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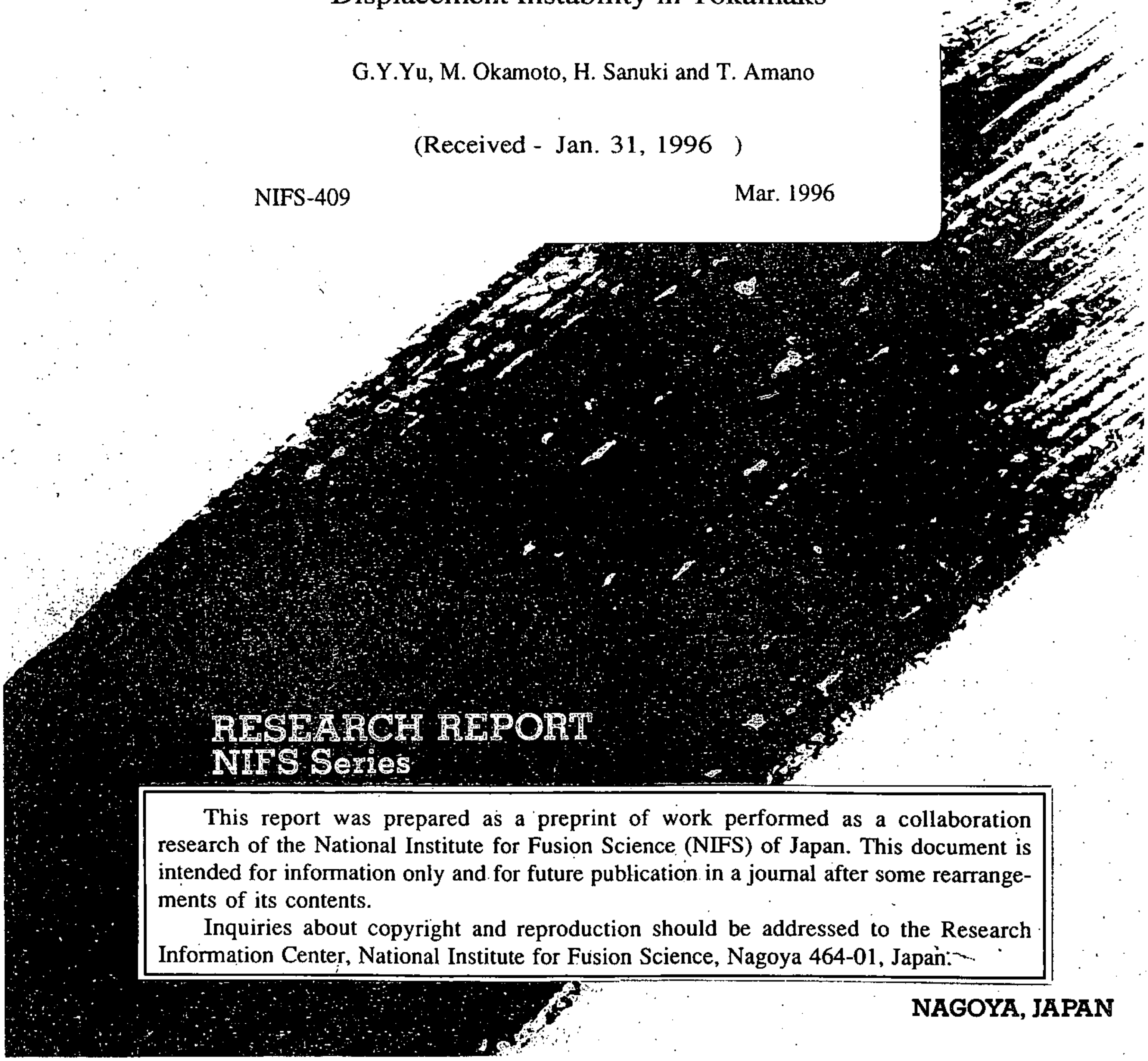

Effect of Plasma Inertia on Vertical Displacement Instability in Tokamaks

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Effect of Plasma Inertia on Vertical Displacement Instability in Tokamaks

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Abstract

The effect of plasma inertia on vertical displacement instability (VDI) is investigated in both linear and nonlinear regimes. In a linear case, the solution of the dispersion shows the existance of a critical elongation for a certain distance of the conducting wall. In the nonlinear case a difference between the cases with and without inclusion of plasma inertia is found. The inclusion of the plasma inertia leads to a "softening" of the critical behavior of the displacement against the distance of the resistive wall. Without inclusion of plasma inertia there exists a critical distance between the wall and the plasma below which VDI can not be stabilized no matter how the parameters in feedback controlling scheme are set. An analysis of the dispersion of linear VDI shows that the plasma inertia plays an important role in the properties of the solutions of dispersion and hence in the behavior of VDI.

