an attractive subject from viewpoint of nonlinear wave phenomena. For a plasma with one ion-species, the linear dispersion relation of a magnetosonic wave propagating perpendicularly to a magnetic field is given as

$$\omega \cong kV_A - ak^3, \tag{1}$$

where the wave number k is assumed to be in the range

$$\frac{\omega_{pe}^2}{c^2} \gg k^2 \gg \frac{\omega_{pi}^2}{c^2}, \tag{2}$$

Here, v_A is the Alfvén speed, c the light velocity, ω_{pe} (ω_{pi}) the electron (ion) plasma frequency, respectively, and the coefficient a in Eq. (1) will be given explicitely later.

It is well known that the nonlinear evolution of a magnetosonic wave in a plasma with one-ion species is described by the Korteweg-de Vries (K-dV) equation [1, 2]. For muti-ion plasma, the dispersion relation may be modified and particularly, the magnetosonic wave in case of two ion-species is split into two modes, namely, the high- and low-frequency modes. Recently, Toida and Ohsawa [3] discussed the nonlinear evolution of these magnetosonic waves (propagating perpendicularly to a magnetic field) in the plasma with two ion-species, and asserted that the high-frequency mode is described by the K-dV equation, although the dispersion branch of high frequency mode has a finite cut-off frequency. The ordering

$$k \sim \varepsilon^{1/2} \tag{3}$$

is applied in the analysis based on the reductive perturbation method. But, this does not always request that the dispersion relation is regular at k=0.

We should note that the dispersion relation and its physical process sensitively depend on the ordering associated with the smallness parameter m_e/m_i, which is a measure of electron-to-ion mass ratio. Although Toida and Ohsawa derived the K-dV equation for the magnetosonic wave under the following ordering for m_e/m_i with smallness parameter ε

$$\frac{m_e}{m_i} \sim \varepsilon^3, \tag{4}$$

it should be noted that the effect of finite cut-off frequency is not correctly involved in the wave equation. Therefore, we have to pay attention to the scaling and ordering with respect to m_e/m_i to discuss accurately the influence of finite cut-off frequency on the high frequency magnetosonic wave. If we assume the ordering

$$\frac{m_e}{m_i} \sim \varepsilon^2, \tag{5}$$

the high-frequency magnetosonic wave under the influence of finite cut-off frequency in the plasma with two ion-species is described by the following dispersion relation

$$\omega \cong kV_A - ak^3 + \frac{b}{k}, \tag{6}$$

where b tends to zero in case of the single ion species and the explicit form of b will be given later. We note that the nonlinear magnetosonic wave on the basis of the dispersion relation (6) can not be described by a conventional K-dV equation based on the dispersion relation (1).

In this paper, we apply a proper ordering and scaling with respect to the mass ratio m_e/m_i and derive a nonlinear wave equation describing nonlinear magnetosonic wave under the effect of finite cut-off frequency on the basis of a fluid equations for a two-ion plasma.

In section 2, we discuss the linear dispersion relation for the high frequency magnetosonic wave under the effect of finite cut-off frequency, and show that its nonlinear behaviour can be described by an extended K-dV equation rather than K-dV equation. In section 3, we reduce this wave equation to the nonlinear Schrődinger type equation by using the reductive perturbation method and examine the propagation characterics of this mode. In section 4, we will have discusions on a stationary solution. The last section is devoted to the summary.

2 Derivation of Model Equation

2. 1 Basic equations

We here study a magnetosonic wave propagating in the x-direction perpendicular to a magnetic field $\vec{B}_0 = B_0\hat{z}$ on the basis of following fluid equations with two ion-species:

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \vec{v}_j) = 0 \tag{7}$$

$$m_j\left(\frac{\partial}{\partial t} + (\vec{v}_j \cdot \nabla)\right)\vec{v}_j = q_j \vec{E} + \frac{q_j}{c}\vec{v}_j \times \vec{B} \tag{8}$$

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \tag{9}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\sum_j q_j n_j \vec{v}_j \tag{10}$$

where the subscript j refers to ion species (two ion species j=a and b) or electrons (j=e), m_j is the mass, q_j the charge, n_j the number density and $\vec{v}_j$ is the velocity. Since we consider the magnetosonic wave propagating in the x-direction perpendicular to B_0, we should note $\partial/\partial y = \partial/\partial z = 0$ in Eqs. (7) – (10).

