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CONTROL OF PFIRSCH-SCHLÜTER CURRENT BY EXTERNAL POLOIDAL MAGNETIC FIELD IN CONVENTIONAL STELLARATORS

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ABSTRACT

The problem is analyzed whether it is possible to create net-current-free equilibrium configurations without Pfirsch-Schlüter current ($j_\zeta = 0$) in conventional stellarators with a planar circular axis. Recently such analysis was presented for the case, when only vertical magnetic field was a control parameter. Here external poloidal field is considered as arbitrary, which must be found from the condition $j_\zeta = 0$. It is shown that such suppression of Pfirsch-Schlüter current by means of external poloidal field is possible, theoretically, in shear-free systems and in stellarators with $\ell \geq 3$ only. It can be said also by other words that in ordinary ℓ=2 stellarators with a shear plasma equilibrium with $j_\zeta = 0$ is impossible at any choice of external poloidal field. In these systems, nevertheless, dipole component of Pfirsch-Schlüter current can be suppressed or strongly reduced by joint effect of vertical and quadrupole fields. It is shown that in this case phase inversion of Pfirsch-Schlüter current could be obtained at realistic conditions. These results explain, in particular, experimental observations of configurations almost insensitive to plasma pressure in Heliotron E.

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Keywords: stellarator, Pfirsch-Schlüter current, magnetic surfaces, vertical magnetic field, quadrupole field, magnetic axis shift, plasma equilibrium

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1. INTRODUCTION

It is well known that it is possible to reduce strongly Pfirsch-Schlüter current in a stellarator by a proper shaping of magnetic surfaces [1,2]. Such a reduction occurs when configurations are optimized to reach quasi-symmetry [2,3], which is the property first pointed out in [4] (see also [5]) as independence of the magnetic field strength on one from two angle Boozer coordinates. Optimization based on this principle was concentrated on configurations with nonplanar axes, because for such systems it could be consistent with demands of plasma stability. Finally, it was possible to reduce Pfirsch-Schlüter current and to get a magnetic well simultaneously in such systems.

Reduction of Pfirsch-Schlüter current results in weaker dependence of configuration on plasma pressure. An impressive example was given in Ref. [2]: magnetic surfaces of Helias-type stellarator practically had not been changed even at β as high as 50%.

Conventional stellarators with a planar axis have not been studied so intensively and systematically from the viewpoint of Pfirsch-Schlüter current reduction, though it is known from the early phase of stellarator research that it is possible, theoretically, to get configuration insensitive to plasma pressure in ordinary ℓ=3 stellarator [6]. But this nice property could be achieved when magnetic axis is shifted inward, into the region of stronger toroidal field, which is unfavorable for plasma stability. Later it was shown analytically [7-9] that combination of two factors is necessary to suppress Pfirsch-Schlüter current in conventional stellarators: strong inward plasma shift and large magnetic hill. Both of them are undesirable, which explains the lack of interest to this subject.

Being evidently below the level of reactor requirements, the inward shifted configurations in conventional stellarators with reduced Pfirsch-Schlüter current remain, nevertheless, to be very attractive from the viewpoint of physics. Also, the interest to this problem has been revived recently by the results from the helical device Heliotron E [10], where configurations showing extremely weak dependence on β have been experimentally found.

Existing theory [6-9] gave only two particular numerical examples of such configurations and qualitative explanation of the phenomenon, which was certainly insufficient to describe the observed effect in Heliotron E or, more general, relevant dependencies in stellarators with a shear. An attempt to get deeper insight has been done in Ref. [11], where the condition of Pfirsch-Schlüter current suppression in conventional stellarators was shown to be equivalent to two-dimensional differential equation with all values expressed through the vacuum magnetic field only. Then this equation has been used to analyze the possibility to suppress Pfirsch-Schlüter current in conventional stellarators using external vertical field only. This was natural, because in Refs. [6,7] the vertical field was the only varying parameter to get the desired effect. Also, in Refs. [8,9] only shift of magnetic surfaces produced by a vertical magnetic field was considered as a tool for Pfirsch-Schlüter current suppression. Finally, in experiments [10] in Heliotron E the operational control parameter was again the vertical field.

However, it is known that quadrupole field also can produce a pronounced effect on stellarator configuration. In stellarators with a shear it is possible to increase equilibrium β limit 1.5~2 times by rather moderate elongation of plasma column, see [12] and references herein. It follows from this that the presence of a quadrupole field in a device can strongly affect the value of Pfirsch-Schlüter current. At least, we can expect that smaller vertical field and related inward shift of plasma column will be needed for Pfirsch-Schlüter current suppression than those estimated in [11], because of partial reduction of this current due to the quadrupole field. The present article is devoted to the problem of Pfirsch-Schlüter current suppression by both vertical and quadrupole external fields.

2. BASIC EXPRESSIONS

Theoretical model to investigate Pfirsch-Schlüter current suppression in conventional stellarators with planar circular axis and helical magnetic field was described recently in Ref. [11]. Here only brief summary of necessary general expressions is given.

