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A New Variable Transformation Technique for the Nonlinear Drift Vortex

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Abstract

The dipole vortex solution of the Hasegawa-Mima equation describing the nonlinear drift wave is a stable solitary wave which is called the modon. The profile of the modon depends on the nonlinearity of the $E \times B$ drift. In order to investigate the nonlinear drift wave more accurately, the effect of the polarization drift needs to be considered. In case of containing the effect of the polarization drift the profile of the electrostatic potential is distorted in the direction perpendicular to the $E \times B$ drift.

KEYWORDS: Drift wave, potential vorticity, Hasegawa-Mima equation, modon

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The Hasegawa-Mima (H-M) equation is one of equations which describe the nonlinear drift wave in an inhomogeneous magnetized plasmas in two dimension, most simple and successful.[1] The solution of the H-M equation, which is obtained by Larichev and Reznik,[2] has the profile of the localized dipole vortex, so-called the modon, in the plane perpendicular to the uniform magnetic field $\boldsymbol{B}_0$. Modons propagate stably in the direction of the electron diamagnetic flow with the phase velocity of linear modes.

Recently, the incorporation of the effects of the shear flow or the magnetic shear, which are not included in the H-M equation, have been discussed.[3-5] This means mathematically to add higher order quantities into the H-M equation. The polarization drift is also one of higher order effects, which is an important factor to consider the time evolution of the nonlinear drift wave. However, even if the effects of the polarization drift is taken into consideration, it is very difficult to solve analytically or to calculate even on the computer the nonlinear drift wave equation. The main subject of the present paper is to show how to avoid the difficulty and to solve the equation in a simple way.

The Charney equation for Rossby waves in a planetary atmosphere is similar to the H-M equation. We have obtained the higher order terms for the Rossby wave similar to the polarization drift velocity for the drift wave.[6] We have also derived previously the equation including those higher order terms by using the transformation of variables in the long wavelength ordering, that is $\rho_s^2\nabla^2 \ll 1$, where ρ_s is the effective Larmor radius and nabla is the derivative with respect to x and y.[6] In present paper we apply a transformation of variables to the drift wave equation in the short wavelength regime.

We assume the cold ion plasma and the electrostatic fields that is $T_i = 0$ and $\boldsymbol{E} = -\nabla\phi$, where the electrostatic potential ϕ is a function of x and y. In this situation the drift wave propagates in the y direction with the drift velocity which is proportional to the unperturbed density gradient of the x direction. For simplicity, we introduce dimensionless variables by $\boldsymbol{r}/\rho_s \to \boldsymbol{r}$, $\omega_{ci}t \to t$, $e\phi/T_0 \to \phi$, where $\rho_s = c_s/\omega_{ci}$, $c_s = \sqrt{T_0/m}$, $\omega_{ci} = eB_0/m$ and T_0 is the mean electron temperature. The basic equations of the drift wave are the equations of motion and continuity for ions, as follows:

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla\phi + \boldsymbol{v}\times\hat{\boldsymbol{z}} \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\boldsymbol{v}) = 0 \tag{2}$$

where $\boldsymbol{v}$ and n are the velocity and density of ions in the two-dimensional plane, respectively. Since the inertia of electrons is negligible and the quasi-neutrality holds, both electrons and ions have the Boltzmann distribution,

$$n = n_0(x)\exp(\phi), \tag{3}$$

where the equilibrium density $n_0(x)$ assumes the form $\ln n_0(x) = \nu_0 x$. Taking the curl of eq.(1), and substituting eq.(2) into it, we obtain the following conservation law,

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right) q = 0$$
$$q = (1+\omega)/n \tag{4}$$

where ω means the z component of the vorticity, $\omega = \hat{\boldsymbol{z}}\cdot\nabla\times\boldsymbol{v}$, and q is called the generalized vorticity or the potential vorticity.

In the short wavelength regime, we introduce the following ordering with the smallness parameter ϵ,

$$\epsilon \sim \frac{\partial}{\partial t} \sim \boldsymbol{v}\cdot\nabla \sim \omega. \tag{5}$$

In addition, the dimensionless electrostatic potential ϕ is regarded as the same order as ϵ, because the electrostatic potential is much less than the electron kinetic energy. The velocity $\boldsymbol{v}$ in the bracket in eq.(4) can be obtained by iteration of eq.(1),

$$\boldsymbol{v} = \boldsymbol{v}_E + \boldsymbol{v}_p, \tag{6}$$

where

$$\begin{cases} \boldsymbol{v}_E = \hat{\boldsymbol{z}}\times\nabla\phi \\ \boldsymbol{v}_p = -\left(\dfrac{\partial}{\partial t} + \boldsymbol{v}_E\cdot\nabla\right)\nabla\phi. \end{cases} \tag{7}$$