2. 2 Linear Dispersion Relation

If we assume $\omega^2 \ll \Omega_e^2$ (Ω_e is the electron cyclotron frequency) and neglect the higher order terms with respect to the smallness parameter m_e/m_i, the linearization of Eqs. (7)-(10) yield the dispersion relation

$$\omega_{\pm}^2 = \frac{A_1 \pm \sqrt{A_1^2 - 4A_0A_2}}{2A_2}, \tag{11}$$

with

$$A_0 = \left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right)\Omega_a^2\Omega_b^2 c^2k^2, \tag{12a}$$

$$A_1 = \left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right)^2 \Omega_a^2\Omega_b^2 + (\omega_{pa}^2 + \omega_{pb}^2)c^2k^2, \tag{12b}$$

$$A_2 = \frac{\omega_{pe}^2}{\Omega_e^2}\left(\omega_{pe}^2 + c^2k^2\right), \tag{12c}$$

where ω_{pj} and Ω_j refer the plasma frequency and cyclotron frequency for ion species (j=a and b) and electrons (j=e), respectively. For the very small wavenumber,

$$k^2 \ll \frac{\omega_{pa}^2}{c^2}, \frac{\omega_{pb}^2}{c^2}, \tag{13}$$

the dispersion relation (11) reduces to

$$\omega_{+}^2 \approx \frac{\Omega_e^2}{\omega_{pe}^4}\left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right)^2 \Omega_a^2\Omega_b^2 + \frac{\Omega_e^2}{\omega_{pe}^4}\frac{(\Omega_a - \Omega_b)^2}{\left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2}\right)}\frac{\omega_{pa}^2\omega_{pb}^2}{\Omega_a^2\Omega_b^2}c^2k^2, \tag{14}$$

on the other hand. in the region of the wavenumber

$$\frac{\omega_{pe}^2}{c^2} \gg k^2 \gg \frac{\omega_{pi}^2}{c^2},$$

we obtain an approximate dispersion relation in the following form

$$\omega_+^2 \approx V_A^2 k^2 \left\{ 1 - \frac{c^2 k^2}{\omega_{pe}^2} \right\} + \frac{(\Omega_a - \Omega_b)^2}{(\omega_{pa}^2 + \omega_{pb}^2)^2} \frac{V_A^2}{c^2} \left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2} \right) \omega_{pa}^2 \omega_{pb}^2. \quad (15)$$

Comparison between Eq. (6) and Eq. (15) gives the coefficients a and b as

$$a = \frac{1}{2} \frac{c^2 V_A}{\omega_{pe}^2}, \quad (16)$$

$$b = \frac{1}{2} \frac{V_A}{c^2} \frac{\omega_{pa}^2 \omega_{pb}^2 (\Omega_a - \Omega_b)^2}{(\omega_{pa}^2 + \omega_{pb}^2)^2} \left(\frac{\omega_{pa}^2}{\Omega_a^2} + \frac{\omega_{pb}^2}{\Omega_b^2} \right), \quad (17)$$

2. 3 Nonlinear Wave Equation

We now derive the model equation for nonlinear magneto-sonic wave characterized by the dispersion relation (6). by using the reductive perturbation method. We here introduce the following stretched space-time variables

$$\xi = \varepsilon^{1/2} (x - Vt), \quad \tau = \varepsilon^{3/2} t \quad (18)$$

and also suppose

$$\frac{m_e}{m_i} \sim \varepsilon^2, \tag{19}$$

instead of the assumption (4). We then expand the plasma variables as

$$B_z = B_0 + \varepsilon B_z^{(1)} + \varepsilon^2 B_z^{(2)} + \cdots, \tag{20a}$$

$$n_j = n_{j0} + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \cdots, \tag{20b}$$