Pfirsch-Schlüter current in conventional stellarators with planar circular axis and helical magnetic field can be expressed as

$$j_\zeta \;=\; 2\pi R p'(\psi)(\Omega \;-\; \langle\Omega\rangle). \tag{1}$$

Here R is the major radius, p is the plasma pressure, ψ is the poloidal flux, Ω is a value characterizing the inhomogeneity of the magnetic field,

$$\Omega \;=\; \frac{<\tilde{\boldsymbol{B}}^2>_\zeta}{B_0^2} + 1 - \frac{R^2}{r^2} \;\cong\; \frac{<\tilde{\boldsymbol{B}}^2>_\zeta}{B_0^2} - \frac{2\rho}{R}\cos u, \tag{2}$$

$\tilde{\boldsymbol{B}}$ is the helical field, B_0 is the toroidal field at geometrical axis, r,ζ,z are usual cylindrical, and ρ,u,ζ are quasi-cylindrical coordinates related with toroidal geometrical axis so that $r=R-\rho\cos u$, Fig.1, brackets $\langle...\rangle$ denote volume averaging, and $\langle...\rangle_\zeta$ stands for averaging over toroidal angle ζ:

$$\langle f\rangle \;\equiv\; \frac{d}{dV}\int_V f d\tau, \qquad\qquad \langle f\rangle_\zeta \;\equiv\; \frac{1}{2\pi}\int_0^{2\pi} f d\zeta.$$

The right-hand side of expression (1) is identically zero when $\Omega=\langle\Omega\rangle$. This is equivalent to the condition

$$\Omega \;=\; \Omega(\psi), \tag{3}$$

which can be rewritten in the differential form as $\nabla\psi\times\nabla\zeta\cdot\nabla\Omega=0$ or

$$\left(\overline{\boldsymbol{B}}_p \;+\; \frac{\nabla\psi_V\times\nabla\zeta}{2\pi}\right)\cdot\nabla\Omega \;=\; 0, \tag{4}$$

see [11]. Here $\overline{\boldsymbol{B}}_p$ is the axisymmetric component of the poloidal field,

$$\overline{\boldsymbol{B}}_p = \frac{1}{2\pi}\nabla(\psi - \psi_V)\times\nabla\zeta, \tag{5}$$

and ψ_V is the poloidal flux of the helical field $\tilde{\boldsymbol{B}}$:

$$\psi_V = \frac{2\pi r^3}{RB_0} < \tilde{B}_z \int \tilde{B}_r \, d\zeta >_\zeta. \tag{6}$$

When Eq. (4) is satisfied, there is no Pfirsch-Schlüter current, and, correspondingly, $\overline{\boldsymbol{B}}_p$ is a vacuum field unaffected by a plasma.

In Eq. (4) there are two functions, ψ_V and $\Omega_0 \equiv < \tilde{\boldsymbol{B}}^2 >_\zeta / B_0^2$, which have to be calculated by given helical field $\tilde{\boldsymbol{B}}$. If at $\overline{\boldsymbol{B}}_p$=0 averaged vacuum magnetic surfaces $\psi_V = const$ are concentric circular toroids, then $\psi_V = \psi_V(\rho)$, $\Omega_0 = \Omega_0(\rho)$. In this case $\nabla\psi_V \times \nabla\zeta \cdot \nabla\Omega_0 = 0$. Also, these both functions can be expressed then through the vacuum rotational transform μ_h:

$$\frac{d\psi_V}{d\rho} = -2\pi B_0 \rho \mu_h(\rho), \qquad \frac{d\Omega_0}{d\rho} = \frac{m\left(\rho^4 \mu_h\right)'}{\ell R^2 \rho^2}. \tag{7}$$

The last equality is valid, if there is only one dominating harmonic of the helical field $\propto sin(\ell u - m\zeta)$. As a result, under these assumptions the left-hand-side of Eq. (4) takes the form

$$X \equiv \left(\overline{\boldsymbol{B}}_p + \frac{\nabla\psi_V \times \nabla\zeta}{2\pi}\right)\cdot\nabla\Omega = \frac{m\left(\rho^4\mu_h\right)'}{\ell R^2\rho^2}\boldsymbol{e}_\rho \cdot \overline{\boldsymbol{B}}_p + 2\frac{\boldsymbol{e}_r \cdot \overline{\boldsymbol{B}}_p}{R} + 2B_0\mu_h \frac{\rho}{R^2}\boldsymbol{e}_\rho \cdot \boldsymbol{e}_z. \tag{8}$$

It was assumed in Ref. [11] that $\overline{\boldsymbol{B}}_p = B_\perp \boldsymbol{e}_z$, which turned the condition $X = 0$ into the equation for $\mu_h(\rho)$. Its formal solution was $\mu_h = \mu_b x^{-(C+4)}$, where μ_b is a constant, $x = \rho / b$ is the dimensionless radius, and $C = 2\ell B_0 / (mB_\perp)$. This solution automatically eliminated ℓ=2 stellarators with a shear from possible candidates for getting $j_\zeta = 0$ all over the radius by means of a vertical field only. Here we will try another chance: parameters of a stellarator will be considered as given, and $X = 0$ will be used to find necessary $\overline{\boldsymbol{B}}_p$.

3. SOLVABILITY OF THE PROBLEM FOR DIFFERENT PROFILES OF ROTATIONAL TRANSFORM

The model described above was used in Ref. [11] to study suppression of Pfirsch-Schlüter current in conventional stellarators with the help of external vertical field, $\overline{\boldsymbol{B}}_p = B_\perp \boldsymbol{e}_z$. Let us consider now more general case:

$$\overline{\boldsymbol{B}}_p = \sum_{n=1} \frac{B_n}{nb^{n-1}} \nabla\left(\rho^n \sin nu\right) = \sum_{n=1} B_n x^{n-1}\left(\boldsymbol{e}_\rho \sin nu + \boldsymbol{e}_u \cos nu\right). \qquad (9)$$

Here B_n is the amplitude of the n-th harmonic of the external vertical field at the distance $\rho = b$ from the geometrical axis (by definition, $B_1 = B_\perp$ is the vertical field). Stellarators are described here in large-aspect-ratio approximation, so toroidal corrections are disregarded in (9).