Keywords: vertical displacement instability, plasma inertia, nonlinear behavior, tokamak

I. Introduction

It is believed that elongated D-shaped configurations show their advantages in achieving higher plasma performance on tokamaks. Nevertheless elongated configurations are unstable against vertical displacement instability (VDI)[1]. VDI is known of importance because it may lead to plasma disruption and even the damage of the device if it can not be well controlled.

Efforts have been put on the study of the effect of various parameters, for example plasma geometry, pressure and profile, on VDI[2,3] as well as the analysis of the VDI behavior in tokamak. It is believed that VDI in tokamak is composed mainly of two components[4], namely a fast oscillation components with a growth rate of 10^5/sec. and a period of few microseconds, as well as a slow shifting component with a characteristic time (or inverse of the growth rate) of the resistive time of the conductors, usually about 10^1 to 10^2 milliseconds. An active feedback scheme is widely used to stabilize VDI in a longer time period.

For weakly elongated plasma the vertical displacement can be considered as a small quantity and a linearized theoretical model is applied. The solution of the dispersion shows the existence of a critical elongation with respect to a certain configuration of conductors. Beyond the critical elongation VDI can not be stabilized no matter how the parameters in active controlling system are set. When one studies the time evolution of the displacement in a period much longer than that of the fast oscillation the plasma inertia is usually neglected[5,6].

For highly elongated case the plasma displacement can no longer be thought to be small. The problem should then be treated in a nonlinear frame of work and a numerical solution is needed.

It is understandable that neglect of plasma inertia would possibly change the properties of both the equations (in linear case, the dispersion) and thus the solution. So in this paper the effect of plasma inertia on VDI is investigated in both linear and nonlinear cases. The critical behavior of VDI with respect to both elongation and the distance of the conductor are shown.

The rest of the paper is arranged as follows. In Sec. II the physical model is introduced. Section III gives the critical condition in a linear case. The results of nonlinear VDI is presented in Sec. IV. Some discrepancy between the cases with and without inclusion of plasma inertia is shown. Finally the conclusion and discussion are given in Sec. V.

II. Physical Model

Figure 1 gives an up-down symmetric tokamak model used in the study. The passive conductors surrounding the plasma are set on the upper side, inside and outside of plasma. The distance of the upper conductor from the equatorial plane of tokamak Zup is used to represent the coupling between the conductors and the plasma. Two pairs of coils are applied for the active control. The plasma torus is assumed to be of D-shape, of which the magnetic configuration and the profiles are obtained from the solution of Grad-Shafranov equation[3]. Both plasma and the passive conductors are filamentized into the toroidal rings. For sake of simplicity a rigid displacement is assumed in the present study.

Plasma motion in vertical direction can be described by the equation

$$m \frac{d^2Z}{dt^2} = -2\,\pi\, R_P\, I_P\, B_R \qquad (1)$$

with B_R the radial component of magnetic field collecting the contribution from the equilibrium field, passive conductors as well as the active coils. Kirchhoff equation is used to describe the coupling between all the conductors, including plasma itself

$$\sum_k \sum_{\beta\neq\alpha} \frac{d}{dt} M_{kj}^{(\alpha\beta)} J_k^{(\beta)} + \frac{d}{dt} L_j^{(\alpha)} J_j^{(\alpha)} + R_j^{(\alpha)} J_j^{(\alpha)} + \sum_k \frac{d}{dt} M_{kP}^{(\alpha P)} I_P = V_j^{(\alpha)} \qquad (2)$$

In Eq. (2) α and β can be set to be S and A. Here the superscripts S, A and P represent the passive conductors, the active controlling coils and the plasma respectively. *L*, *R* and *M* are the inductance, resistance and mutual-inductance of conductors. J_j is the current in the j-th conductors and V_j the applied voltage in the coils. In our case a feedback scheme is applied so

$$V^{(A)} = g\,[Z(t) - Z_0] \tag{3}$$

with g the gain factor and Z_0 the reference displacement used.

For finite vertical displacement the current distribution induced in the conductors depends on the displacement so Eqs. (1) and (2) are nonlinear. Numerical solution is necessary.

III. Critical condition in linear case

Before performing the numerical solution of Eqs. (1) to (3), the solution of the dispersion of linearized Eqs. (1) - (3) is briefly presented below. The dispersion deduced as

$$\Delta = \gamma^4 + a\,\gamma^3 + b\,\gamma^2 + c\,\gamma + d = 0 \tag{4}$$

gives the solutions of the growth rate and the frequency of the displacement. In Eq. (4) a,b,c and d are the quantities related to plasma geometry, plasma density and conductors location etc. Figure 2 gives the dependence of the two solutions of the dispersion for a certain elongation (1.6 in the case of Fig.2) upon Zup value. It is shown that one solution, with growth rate of Gama1 and frequency of Omg1, is essentially a purely growing mode while the another with growth rate of Gama2 and frequency of Omg2 is a fast oscillating and decaying mode. The critical behavior occurring around a certain value of Zup (.75 as indicated in Fig.2) is characterized by the following facts: for the purely growing mode, the growth rate increases rapidly (from $10^0/\tau_S$ to $10^4/\tau_S$); for the oscillating mode the frequency drops to zero and the decay rate (negative growth rate) increases very fast. Here τ_S is the resistive time of the conductor and is about 40 ms in the case calculated. As known from the linear control theory, beyond the critical condition VDI can not be stabilized no matter how the parameters in the active controlling system are set. Figure 3 gives the growth rate for purely growing mode as functions of both Zup and e (elongation of plasma). The critical behavior for different e occurs at different Zup value.