$$E_x = \varepsilon^{1/2}[E_x^{(1/2)} + \varepsilon E_x^{(3/2)} + \cdots], \tag{20c}$$

$$E_y = \varepsilon \ \ [E_y^{(1)} + \varepsilon E_y^{(2)} + \cdots], \tag{20d}$$

$$v_{xj} = \varepsilon v_{xj}^{(1)} + \varepsilon^2 v_{xj}^{(2)} + \cdots, \tag{20e}$$

$$v_{yj} = \varepsilon^{1/2} v_{yj}^{(1/2)} + \varepsilon^{3/2} v_{yj}^{(3/2)} + \cdots. \tag{20f}$$

Here we note that the expansion of these plasma variables depends sensitively on the ordering of m_e/m_i. Substituting Eqs. (20a) – (20f) with Eqs. (18) and (19) into the set of fundamental equations (7) – (10), we finally arrive at the nonlinear evolution equation for magnetosonic wave under the influence of the finite cut-off frequency in the form of an extended K-dV equation

$$\frac{\partial}{\partial \xi}\left\{\frac{\partial u}{\partial \tau} + \beta u \frac{\partial u}{\partial \xi} + \gamma \frac{\partial^3 u}{\partial \xi^3}\right\} - \delta u = 0 \ , \tag{21}$$

with the abbreviations of

$$\beta = \frac{3}{2} \frac{\omega_{pe}{}^2(\omega_{pa}{}^2\Omega_a + \omega_{pb}{}^2\Omega_b)}{\Omega_e(\omega_{pa}{}^2 + \omega_{pb}{}^2)^2}, \ \gamma = \frac{c^2 V}{2\omega_{pe}{}^2}, \tag{22}$$

$$\delta = \frac{V}{2c^2} \frac{\omega_{pa}{}^2\omega_{pb}{}^2(\Omega_a - \Omega_b)}{(\omega_{pa}{}^2 + \omega_{pb}{}^2)} \left\{\frac{\omega_{pa}{}^2}{\Omega_a{}^2} + \frac{\omega_{pb}{}^2}{\Omega_b{}^2}\right\}, \tag{23}$$

$$V^2 = c^2 \frac{\Omega_e{}^2}{\omega_{pe}{}^2} \frac{(\omega_{pa}{}^2 + \omega_{pb}{}^2)}{\omega_{pe}{}^2}, \tag{24}$$

where Ω_j (j=a, b, e) is the cyclotron frequency for j-th species and we put $u = v_{ye}^{(1)}$. If we neglect the δ-term in Eq. (21) (this case is realized in a plasma with one-ion species in which δ tends to zero) , equation (21) reduces to so-called K-dV equation, which has been discussed by Toida and Ohsawa. In the present paper, we restrict our discussions to a case with small but finite k (see, Eqs. (2) and (18)) because Eq. (21) appears not to be valid in the long wavelength limit, which is seen from Eq. (6). We note that the characteristics of Eq. (21) sensitively depends on the competing effect between γ-and δ-terms and this situation is essentially different from the previous results [3].

We here discuss some property of Eq. (21) compared with the K-dV equation. When we consider a periodic solution or a solution which tends to zero for $|\xi|\to\infty$, we obtain the following boundary condition by intergrating Eq. (21) with respect to ξ,

$$\int_{-\infty}^{\infty} u d\xi = 0, \tag{25}$$

which is different from the conventional boundary condition for the K-dV equation. If we introduce a new variable Ψ through

$$u = \frac{d^2\Psi}{d\xi^2} \quad , \tag{26}$$

eqation (21) reduces to the following nonlinear equation

$$\frac{\partial u}{\partial \tau} + \beta u \frac{\partial u}{\partial \xi} + \gamma \frac{\partial^3 u}{\partial \xi^3} - \delta \frac{\partial \psi}{\partial \xi} = 0 \, . \tag{27}$$

Furthermore, the substitution of Eq. (26) into Eq. (27) yeilds the following equation

$$\frac{\partial^2 \psi}{\partial \tau \partial \xi} + \frac{\beta}{2} u^2 + \gamma \frac{\partial^2 u}{\partial \xi^2} - \delta \psi = 0 \, . \tag{28}$$