When $\overline{\boldsymbol{B}}_p$ in such a form is substituted into the expression (8), it is reduced to

$$X = 2\frac{b}{R^2} B_0 x \left\{ \mu_h \sin u + \sum_{n=1} \left[\frac{m\left(x^4 \mu_h\right)'}{2\ell x^3} \frac{B_n}{B_0} - \frac{R B_{n+1}}{b B_0} \right] x^{n-1} \sin nu \right\}. \qquad (10)$$

Here prime denotes the derivative over the dimensionless minor radius $x = \rho / b$.

Radial dependence of coefficients of this Fourier expansion is governed by $\mu_h(x)$. One basic characteristic of μ_h is always important: the ratio μ_0 / μ_b, where μ_0 is the value of μ_h at the axis (x=0) and μ_b is its value at the boundary (x=1). There are three clearly different cases: $\mu_0 / \mu_b = 0$, which includes all stellarators with $\ell \geq 3$; $0 < \mu_0 / \mu_b < 1$, which represents $\ell = 2$ stellarators with shear; and $\mu_0 / \mu_b = 1$, which is for shear-free systems. Let us analyze, first, how this difference shows itself in the expression (10).

In $\ell = 2$ stellarators without shear and in stellarators with $\ell \geq 3$ rotational transform can be approximated by

$$\mu_h = \mu_b x^{2(\ell-2)}. \qquad (11)$$

In this case

$$X = 2\frac{b}{R^2}B_0x\left\{\left[\mu_h\left(1 + \frac{mB_\perp}{B_0}\right) - \frac{RB_2}{bB_0}\right]sinu + \sum_{n=2}\left(m\mu_h\frac{B_n}{B_0} - \frac{RB_{n+1}}{bB_0}\right)x^{n-1}sinnu\right\}. \quad (12)$$

It follows that $X = 0$ when there is a vertical field

$$B_\perp = B_\perp^0 \equiv -B_0 / m. \quad (13)$$

This is the result of Ref. [11]. Pfirsch-Schlüter current is suppressed then all over the radius, and fields of higher multipolarity ($n \geq 2$) are not needed for this purpose. Moreover, for stellarators with $\ell \geq 3$ this is the only solution of the problem, because at $\mu_h' \neq 0$ it is impossible to suppress harmonics with $n \geq 2$ in the expansion (12), if any B_n is nonzero.

For stellarators without shear, however, solution of the equation $X = 0$ is not unique. One can see from (12) that $X = 0$ at

$$B_2 = \mu_b\frac{b}{R}B_0\left(1 + \frac{mB_\perp}{B_0}\right), \quad (14)$$

and

$$B_{n+1} = \mu_b\frac{mb}{R}B_n \quad (15)$$

for $n \geq 2$. These both expressions are equivalent to

$$B_n = \left(\mu_b\frac{mb}{R}\right)^{n-2}\left(1 + \frac{mB_\perp}{B_0}\right)B^*, \quad (16)$$

where $n \geq 2$ and

$$B^* = \mu_b\frac{b}{R}B_0. \quad (17)$$

The constant B^* can be called "effective poloidal field".

To get $X = 0$, one possible choice is the same as needed for $\ell \geq 3$ case: B_n=0 for all $n \geq 2$. It leads again to (13), which was already discussed in Ref. [11]. The second solution reserves more freedom: amplitude of one harmonic may be arbitrarily given, and all others are determined iteratively through Eqs. (14) and (15). It is very natural for stellarators without shear to have $\mu_b mb / R << 1$. Under this condition $B_{n+1} << B_n$, so this new solution seems to be reasonable. At least, from theoretical point of view. This is a formal solution of the problem. It will be discussed in more details in the next section.

In the most interesting case, for ℓ=2 stellarators with a shear, instead of (11) another expressions for μ_h should be used:

$$\mu_h = \mu_0 + (\mu_b - \mu_0)x^2. \tag{18}$$

For such profile of the rotational transform we get from Eq. (10)

$$X = 2\frac{b}{R^2}B_0\sum_{n=1}(a_n + b_n x^2)x^n \sin nu, \tag{19}$$

where

$$a_1 = \mu_0 + \frac{2m}{\ell}\mu_0\frac{B_\perp}{B_0} - \frac{RB_2}{bB_0}, \tag{20}$$

$$b_1 = (\mu_b - \mu_0)\left(1 + \frac{3mB_\perp}{\ell B_0}\right), \tag{21}$$

and higher-order coefficients ($n \geq 2$) are

$$a_n = \frac{2m}{\ell}\mu_0\frac{B_n}{B_0} - \frac{RB_{n+1}}{bB_0}, \tag{22}$$

$$b_n = \frac{3m}{\ell}(\mu_b - \mu_0)\frac{B_n}{B_0}. \tag{23}$$

It can be easily seen from (19)-(23) that at $0 < \mu_0 / \mu_b < 1$ it is impossible to make $X = 0$ by any choice of the external magnetic field. Indeed, it is necessary to put $B_n = 0$ for $n \geq 2$ to make $b_n = 0$ in (23). Then, automatically, $a_n = 0$ also. But after this only one free parameter, $B_\perp$, is left, which does not allow to satisfy two different remaining equations $a_1 = 0$ and $b_1 = 0$.

Expressions (19)-(23) clearly show the difference between this and two other limiting cases: $\mu_0=\mu_b$ (no shear) and $\mu_0=0$ ($\ell=3$ stellarator). If there is no shear, one restriction on B_n disappears, because all b_n vanish identically. Only $a_n=0$ must be satisfied, which is possible even with one B_n arbitrary. In the opposite case, if $\mu_0=0$, then a_n contains only B_{n+1}, while b_n depends on B_n. The absence of the term with $B_\perp$ in a_1 in this case is the main factor allowing to make zero all coefficients in X.