IV. Solution of nonlinear VDI

In case of large elongation and finite displacement the nonlinear solution of Eqs. (1) to (3) is conducted to get the evolution of the displacement in a time period much longer than the resistive time of the conductors[4]. Usually for longer time period the slow shifting component is taken into account only and the fast oscillating component is omitted. This corresponds to neglect the inertia term in Eq. (1), as done in some of the previous investigations[5,6]. Eq. (1) then becomes to be

$$2 \pi R_P I_P B_R = 0 \qquad (5)$$

By numerically solving Eqs. (5), (2) and (3) the time evolution of the displacement is obtained. In this paper the final solution is determined by using an optimizing technique. That is by optimizing the gain factor g in (3) to obtain the minimum displacement within the calculated time period. By varying Zup value a dependence of the minimum displacement Zm as a function of Zup can be obtained, as shown in Fig.4. It is seen that the dependence of Zm vs Zup jumps suddenly as Zup is below a certain value. This is in contrary with the intuition.

However when one analyzes the behavior of dispersion it can be found that there exists an essential difference between the solutions of the dispersion in cases with and without inertia term. This means that the neglect of the inertia term will change the property of the dispersion equation (4) and hence lead to the solutions which are probably quite different from those of the dispersion with plasma inertia. Figure 5 gives the growth rate, being a solution of dispersion without inertia term, as a function of Zup. It is seen that at small Zup value a positive growth rate exists and increases with Zup value followed by a sharp drop to a negative value at a certain Zup value. We believe that this solution leads to the behavior of Zm vs Zup in Fig.4.

By carrying out the same procedures as stated above to get the numerical solution of Eqs. (1) - (3) the dependence of the resultant Zm upon Zup is obtained and plotted in Fig. 6. The result seems reasonable. Physically in case without plasma inertia a purely growing mode exists. The response of conductors to such a mode is not sufficient to stabilize it. The active feedback system can not offer an effective stabilization to this mode due to the coupling between the coils and other conductors, even the instataneous response of the applied voltage to plasma displacement is adopted in our calculation. This leads to a larger minimized displacement obtained in the calculation as Zup is less than some value (Fig.4). Plasma inertia contributes a high frequency (about 10^5/sec) oscillating mode of VDI, as shown in Fig.2. For such a short time scale the conductors can be considered to be ideal. The response of the conductor to this fast oscillating mode is always stabilizing. Hence the results in Fig.6 are obtained. The numerical result obtained in our calculation does not appear to be so smoothing. Nevertheless it is clearly shown that both Zm and the rate of change of Zm vs Zup is increasing with Zup.

If one compares the critical behavior in both linear and nonlinear cases it can be found that due to the nonlinear coupling of the conductors-plasma as well as conductors-conductors the critical behavior in nonlinear case is "softened".

V. Conclusion and Discussion

Based upon the simple model considered in this investigation it can be concluded that the plasma inertia plays a crucial role in the stabilization of nonlinear VDI. So in analyzing VDI on tokamak one should be careful to check if the plasma inertia is negligible.

On the other hand, also based upon the model in the present study the results show that one of the differences between linear and nonlinear VDI is that the critical behavior is softened in the nonlinear case due to the coupling among the conductors.

The present study is carried out with a rigid displacement model. The plasma deformation during displacing will also change the behavior of VDI[7]. For the further investigation a non-rigid model is interesting to be applied.

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Figure Captions

Fig. 1 Tokamak model used. The passive conductors are assumed to be at the above, inside and outside of the plasma. Two pairs of active coils are placed. The distance of the upper passive conductor from the equatorial plane of tokamak, Zup, is used to represent the wall-plasma interaction.

Fig. 2 Two pairs of the dispersion solutions of linear VDI as functions of Zup (in m). Gama-i and Omg-i (i=1,2) are the growth rates and the frequencies respectively. i=1 is for purely growing mode; i=2 is for oscillating and decaying mode. The critical behavior occurs around Zup=.75.

Fig. 3 Growth rates (Gama) of purely growing mode of linear VDI with plasma inertia.
3a) Gama as a function of Zup with e ranging from 1.2 to 2.0.
3b) Gama as a function of e with Zup ranging from .60 to 1.00 m.
The figures show that the different critical e occurs at different Zup.

Fig. 4 Minimum displacement Zm (in cm) of nonlinear VDI as a function of Zup in the case with active feedback controlling system but without plasma inertia.