Here $\boldsymbol{v}_E$ and $\boldsymbol{v}_p$ are the $E\times B$ drift and the polarization drift, respectively. If we consider only the lowest order of eq.(4), we obtain the H-M equation,

$$\frac{\partial q}{\partial t} + \{\phi, q\} = 0 \tag{8}$$

where the bracket of the second term on the left hand side is the Poisson's bracket with respect to x and y. The equation, which we will derive, has to include the effects of the polarization drift which is not included in eq.(8). It would be possible to derive the equation including the polarization drift from eq.(4). However, it has too much nonlinear terms and higher derivatives to be solved directly. In order to derive a simpler equation, we introduce the transformation of variables, as follows:

$$\begin{cases} \boldsymbol{x} & \rightarrow & \boldsymbol{X} = \boldsymbol{x} + \nabla\phi \\ t & \rightarrow & \tau = t \\ \phi & \rightarrow & \Phi = \phi + (\nabla\phi)^2/2 \\ q & \rightarrow & Q = q \end{cases} \tag{9}$$

where $\boldsymbol{X}$ is the two dimensional new coordinates' vector (X, Y).

We note that the (X, Y) transformed by the transformation of variables (9) are not the normal coordinates because X and Y include the variable ϕ. The introduction of new variables (X, Y) is nothing but a method to simplify the equation including higher order terms.

The potential vorticity q is transformed to Q which is described by X, Y and Φ. It is to be noted that the transformation of variables (9) for the short wavelength are up to the second order and depend on the time, while the transformation for the long wavelength, which we derived in the previous report,[6] was under the stationary condition up to the third order in ϵ. With use of the transformation of variables (9) and eqs.(6) and (7), equation(4) is rewritten as,

$$\frac{\partial Q}{\partial \tau} + \{\Phi, Q\} = 0, \tag{10}$$

where now, the Poisson's bracket in eq.(10) is with respect to X and Y. It is easy to find that eq. (10) is the same form as the H-M equation (8). In the lowest order, Q is the same as q in the H-M equation. Therefore, we obtain the equation of the same form as the H-M equation written in term of X, Y and Φ as follows:

$$\frac{\partial}{\partial \tau}(\nabla_X^2 \Phi - \Phi) + \{\Phi, \nabla_X^2 \Phi - \Phi\} + \nu_0 \frac{\partial \Phi}{\partial Y} = 0 \tag{11}$$

where ∇_X is the derivative with respect to X and Y, and $\nu_0 = \frac{d}{dX} \ln n_0$. As mentioned above, the stationary solution Φ_0 of eq.(11) is the modon obtained by Larichev and Reznik,[2] which travels to the Y direction with the constant speed u as follows:

$$\Phi_0(R, \theta) = \begin{cases} [A\mathrm{J}_1(kR) + \alpha R] \cos\theta & (R \le r_0) \\ B\mathrm{K}_1(pR) \cos\theta & (R > r_0) \end{cases} \tag{12}$$

where r_0 is the separatrix radius, $R = (X^2 + (Y - u\tau)^2)^{1/2}$, $A = -(u + \nu_0)r_0/k^2\mathrm{J}_1(kr_0)$, $B = ur_0/B\mathrm{K}_1(pr_0)$, $\alpha = u + (u + \nu_0)/k^2$, $p^2 = 1 + \nu_0/u$ and k is decided by continuity of $\partial\Phi_0/\partial R$ on $R = r_0$, which is written as,

$$\frac{\mathrm{J}_2(kr_0)}{\mathrm{J}_1(kr_0)} + \frac{k\mathrm{K}_2(pr_0)}{p\mathrm{K}_1(pr_0)} = 0. \tag{13}$$

The profile of eq.(12) is shown in Fig. 1 in (X, Y) space. However, it is described by X and Y so that we need to transform it to the real (x, y) space by using the inverse transformation of the transformation of variables (9). The electrostatic potential ϕ is described by the modon solution Φ_0 as follows:

$$\phi(x, y) = \Phi_0(\boldsymbol{x} + \nabla\Phi_0(\boldsymbol{x})) - (\nabla\Phi_0(\boldsymbol{x}))^2/2. \tag{14}$$

The profile of eq.(14) shown in Fig. 2 is distorted toward the x direction as a result of the transformation of variables (9). The potential ϕ has a distorted profile as shown in Fig. 2.

We note that the direction of the polarization drift $\boldsymbol{v}_p$ is perpendicular to contour lines in Fig. 1.

Figure 3 shows the case $r_0 = 2$ with other parameters same as in Fig. 2. As the separatrix radius r_0 increases, the gradient of Φ_0 around the separatrix $R = r_0$ becomes steeper and the profile of ϕ in Fig. 3 is distorted stronger than in Fig. 2. This means that the magnitude of the separatrix radius has the strong influence on the distortion of the potential ϕ.

The transformation of variables in the long wavelength ordering, in which the effect of the polarization drift is of the order of ϵ^3, is also identical to in the short wave length ordering. However, in this case, the effect of the polarization drift is less than the ones of other quantities which are of the order of ϵ^2, for example, the shear flow and the shear magnetic field. On the other hand, it becomes indispensable when we investigate these effects in the short wavelength ordering.