This analysis shows that possibility to suppress Pfirsch-Schlüter current j_ζ by means of external poloidal field in a stellarator is essentially determined by the profile of its vacuum rotational transform μ_h. It is possible to make $j_\zeta=0$ in the case only when μ_h is described by one-parameter dependence (11). Inside this family there is a great difference between stellarators with $\ell\geq 3$ and stellarators without shear. There is only one solution for $\ell\geq 3$, which is given by (13). But for shearless systems there is more general solution (16), where, formally, one parameter ($B_\perp$, for example) is free. Let us discuss the latter case.

4. SHEARLESS CONFIGURATIONS WITHOUT PFIRSCH-SCHLÜTER CURRENT

It is a true fact that one parameter from B_n and $B_\perp$ in (14) and (15) is free. However, in a general case there always exists a potential danger to destroy magnetic surfaces $\psi=const$ by the external magnetic field, if it is strong enough, see Ref. [13]. Just this mere fact compels further analysis of the problem. In this particular case it will be useful for understanding the nature of the mentioned above degree of freedom.

To find external field necessary for Pfirsch-Schlüter current suppression, we have used Eq. (4), which is the differential equivalent of the condition (3). Function ψ could be found then by integrating (5). For shear-free stellarators we can show also another elegant way to solve the problem, starting from Eq. (3).

Condition (3) means that, in reverse, ψ is function of Ω only. At the same time this function must satisfy the equation

$$div \frac{\nabla(\psi - \psi_V)}{r^2} = -\frac{2\pi}{r} j_\zeta, \tag{24}$$

which is a consequence of Maxwell equation $\boldsymbol{j} = rot\boldsymbol{B}$ for the poloidal magnetic field given by Eq. (5). When $\psi = \psi(\Omega)$, its right-hand side is zero (no Pfirsch-Schlüter current), so in large-aspect-ratio approximation it turns into

$$\nabla^2 \psi(\Omega) = \nabla^2 \psi_V. \tag{25}$$

Two functions must be known to solve this equation: ψ_V and Ω. If rotational transform is given by (11), then one can easily get from (7)

$$\Omega_0 = -\frac{m}{\pi R^2 B_0} \psi_V. \tag{26}$$

In shearless case ($\mu_0 = \mu_b$)

$$\psi_V = -\pi B_0 \mu_0 \rho^2, \tag{27}$$

therefore the function Ω given by Eq. (2) can be expressed as

$$\Omega = \frac{d}{R}\left(\xi^2 - 2\xi \cos u\right), \tag{28}$$

where $\xi \equiv \rho / d$ is the polar radius normalized by distance d:

$$d = \frac{R}{m\mu_0}. \tag{29}$$

When expressions (27) and (28) are substituted into Eq. (25), it takes the form

$$\eta \psi''(\eta) + \psi'(\eta) = -\pi B_0 \mu_0 d^2, \tag{30}$$

where

$$\eta = \xi^2 - 2\xi \cos u + 1. \tag{31}$$

Solution of the Eq. (30) is

$$\psi = C_1 - \pi B_0 \mu_0 d^2 \eta + C_2 ln\,\eta, \quad (32)$$

where C_1 and C_2 are arbitrary constants.

By definition (5), function ψ here is the poloidal flux due to the helical and poloidal fields. To identify the sources necessary to create such a flux, we can represent solution (32) as

$$\psi = \psi_V + \psi_\perp\Big|_{B_\perp = B_\perp^0} + \psi_J, \quad (33)$$

where

$$\psi_\perp = \pi r^2 B_\perp = \pi(R - \rho cos u)^2 B_\perp \cong -2\pi R\rho B_\perp cos u + const \quad (34)$$

is the flux of homogeneous vertical field, $B_\perp^0 = -B_0/m$ is the value introduced earlier by Eq. (13), and

$$\psi_J = RJ\,ln\frac{1}{\sqrt{\xi^2 - 2\xi cos u + 1}}. \quad (35)$$

One can recognize in (35) the flux of a ring filament current J of radius $R-d$ lying in the equatorial plane ($\rho = d$, $u = 0$). Value J is arbitrary in (35).

Thus, to get $\Omega = \Omega(\psi)$ in shearless stellarator, it is necessary, according to (33), to apply vertical field $B_\perp^0 = -B_0/m$. After this $\Omega = const\cdot\psi$, which guarantees that Pfirsch-Schlüter current will not appear in this configuration. This property will not be affected then, if ring current placed at $r = R-d$, $z = 0$ will be added. The value of this current is the only degree of freedom to scan the whole family of configurations without Pfirsch-Schlüter current in stellarators without shear.

This is, of course, the same degree of freedom which manifested itself in the expression (16) in terms of the external field harmonic amplitudes, one from which could be considered as arbitrary. Solution (33) is equivalent to (16). At the same time it is much more convenient in showing separately two distinctively different parts of the external poloidal magnetic field, which allows to get

$\Omega = \Omega(\psi)$ in shear-free systems.

Magnetic surfaces are shifted under the action of a vertical field by

$$\Delta_\perp = \frac{RB_\perp}{\mu B_0}. \tag{36}$$

In shear-free stellarators this shift is the same for any surface. When $B_\perp = B_\perp^0$, we get $\Delta_\perp = -d$, where d is the distance introduced by Eq. (29). Minus here shows that it is an inward shift. The location of the magnetic axis is then $r_{ax} = R - d$. This case was already discussed in Ref. [11]: the shift $\Delta_\perp = -d$ is too large to be accepted. Also, this solution requires $\mu >> 1$, which is unusual for typical stellarators.