Fig. 5 Growth rate and frequency of linear VDI as a function of Zup in the case without plasma inertia.

Fig. 6 Minimum displacement Zm of nonlinear VDI as a function of Zup in case with both active feedback controlling system and plasma inertia.

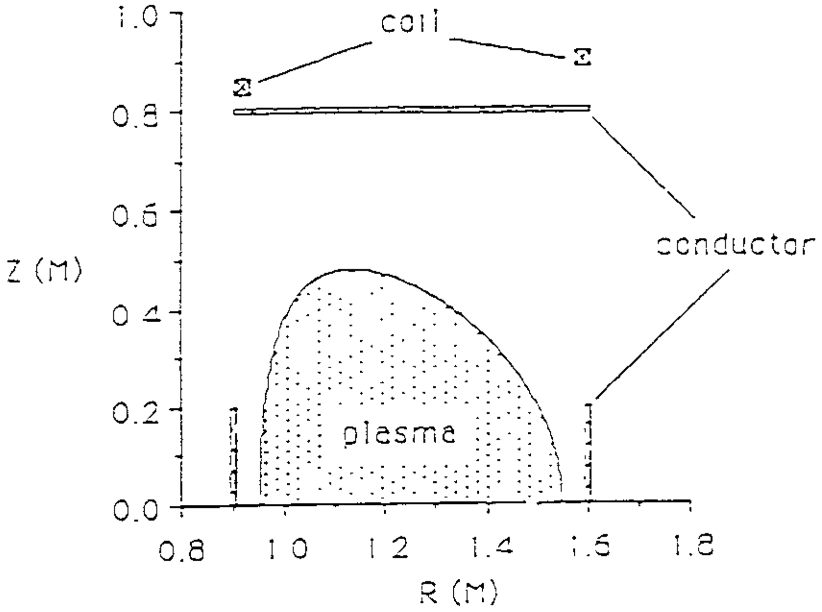


Fig. 1

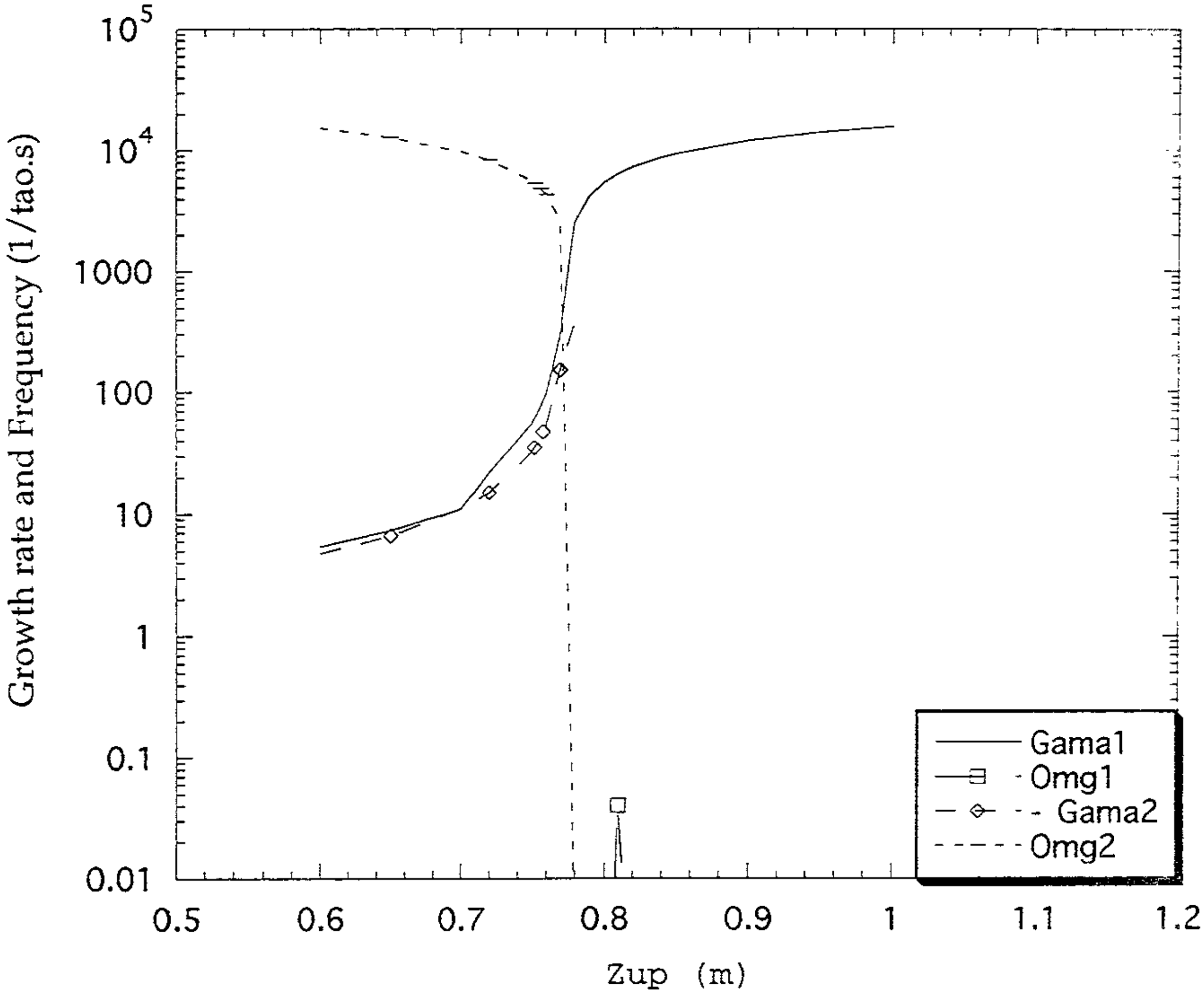

Growth rate and Frequency (1/tao.s)
10^5
10^4
1000
100
10
1
0.1
0.01
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Zup (m)
Gama1
Omg1
Gama2
Omg2