When we regard Eqs. (27) and Eq. (28) as the coupled equations, these represent the two wave interaction process between u and Ψ. Although K-dV equation has infinite number of conserved quantities [4, 5], it seems that these coupled equations have only three conserved ones, and a conservation law is generally expressed in the form

$$\frac{\partial T^{(i)}}{\partial t}+\frac{\partial X^{(i)}}{\partial x}=0 \ , \tag{29}$$

where T, the conserved density, and -X, the flux of T, are functionals of u and Ψ. For Eq. (21) we have the following three conserved quantities (here, the subscript ξ denotes the differential with respect to ξ)

$$T^{(1)}=u, \tag{30a}$$
$$X^{(1)}=\beta u^2/2+\gamma u_{\xi\xi}-\delta\psi, \tag{30b}$$

$$T^{(2)}=u^2/2, \tag{31a}$$
$$X^{(2)}=\beta u^3/3+\gamma u u_{\xi\xi}-\gamma u_\xi{}^2/2-\delta\psi_\xi{}^2/2, \tag{31b}$$

$$T^{(3)}=u^3/3-\gamma u_\xi{}^2/\beta+\delta\gamma\psi_\xi{}^2/\beta^2, \tag{32a}$$
$$X^{(3)}=\beta u^4/4+\gamma u^2 u_{\xi\xi}-2\gamma u u_\xi{}^2-2\gamma^2 u_\xi u_{\xi\xi\xi}/\beta+\gamma^2 u_{\xi\xi}{}^2/\beta \tag{32b}$$
$$+2\delta\left\{\left(\gamma^2/\beta^2\right)\psi_\xi\psi_{\xi\xi\xi}-\delta\psi^2/2\beta\right\},$$

provided Ψ is bounded and mean squre integrable.

3. Envelop Soliton Solution

In this section, we derive the nonlinear Schrödinger (NS) equation on the basis of conventional reductive perturbation method and study the propagation characteristics. Again, the variable transformation is introdueed by

$$\eta = \varepsilon(\xi - \lambda\tau),$$
$$\zeta = \varepsilon^2\tau.$$

Here ε is a parameter specifying the smallness of amplitude and we assume the following solution;

$$u = \sum_{n=1} \varepsilon^n \sum_{l=-\infty} u_l^{(n)}(\zeta,\eta)\exp[il(K\xi - \Delta\tau)]. \tag{33}$$

As the reality condition, we have

$$u_l^{(n)} = u_{-l}^{(n)*}, \tag{34}$$

where the asterisk denotes the complex conjugate. Moreover, for the first order of u, $u_l^{(1)}$, we assume

$$u_l^{(1)} = 0 \ \text{ for } |l| \neq 1. \tag{35}$$

This means that a modulation of the plane wave with the frequency Δ and the wave number K is now under consideration. Substitution of Eq. (33) into Eq. (21) yields the n-th order equation

$$
\begin{aligned}
&[l^2K\Delta + \gamma l^4K^4 - \delta]u_l^{(n)} - il[\Delta + \lambda K + 4l^2K^3\gamma]\frac{\partial u_l^{(n-1)}}{\partial\eta} \\
&-\lambda\frac{\partial^2 u_l^{(n-2)}}{\partial\eta^2} + ilK\frac{\partial u_l^{(n-2)}}{\partial\zeta} - 6\gamma l^2K^2\frac{\partial^2 u_l^{(n-2)}}{\partial\eta^2} + \frac{\partial^2 u_l^{(n-3)}}{\partial\zeta\partial\eta} \\
&4i\gamma lK\frac{\partial^3 u_l^{(n-3)}}{\partial\eta^3} + \gamma\frac{\partial^4 u_l^{(n-4)}}{\partial\eta^4} + \beta\sum_{l'}\sum_{n'}\{-l(l-l')K^2u_{l'}^{(n-n')}u_{l-l'}^{(n')} \\
&+2iKlu_{l'}^{(n-n'-1)}\frac{\partial u_{l-l'}^{(n')}}{\partial\eta} + u_{l'}^{(n-n'-2)}\frac{\partial^2 u_{l-l'}^{(n')}}{\Delta\eta^2} + \frac{\partial u_{l'}^{(n-n'-2)}}{\partial\eta}\frac{\partial u_{l-l'}^{(n)}}{\partial\eta}\} = 0.
\end{aligned}
\tag{36}
$$