Now we can see that, unfortunately, these requirements, first, cannot be eased by using some additional external magnetic field. And, second, the only new possibility "to improve" this configuration with $j_\zeta = 0$ is, according to the general solution (33), to let any current J flow along the ring $r = R - d$, which is the position of the shifted magnetic axis. This ring-like internal current is not necessary for getting $j_\zeta = 0$. But nothing more, besides the magnetic field of this current, can be introduced without breaking this property.

All this means that configurations without Pfirsch-Schlüter current in conventional shear-free stellarators are out of the range of present-day practical accessibility. They may be considered as representing only the ideal case, which turned out to be too far from realistic systems.

This conclusion is pessimistic, but it is definite, which is one of theoretical merits of the analysis. Among others we can mention that (33) is the exact (in the frame of the model used) solution of the problem. Also, it is a general solution, which means that there are no other chances to improve the situation in this family of stellarators with ψ_V described by (27). Then, we should keep in mind that (33) represents the case, when there are no Pfirsch-Schlüter current at all. Though such a state cannot be reached in a real device, it is useful, anyway, to know about this state in attempts to optimize conventional planar-axis stellarators.

If complete suppression of the Pfirsch-Schlüter current is impossible, then another less challenging, but practical goal remains: to reduce this current and to increase by this equilibrium β limit. One tendency is obvious: current j_ζ is reduced when configuration is moved toward the state with $j_\zeta = 0$, which does exist for stellarators with rotational transform approximated by (11). In other words, by inward shift, as it was demonstrated by experiments in Heliotron E [10]. This effect can be illustrated by

$$\Omega - \langle\Omega\rangle = -\frac{2a}{R}\left[1 + \frac{m\left(a^4\mu_h\right)'}{2\ell R a^3}\Delta\right]\cos\theta, \qquad (37)$$

which becomes very simple for shear-free stellarators:

$$\Omega - \langle\Omega\rangle = -\frac{2a}{R}\left[1 + \frac{\Delta}{d}\right]\cos\theta. \qquad (38)$$

At realistic assumptions the distance d is rather large in this case, so $|\Delta|/d << 1$, and change of $\Omega - \langle\Omega\rangle$ due to the shift cannot be significant.

But to make smaller the value of $\Omega - \langle\Omega\rangle$ is only one of two possibilities to reduce j_ζ. The second, as it follows from Eq. (1), is to make smaller $p'(\psi)$ at given p due to the change of ψ. It can produce essential effect at $\Omega - \langle\Omega\rangle \neq 0$ even at outward plasma shift [14]. For a further more detailed study of this second chance we can propose to use the described above filament current. The influence of this current on vacuum magnetic surfaces $\psi = const$ can be seen from

$$\psi = \psi_V + \psi_J = -\pi d^2 B_0 \mu\left[\xi^2 - \frac{mB_z}{B_0}\ln\left(\xi^2 - 2\xi\cos u + 1\right)\right], \qquad (39)$$

where $B_z = -J/(2\pi d)$ is the vertical field created by the ring $(r = R - d)$ current J at geometrical axis $(r = R)$. At $B_z > -B_0/(4m)$ it is possible to shift magnetic surfaces inward or outward by varying J value. But at the same time this does not affect the value $\nabla\psi \times \nabla\Omega$ because $\nabla\psi_J \times \nabla\Omega = 0$.

5. ORDINARY $\ell=2$ STELLARATORS

As was already pointed out, it is impossible to get equilibrium configuration with $j_\zeta=0$ in $\ell=2$ stellarators with shear. But properly chosen external poloidal field can provide rather strong (5-6 times) reduction of Pfirsch-Schlüter current.

In the case analyzed in Ref. [11] it was done by vertical field only. The mechanism of this reduction is clearly seen from Eqs. (20) and (21): both values a_1 and b_1 become smaller at $B_\perp<0$. The best result is achieved, when $a_1+b_1x^2$ in the expression (19) vanishes at some x_0 inside the plasma. To get $b_1=0$, we need $mB_\perp/B_0=-2/3$. Such value of $B_\perp$ is rather large, but it is not yet sufficient: it is necessary to make $b_1<0$. But to make $a_1=0$, even more stronger vertical field is needed: $mB_\perp/B_0=-1$.

Quadrupole field allows to get similar reduction of Pfirsch-Schlüter current at smaller values of $B_\perp$. It is discussed below. But, first, we must mention another more interesting new chance: it is possible to get simultaneously $a_1=0$ and $b_1=0$ (which could not be done at $B_2=0$) at

$$B_\perp = -\frac{2}{3m}B_0, \qquad B_2 = \frac{1}{3}\mu_0\frac{b}{R}B_0. \tag{40}$$

The first term in expansion (19) is eliminated by this. It means that main cosine harmonic of Pfirsch-Schlüter current can be suppressed by such combination of $B_\perp$ and B_2. Then instead of $X=X_1\sin u$ we get $X=X_2\sin 2u$. In other words, the structure of Pfirsch-Schlüter current is changed from ordinary dipole to unusual quadrupole, when this external field is applied. Also, the amplitude of Pfirsch-Schlüter current becomes smaller, because at typical parameters $|X_2|<<|X_1^0|$, where $X_1^0=a_1^0+b_1^0x^2$ is the value of X_1 at $B_\perp=B_2=0$. This can be seen from

$$\frac{a_2}{a_1^0} = \frac{b}{3d}, \qquad \frac{b_2}{b_1^0} = \frac{b}{2d}, \tag{41}$$

which is obtained from (20)-(23) for $B_\perp$ and B_2 given by (40). Here d is the distance defined by Eq. (29). In all existing stellarators $d > b$.