Fig. 2

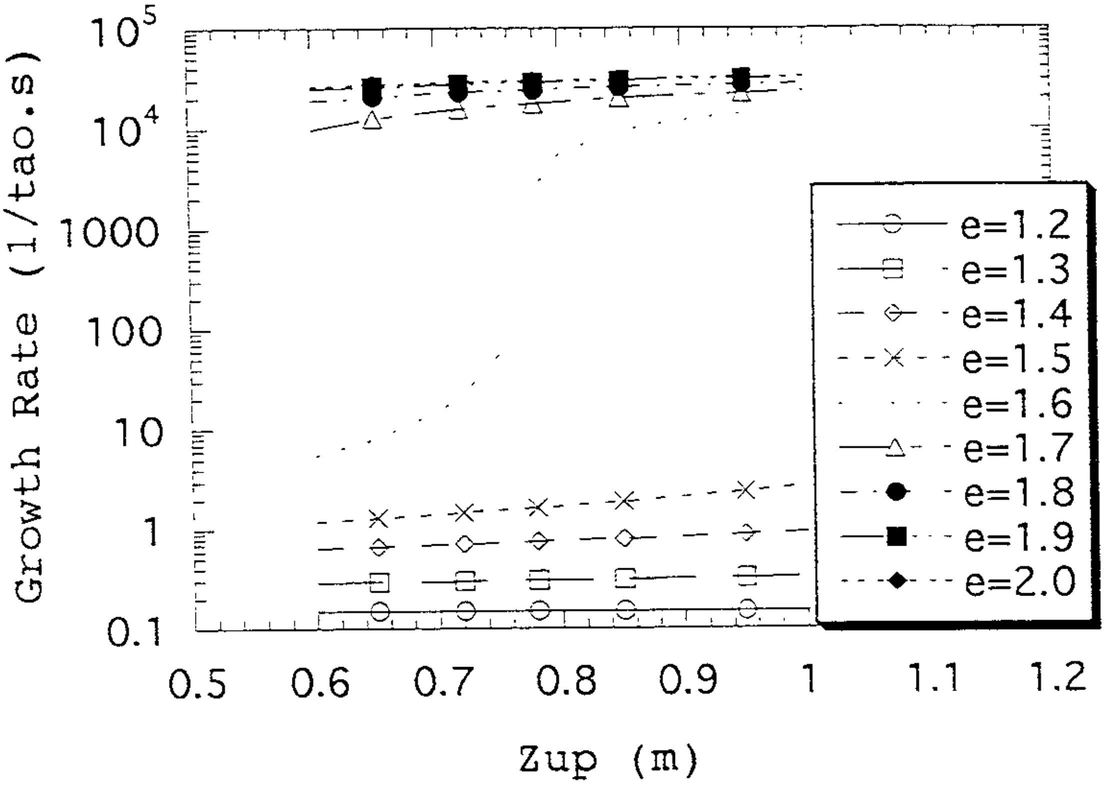