In conclusion, we find that the transformation of variables (9) gets rid of complicated nonlinear terms and higher derivatives in the nonlinear drift wave equation and makes the nonlinear analysis extremely transparent as compared with the straightforward calculation. The solution (14) of simplified equation (11) with use of the transformation of variables (9) propagates stablely as the modon and the simulation of it is a future subject.

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Figure Caption

Fig. 1. An electrostatic potential ϕ for the modon, where $\nu_0 = 0.1$, $u = 0.1$ and $r_0 = 1$ in the dimensionless coordinates system. The interval of contour lines is 0.02 ranging from -0.16 to 0.16. The modon propagates to the positive y direction.

Fig. 2. An electrostatic potential ϕ of eq.(14). The interval of contours and parameters are the same as them in Fig. 1. The interval of contour lines is 0.04 ranging from -0.20 to 0.16.

Fig. 3. An electrostatic potential ϕ of eq.(14), where $r_0 = 2$ and other parameters is same as them in Fig. 1. The interval of contour lines is 0.125 ranging from -0.5 to 0.5.

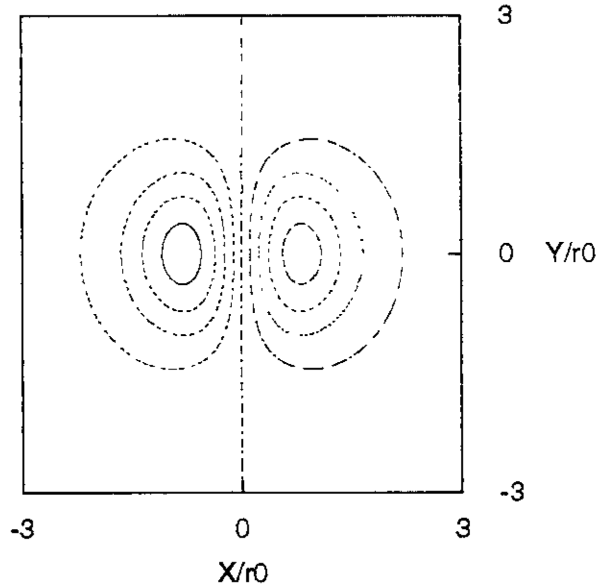

Fig. 1.

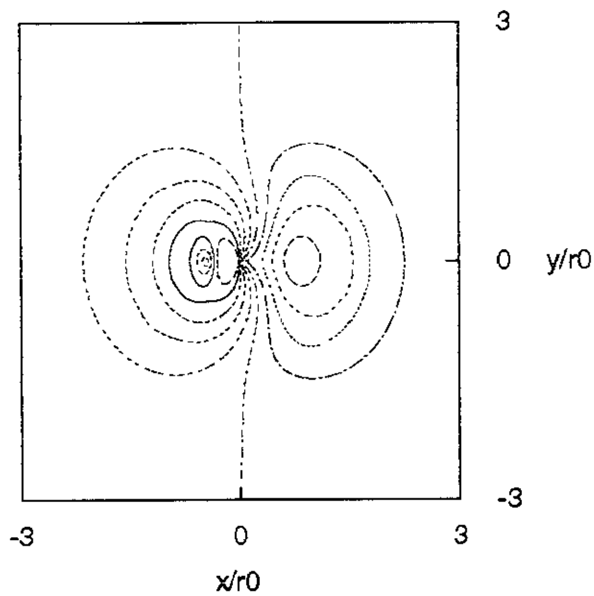

Fig. 2.

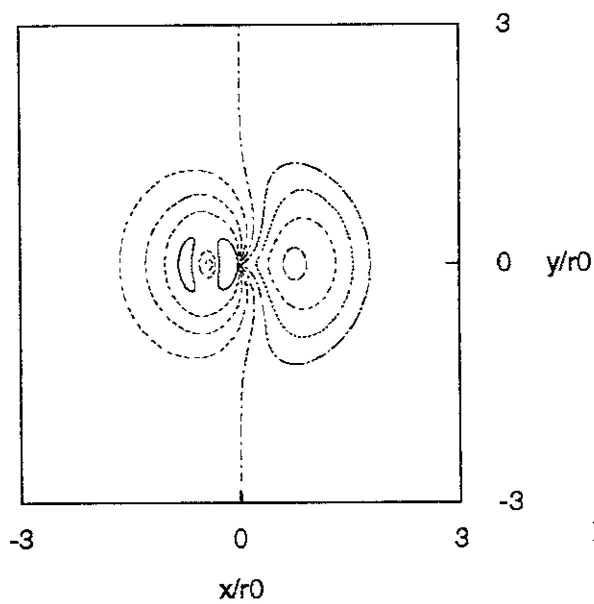

Fig. 3.

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