If both vertical and quadrupole fields are imposed on vacuum stellarator configuration, its geometry is described by the equation

$$-\frac{\psi}{\pi B_0 b^2} = \mu_0 x^2 + (\mu_b - \mu_0)\frac{x^4}{2} + \frac{RB_2}{bB_0}x^2 \cos 2u + 2\frac{RB_\perp}{bB_0} x \cos u = const. \quad (42)$$

Magnetic axis is shifted then relative to geometrical axis by

$$\Delta_{ax} = \frac{RB_\perp}{\mu_{ax}B_0 + AB_2} = \frac{d}{\mu_{ax}/\mu_0 + B_2/B_2^{cr}} \frac{mB_\perp}{B_0}, \quad (43)$$

where $\mu_{ax} = \mu_h(\Delta_{ax})$, $A = R/b$ is the aspect ratio, and B_2^{cr} is the "critical amplitude" of the quadrupole field

$$B_2^{cr} = \mu_0 \frac{b}{R} B_0. \quad (44)$$

It is known that at $B_2 < B_2^{cr}$ magnetic surfaces in stellarators are elongated under the action of the quadrupole field, but topology of the configuration is not changed. But at $B_2 = B_2^{cr}$ magnetic axis is splitted, and at $B_2 > B_2^{cr}$ internal figure-eight separatrix appears, see [12,13]. In our case, as defined by Eq. (40), $B_2 / B_2^{cr} = 1/3$, which is not large.

If difference between μ_{ax} and μ_0 could be disregarded, we would get for parameters (40)

$$\Delta_{ax} = -d/2. \quad (45)$$

This gives an upper estimate for $|\Delta_{ax}|$, because always $\mu_{ax} > \mu_0$. Nevertheless, this value is already two times smaller than was necessary to suppress Pfirsch-Schlüter current by vertical field only in shear-free case, see Eq. (38).

It illustrates the positive effect of the quadrupole field. For Heliotron E with R=220 cm, $m = 19$, $\mu_0 = 0.53$ [10] we would get from (45) $\Delta_{ax} = -11$ cm. It is approximately half of minor radius (b=21 cm). This is the reason why very strong influence of plasma position on its equilibrium was observed in Heliotron

E [10]. In fact, due to the high shear in Heliotron E, real $|\Delta_{ax}|$ found from Eq. (43) is even smaller than this estimate: $\Delta_{ax} \approx -7.6$ cm. This could be certainly within the operational range of Heliotron E, if not only vertical, but also external quadrupole field would be optimized.

This is the "best" case, when it possible in experiment to get close to the state where j_ζ has no dipole component. But, for comparison, in CHS $R=100$ cm, m=8, $\mu_0 = 0.3$, b=20 cm, and, accordingly, $|\Delta_{ax}| \approx b$. For LHD $|\Delta_{ax}|/b$ is smaller, but also of the order of unity: $R=375$ cm ("the standard configuration" [15]), m=10, $\mu_0 = 0.36$, which gives d=104 cm, and $d/(2b) \approx 0.9$.

We can conclude that, theoretically, it is possible to suppress the main dipole component of Pfirsch-Schlüter current by means of external vertical and quadrupole fields. But rather large values of $|\Delta_{ax}|/b$ are necessary for that. It means that condition $a_1 = b_1 = 0$ is too ambitious for conventional stellarators (except, maybe, Heliotron E). Let us analyze less restrictive opportunities.

For existing devices the range of acceptable $B_\perp$ and B_2 is limited by admissible values of $|\Delta_{ax}|/b$. In LHD, for example, b=60 cm, and position of magnetic axis can be varied from R_{min}=3.6 m to R_{max}=3.9 m [15]. Then only $|\Delta_{ax}|/b$<0.25 can be of practical interest in this case, which is approximately 3 times smaller than given by (45).

It follows from (43) that smaller $|B_\perp|$ and larger B_2 are better for getting smaller $|\Delta_{ax}|$. Parameters (40) are optimal in a sense that they make $a_1 = b_1 = 0$. Smaller $|B_\perp|$ makes $b_1 > 0$. Then also $a_1 > 0$, but by increasing B_2 it is possible, as one can see from (20), to keep $a_1 = 0$ or even to make a_1 negative. If $a_1 < 0$, but $a_1 + b_1 > 0$, we get the result similar to that described in Ref. [11]: the amplitude of the main dipole harmonic of Pfirsch-Schlüter current changes its sign somewhere inside the plasma. In other words, at some magnetic surfaces it vanishes and, besides, inside and outside this surface it flows in opposite directions. Because of the mutual cancellation of the magnetic fields produced by such opposite currents, their integral effect on equilibrium configuration is much weaker than usual.

If $a_1 + b_1 x_0^2 = 0$ at some x_0, then, as it follows from (20) and (21), three parameters x_0, $B_\perp$ and B_2 are related by

$$\frac{mB_\perp}{B_0} = \frac{\mu_0\left(B_2 / B_2^{cr} - 1\right) - (\mu_b - \mu_0)x_0^2}{\mu_0 + 1.5(\mu_b - \mu_0)x_0^2}. \tag{46}$$

We can see from here that in this case $B_\perp$ must be negative at $B_2 \leq B_2^{cr}$. To make $|B_\perp|$ smaller and to minimize $|\Delta_{ax}|$, let us put $B_2 = B_2^{cr}$. Then

$$\Delta_{ax} = \frac{RB_\perp}{B_0(\mu_0 + \mu_{ax})} \approx -\frac{d}{2}\frac{(\mu_b - \mu_0)x_0^2}{\mu_0 + 1.5(\mu_b - \mu_0)x_0^2}, \tag{47}$$

where the latter expression is the estimate similar to (45).