From the first order equation with $l = \pm 1$ of Eq. (36), we obtain

$$[K\Delta + \gamma K^4 - \delta]u_{\pm 1}^{(1)} = 0, \tag{37}$$

which gives the frequency shift as

$$\Delta = -\gamma K^3 + \delta/K, \tag{38}$$

because $u_{\pm 1}^{(1)} \neq 0$. The second order equation for $l = \pm 1$ is given as

$$i[\Delta + \lambda K + 4K^3\gamma]\frac{\partial u_{l=\pm 1}^{(1)}}{\partial\eta} = 0, \tag{39}$$

which yields the relation

$$\lambda = -3K^2\gamma - \frac{\delta}{K^2} \equiv \frac{\partial\Delta}{\partial K}, \tag{40}$$

in order for Eq. (39) to have a non-trivial solution. Thus, λ corresponds to the group velocity. From the second order equation for $l=\pm 2$, we have

$$\begin{pmatrix} u_2^{(2)} \\ u_{-2}^{(2)} \end{pmatrix} = \frac{2\beta K^2}{12\gamma K^4 + 3\delta} \begin{bmatrix} u_1^{(1)} u_1^{(1)} \\ u_{-1}^{(1)} u_{-1}^{(1)} \end{bmatrix}. \tag{41}$$

Using Eqs. (38), (40) and (41), we obtain from the third order equation with $l=1$

$$i\frac{\partial u_1^{(1)}}{\partial \zeta} + A\frac{\partial^2 u_1^{(1)}}{\partial \eta^2} + B|u_1^{(1)}|^2 u_1^{(1)} = 0, \tag{42}$$

where

$$A = -[3\gamma K - \frac{\delta}{K^3}] = \frac{1}{2}\frac{\partial \lambda}{\partial K}, \tag{43}$$

$$B = -\frac{2\beta^2 K^3}{12\gamma K^4 + 3\delta}, \tag{44}$$

which is the well-known nonliner Schrödinger equation. Solutions of the nonlinear Schrödinger equation have been studied in detail [6]. In particular, if A and B take the same sign, the solution which tends to zero for $|\eta| \to \infty$, is an envelop soliton,

$$u_1^{(1)}(\eta,\zeta) = a\cdot \mathrm{sec}\,h[\kappa(\eta - V\zeta)]\cdot \exp[i\frac{A}{2}\{\frac{V}{A^2}\eta - (\frac{V^2}{2A^2} - 2\kappa^2)\zeta\}], \tag{45}$$

where $\kappa^2 = (B/2A)\, a^2$ and (45) represents the soliton moving with the velocity V. If $u_1^{(1)}$ approarches a constant $u_0^{(1)}$ at infinity, we have the plane-wave solution,

$$u_1^{(1)}(\eta,\zeta) = u_0^{(1)} \exp[i\{K\eta - (AK^2 - Bu_0^{(1)2})\zeta\}]. \tag{46}$$

However, the plane wave is not stable but subject to the modulational instability. On the other hand, the plane wave is stable if A and B take the opposite signs. It should be noted that the coefficient A may change the sign due to the influence of cutt-off frequency, δ-term. Namely, A is positive for $K<K_c \equiv (\delta/3\gamma)^{1/4}$ and A and B take the opposite sign. We here restrict our discussions to the case with $A \cdot B>0$. Noting that the solution (45) has an ambiguousness for choice of the phase factor, we describe the solution in the form

$$u_1^{(1)}(\eta,\zeta) = a \cdot \mathrm{sech}[\kappa(\eta - v\zeta)] \exp[i(\psi + f)], \tag{47}$$

with

$$\psi = \frac{A}{2}\left[\frac{V}{A^2}\eta - \left(\frac{V^2}{2A^2} - 2\kappa^2\right)\zeta\right], \tag{48}$$

where the phase factor f must be choosen in order for the boundary condition (25) to be satisfied, i. e., $f=\pi/2$. Substitution of (47) with (48) into (43) gives the first order solution as

$$u^{(1)} = 2a \cdot \mathrm{sec}\,h[\kappa(\eta - V\zeta)]\sin[\psi + \phi], \tag{49}$$

where $\phi = K\xi - \Delta\tau$.