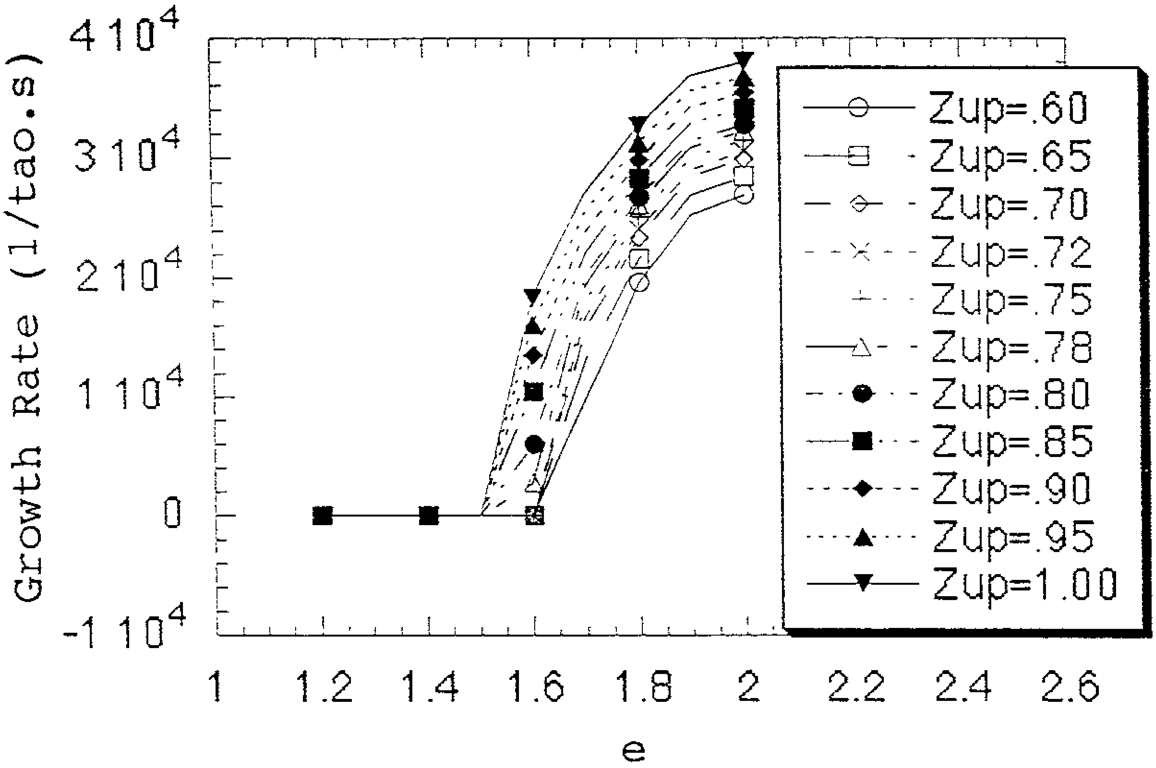
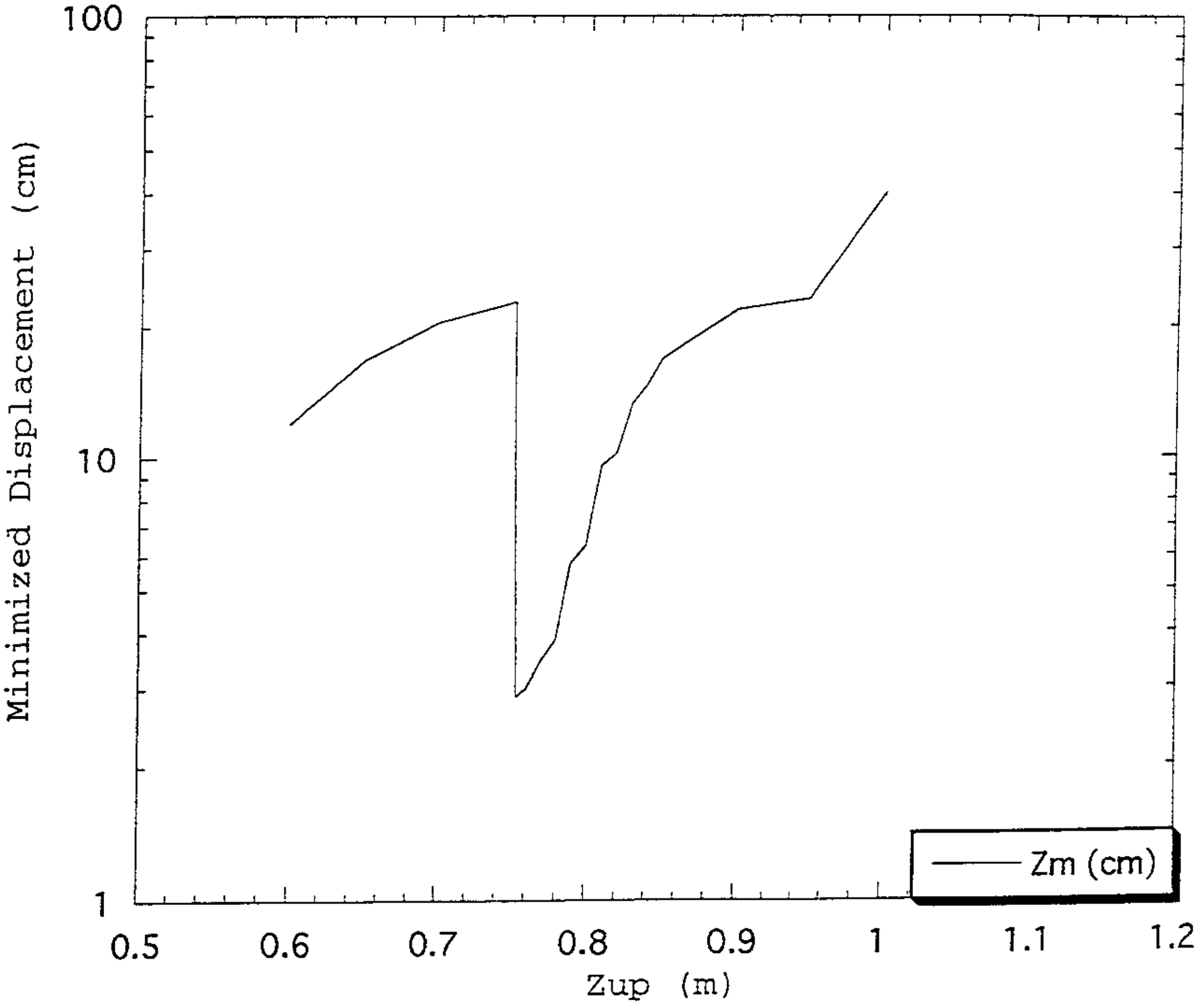