This value is smaller than (45) for any x_0. To get integral independence on β, it is necessary to suppress cosine component of j_ζ at some intermediate magnetic surface inside the plasma. If $x_0 = 0.5$, $|\Delta_{ax}|$ estimated from (47) is 3.5 smaller than (45) for stellarator with $\mu_0 / \mu_b = 1/3$, see Fig. 2, and 5.5 times smaller for $\mu_0 / \mu_b = 1/2$. Such values of $|\Delta_{ax}|$ are quite realistic. It means that it might be possible, in principle, to observe in conventional stellarators the effect similar to that observed in Heliotron E [10]: pressure-induced plasma column shift, Δ_β, becomes weak and almost vanishes, when plasma column is shifted inward by external vertical field. Quadrupole field allows to reach such a state at reasonable values of $|B_\perp|$.

Here vertical and quadrupole magnetic fields have been discussed, because the main dipole term in (19) is determined by $B_\perp$ and B_2 only. In general, only vertical and quadrupole magnetic fields can affect entirely the whole stellarator configuration, but poloidal fields of higher multipolarity cannot "penetrate" up to the magnetic axis, see [12]. In our case it is evident from (19)-(23). Higher-order harmonics can be useful for the control of the peripheral part of plasma column. Maybe, it will be necessary at more refined level of optimization of planar-axis conventional stellarators.

6. CONCLUSION

Reduction of Pfirsch-Schlüter current is one of the natural ways of stellarator optimization. The merits of such reduction have been already convincingly demonstrated for Helias-type systems [16]. However, in this respect traditional conventional stellarators with planar circular axis have not been yet optimized to similar extent. Existing theory even did not give an answer to the principal question: whether it is possible or not to get plasma equilibrium with diamagnetic current only.

We analyzed here this problem using equation (4) as a starting point. When it is satisfied, there is no Pfirsch-Schlüter current at all. If $j_\zeta = 0$ all over plasma cross-section, there is no contribution to the poloidal magnetic field from equilibrium plasma currents. Then $\overline{\boldsymbol{B}}_p$ in Eq. (4) is the vacuum field. Because of this there is no need to solve equilibrium problem in searching for configurations without Pfirsch-Schlüter current. This is a great advantage of this approach. It allows to prove existence or nonexistence of such equilibria. In the latter case we can clearly see the reason, why the condition $j_\zeta = 0$ cannot be fulfilled

It follows from the analysis that plasma equilibrium without Pfirsch-Schlüter current can exist, theoretically, in two cases only: in stellarators with $\ell \geq 3$ and stellarators without shear. To obtain such equilibrium state, in both cases vertical field $B_\perp = -B_0 / m$ is necessary. This is the only solution for $\ell \geq 3$, but there is one additional degree of freedom for stellarators without shear: magnetic field of filament current with $r = R - d$, $z = 0$ does not break the property $j_\zeta = 0$ achieved after imposing $B_\perp = -B_0 / m$. This result, however, seems to be of theoretical importance only, because this current must be exactly at the magnetic axis shifted by the vertical field to $r = R - d$.

There are no other chances to get plasma equilibrium with $j_\zeta = 0$ in conventional stellarators, which have $\psi_V = \psi_V(\rho)$ at $\overline{\boldsymbol{B}}_p$=0. Also, it is impossible to get such equilibrium at any choice of external poloidal field $\overline{B}_p$ in ordinary ℓ=2 stellarators with a shear.

Then only less ambitious opportunity remains for these systems: to reduce the amplitude of Pfirsch-Schlüter current and to control its structure. For this purpose combination of external vertical and quadrupole fields is much better than vertical field alone.

It is shown that by proper choice of $B_\perp$ and B_2, see Eq. (40), it is possible to suppress the main dipole component of Pfirsch-Schlüter current all over plasma cross-section. Under the action of such magnetic field instead of usual $j_\zeta \propto \cos\theta$ we would get $j_\zeta \propto \cos 2\theta$. According to estimates, at present it could be done in Heliotron E only, where conditions (40) are acceptable. This result explains the observed reduction of pressure-induced plasma shift in recent experiments [10].

This state, however, is beyond the operational range of other existing devices, because too large inward plasma shift is needed. But another interesting phenomenon certainly might be observed at realistic conditions: Pfirsch-Schlüter current phase inversion inside the plasma. In this case dipole harmonic of j_ζ vanishes at some magnetic surface, where this inversion happens. Inside and outside this surface equilibrium current flows in opposite directions. Also, its amplitude is strongly reduced. As a result, dependence on β must be weak then.