4. Dissussions

We here discuss briefly a stationary traveling wave solution of Eq. (21). If we introduce

$$\partial u/\partial t = -C\ \partial u/\partial \xi \quad , \tag{50}$$

equation (21) reduces to the following coupled equations

$$u = \frac{d^2\Psi}{d\xi^2} \quad , \tag{51}$$

$$-Cu + \frac{\beta}{2}u^2 + \gamma\frac{d^2u}{d\xi^2} - \delta\Psi = 0 \quad . \tag{52}$$

We then obtain the conserved quantity as

$$\frac{d}{d\xi}\left\{-\frac{C}{2}u^2 + \frac{\beta}{6}u^3 + \frac{\gamma}{2}\left(\frac{du}{d\xi}\right)^2 - \delta\Psi u + \frac{\delta}{2}\left(\frac{d\Psi}{d\xi}\right)^2\right\} = 0 \quad . \tag{53}$$

Furthermore, on introducing $u \equiv \nu - \delta\Psi/C$, we obtain the following coupled equations

$$\frac{d^2\Psi}{d\xi^2} + \frac{\delta}{C}\Psi = \nu \quad , \tag{54}$$

$$\gamma\frac{d^2\nu}{d\xi^2} = C\nu - \frac{\beta}{2}(\nu - \frac{\delta}{C}\Psi)^2 + \frac{\delta\gamma}{C}(\nu - \frac{\bar{\delta}}{C}) \quad , \tag{55}$$

which stand for the interaction process between solitary wave and a wave with wavelength $\sqrt{C/\delta}$.

In section 3, we derived the envelop soliton solution of the extended K-dV equation (21). But it is not stationary. It is not known nor readily seen whether the extended K-dV equation prosseses the stationary solutions under the boundary condition (25). We might be able to study the characteristics of the stationary solutions on the basis of the coupled equation (54) and (55).

5. Summary

In this article, we studied the nonlinear magnetosonic waves propagating perpendicularly to a magnetic field on the basis of fluid equations including two-ion species. Magnetosonic waves in case of two-ion plasma are split into two typical modes, namely, the high- and low- frequency modes. It shloud be noted that the dispersion branch of high frequency mode has a finite cut -off frequency. It turned out that the high frequency magnetosonic wave under the influence of finite cut-off frequency can be described by an extended K-dV equation rather than the conventional K-dV equation. It seems that the extended K-dV equation has only three conserved quantities though the conventional K-dV equation has infinite number of conserved ones.

Based on the reductive perturbation technique, we reduced the extended K-dV equation into the nonlinear Schrődinger type equation and discussed the propagation characteristics of this mode. We showed that the critical wave number K_c may exist due to the finite cutt-off frequency effect. This fact affects the situation whether the mode is modulationally stable or not. Detailed discussions on stationary solutions awaits futher investigations.

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References

[1] Kakutani T., Ono H., Tniuti T. and Wei C. C., J. Phys. Soc. Jpn. 24, 1159 (1968).

[2] Ohsawa Y., Phys. Fluids 29, 1844 (1986).

[3] Toida M. and Ohsawa Y., J. Plasma Fusion Res. 69, 1341 (1993).

[4] Miura R. M., J. Math. Phys. 9, 1204 (1968).

[5] Wadati M., Sanuki H. and Konno K., Progr. Theor. Phys. 53, 419 (1975).

[6] Taniuti T., Supplement of Progr. Theor. Phys. No. 55, 1 (1974).

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