Growth Rate (1/tao.s)
10^5
10^4
1000
100
10
1
0.1
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Zup (m)
e=1.2
e=1.3
e=1.4
e=1.5
e=1.6
e=1.7
e=1.8
e=1.9
e=2.0

Fig. 3a



Growth Rate (1/tao.s)
4 10^4
3 10^4
2 10^4
1 10^4
0
-1 10^4
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
e
Zup=.60
Zup=.65
Zup=.70
Zup=.72
Zup=.75
Zup=.78
Zup=.80
Zup=.85
Zup=.90
Zup=.95
Zup=1.00

Fig. 3b



100
10
1
Minimized Displacement (cm)
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Zup (m)
Zm (cm)

Fig. 4

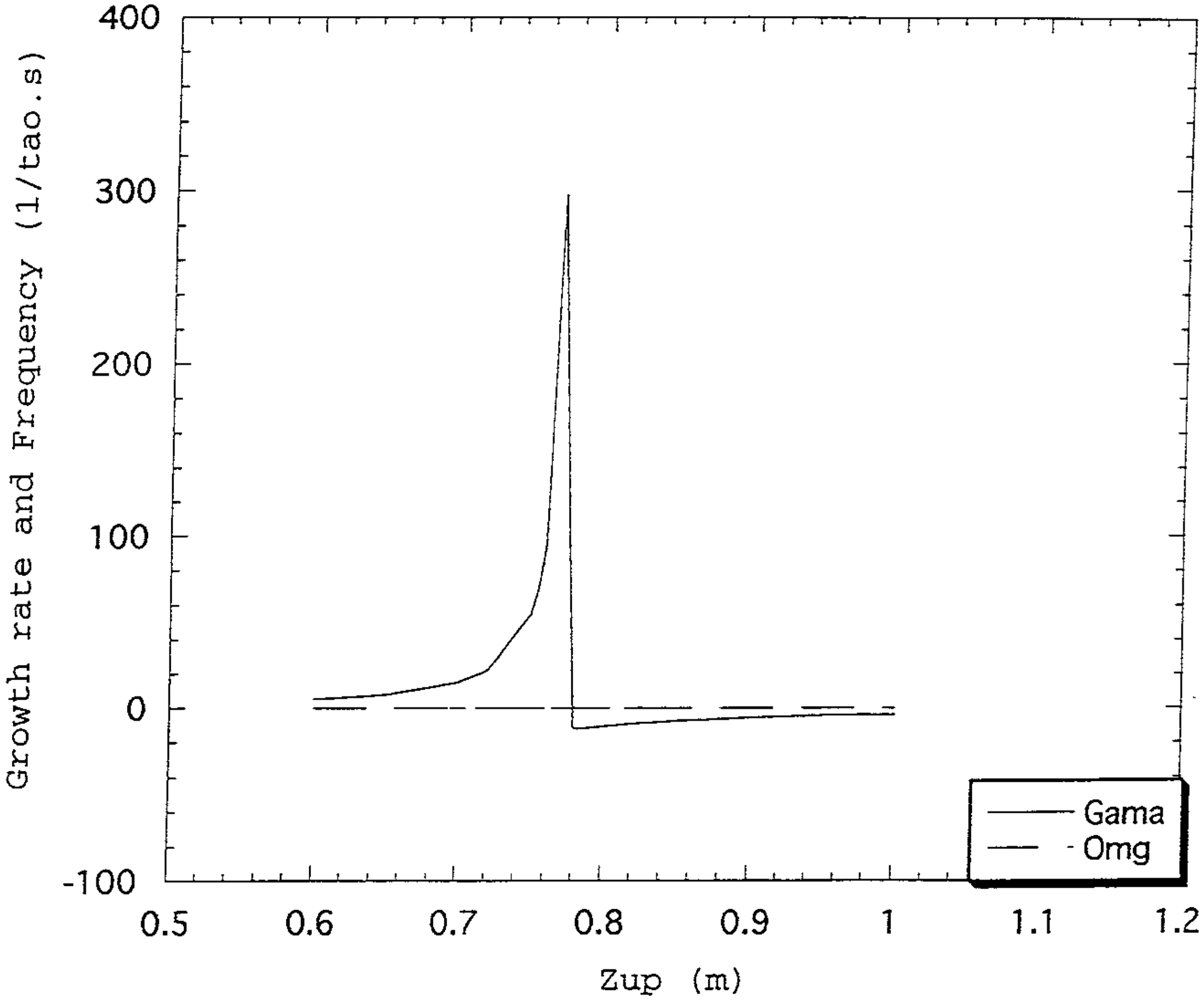

400
300
200
100
0
-100
Growth rate and Frequency (1/tao.s)
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Zup (m)
Gama
Omg

Fig. 5

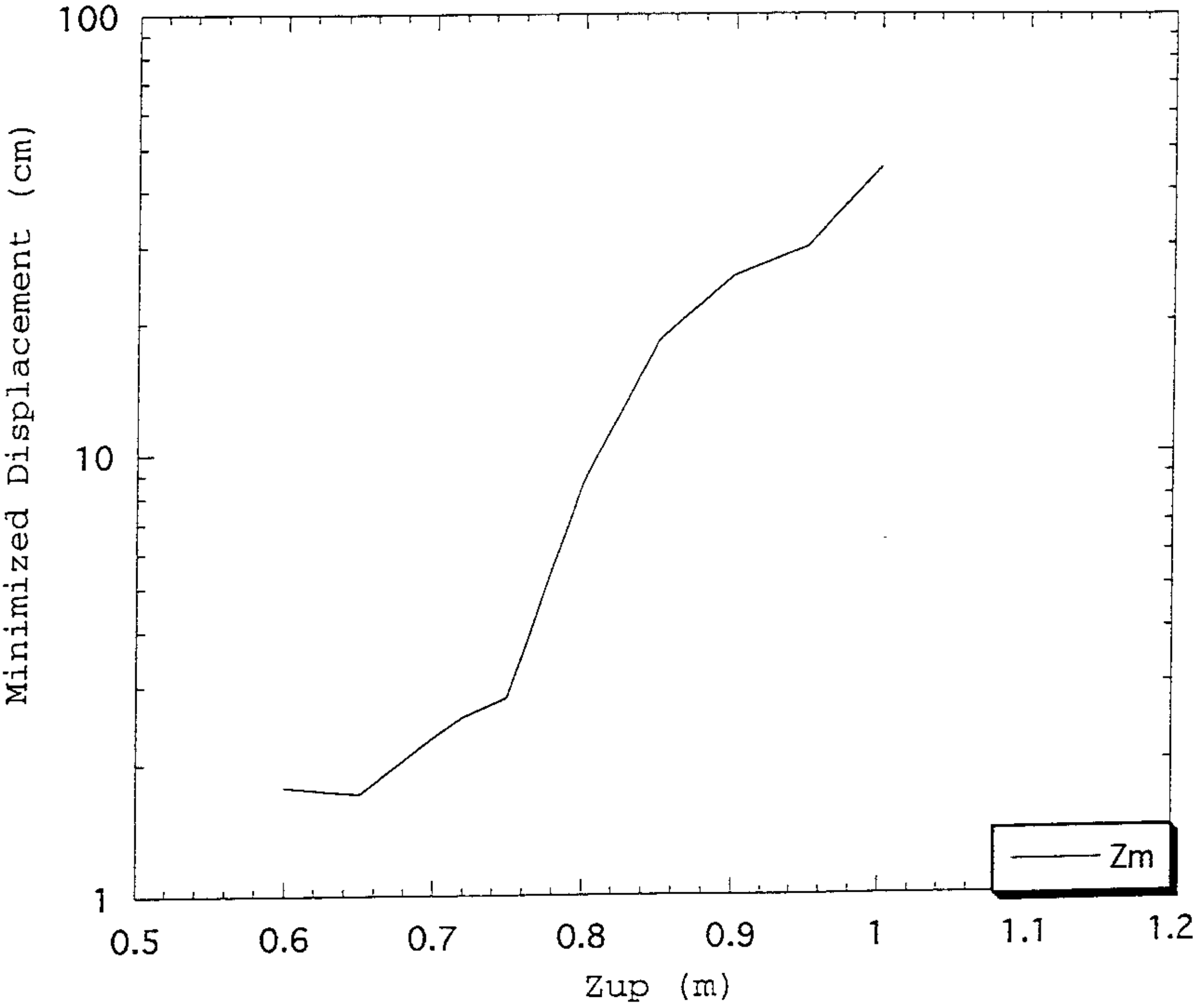

100
10
1
Minimized Displacement (cm)
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Zup (m)
Zm

Fig. 6

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