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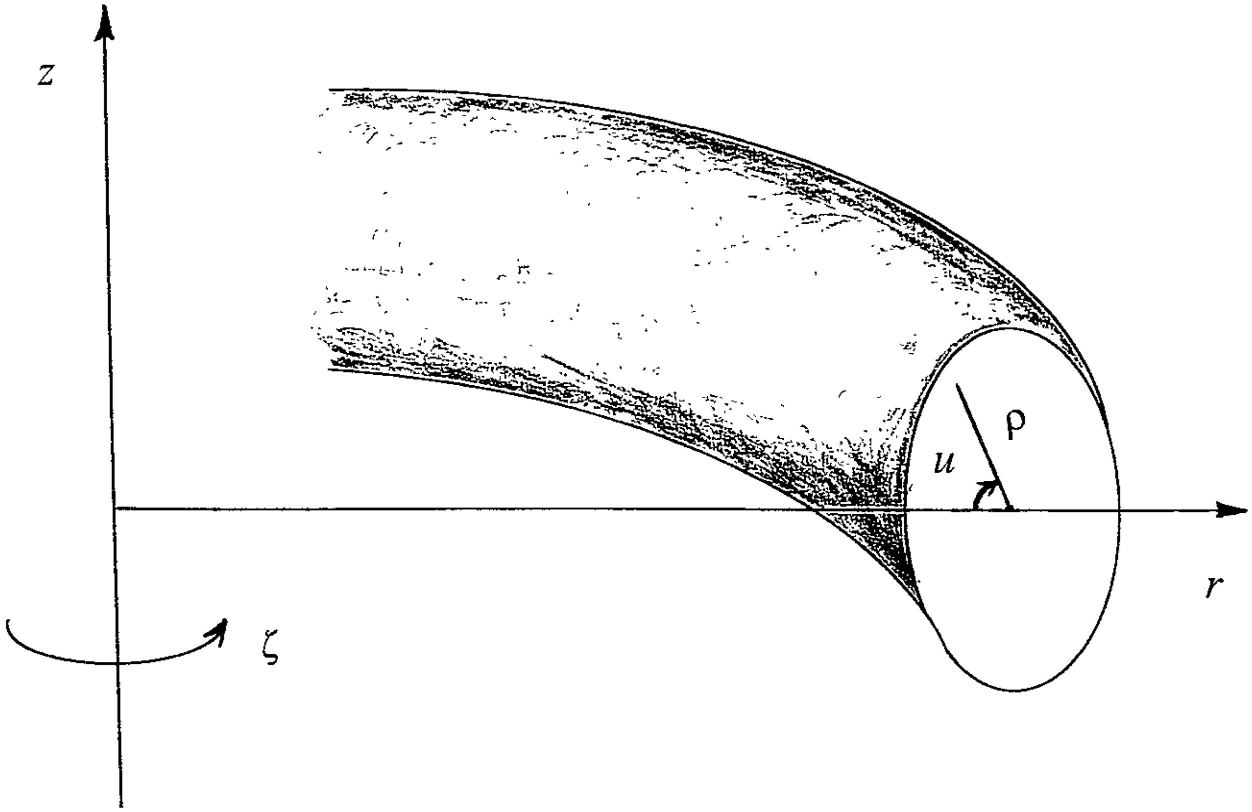

Fig. 1. Coordinates used in the analysis

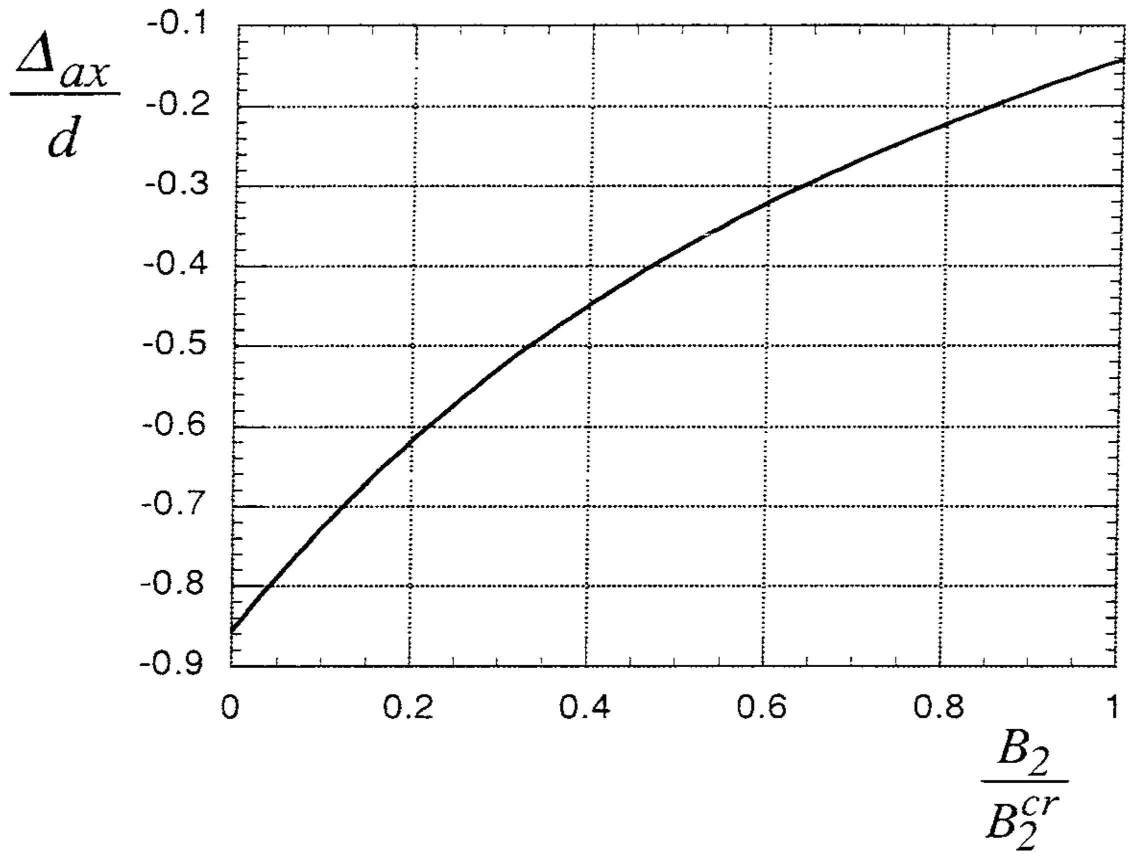

Fig. 2. Reduction of magnetic axis shift due to the quadrupole field for the case $x_0 = 0.5$ in ℓ=2 stellarator with $\mu_0 / \mu_b = 1/3$

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