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(Received - Aug. 6, 1996 )

NIFS-435

Aug. 1996

### RESEARCH REPORT NIFS Series

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# Transport processes and entropy production in toroidally rotating plasmas with electrostatic turbulence

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## Abstract

Transport processes and resultant entropy production in magnetically confined plasmas are studied in detail for toroidally rotating systems with electrostatic turbulence. A new gyrokinetic equation is derived for rotating plasmas with large flow velocities on the order of the ion thermal speed. Neoclassical and anomalous transport of particles, energy, and toroidal momentum are systematically formulated from the ensemble-averaged kinetic equation with the gyrokinetic equation. As a conjugate pair of the thermodynamic force and the transport flux, the shear of the toroidal flow, which is caused by the radial electric field shear, and the toroidal viscosity enter both the neoclassical and anomalous entropy production. The interaction between the fluctuations and the sheared toroidal flow is self-consistently described by the gyrokinetic equation containing the flow shear as the thermodynamic force and by the toroidal momentum balance equation including the anomalous

viscosity. Effects of the toroidal flow shear on the toroidal ion temperature gradient driven modes are investigated. Linear and quasilinear analyses of the modes show that the toroidal flow shear decreases the growth rates and reduces the anomalous toroidal viscosity.

Keywords: rotating plasma, anomalous transport, neoclassical transport, entropy production, ion temperature gradient driven modes

## I. INTRODUCTION

In most of magnetically confined toroidal plasmas, the observed particle and heat fluxes across the magnetic flux surfaces are dominated by the turbulent or anomalous transport,<sup>1</sup> which greatly exceed the predictions of the neoclassical transport theory.<sup>2-4</sup> However, in some operational regions of tokamak plasmas such as high-confinement modes (H-modes)<sup>5</sup> and reversed shear configurations,<sup>6</sup> there have been observed transport barriers with significant reduction of anomalous transport. Generally, large radial electric field shear (or sheared flow) is considered as a cause of such reduction of the transport level. Determination of profiles of the radial electric fields or the sheared flows requires relevant analysis of the momentum balance equations. Since the viscosity involved in the neoclassical transport gives a significant contribution in the momentum balance, we should consider both the neoclassical and anomalous transport processes simultaneously in order to investigate the interaction of the sheared flow and the turbulent fluctuations.

In our previous works,<sup>7,8</sup> the neoclassical and anomalous transport are formulated in the synthesized framework, although that theory treats the  $\mathbf{E} \times \mathbf{B}$  flow velocity as on the order of the diamagnetic drift velocity. In the present work, our theory is extended to that for the rotating plasma with large flow velocities on the order of the ion thermal speed. Then, it is not valid to use conventional drift-kinetic and gyrokinetic equations,<sup>9-15</sup> in which the flow velocities are assumed to be  $\mathcal{O}(\delta v_{Ti})$ . Here  $v_{Ti} \equiv (2T_i/m_i)^{1/2}$  denotes the ion thermal velocity,  $\delta \equiv \rho_i/L$  the drift ordering parameter,  $\rho_i \equiv v_{Ti}/\Omega_i$  the ion thermal gyroradius, and  $L$  the equilibrium scale length. Hazeltine and Ware derived the drift-kinetic equation for the plasma with large flows on the order of the ion thermal speed.<sup>16</sup> That equation has a complicated structure including the gyroviscosity term although it reduces to a simplified form in the case of the toroidally rotating axisymmetric plasma, for which the ion neoclassical transport coefficients were obtained by Hinton and Wong<sup>17</sup> and by Catto *et al.*<sup>18</sup> Artun and Tang<sup>19,20</sup> derived the gyrokinetic equation in the case where the large equilibrium flows exist. In the present work, we derive the new form of the gyrokinetic equation for the large

flow case. Our gyrokinetic equation contains the term responsible for the perpendicular anomalous viscosity (or Reynolds stress), which is not included in the equation by Artun and Tang. This gyrokinetic equation, which is written in a compact form for the toroidally rotating axisymmetric system, is useful to express the anomalous transport and the resultant anomalous entropy production. It is emphasized that, in the rotating plasma, the shear of the toroidal flow or the radial electric field shear enters both the neoclassical and anomalous transport equations as an additional thermodynamic force, and that the products of the toroidal flow shear and the conjugate neoclassical and anomalous toroidal viscous fluxes make significant contributions to the total entropy production. These contributions of the toroidal flow shear and viscosities are considered as higher-order small quantities in  $\delta$  by conventional treatments. The turbulent fluctuations and the resultant anomalous transport are influenced by the flow shear or the radial electric field shear contained as the additional thermodynamic force in the gyrokinetic equation.

Taking account of all the neoclassical and anomalous transport processes, we obtain balance equations for the particles, energy, toroidal momentum, entropy, and the fluctuation amplitude. Since the toroidal momentum is directly related to the radial electric field, the toroidal momentum balance equation describes the temporal evolution of the radial electric field. Through the anomalous viscosity term in the toroidal momentum balance equation, the fluctuations affect the flow and the associated radial electric field as a reaction to the sheared flow effect on the fluctuations. Thus, our extended theory gives a self-consistent description of the interaction between the fluctuations and the sheared flow based on the rigorous statistical kinetic foundation.

As in our previous works,<sup>7,8</sup> let us start from an ensemble-averaged kinetic equation for species  $a$ :

$$\frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \nabla f_a + \frac{e_a}{m_a} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f_a}{\partial \mathbf{v}} = \langle C_a \rangle_{\text{ens}} + \mathcal{D}_a + \mathcal{I}_a \quad (1)$$

where  $C_a$  is a collision term,  $\mathcal{I}_a$  is a term representing the effects of external sources such as neutral beam injection, and  $\mathcal{D}_a$  is a fluctuation-particle interaction term defined by

$$\mathcal{D}_a = -\frac{e_a}{m_a} \left\langle \hat{\mathbf{E}} \cdot \frac{\partial \hat{f}_a}{\partial \mathbf{v}} \right\rangle_{\text{ens}}$$

$$\hat{\mathbf{E}} = -\nabla \hat{\phi} \quad (2)$$

Here  $\langle \cdot \rangle_{\text{ens}}$  denotes the ensemble average and we divided the distribution function (the electric field) into the ensemble-averaged part  $f_a(\mathbf{E}, \Phi)$  and the fluctuating part  $\hat{f}_a(\hat{\mathbf{E}}, \hat{\phi})$ . Throughout this paper, the magnetic fluctuations  $\hat{\mathbf{B}}$  are not considered although generalization to the case with the magnetic fluctuations is straightforward. The source term  $\mathcal{I}_a$  is assumed to be a quantity of  $\mathcal{O}(\delta^2)$ .<sup>17</sup> Then we should note that the linearized drift-kinetic equation and the gyrokinetic equation are not affected by  $\mathcal{I}_a$ . Thus the neoclassical and anomalous transport equations are not changed by the source term, although the balance equations of particles, energy, and momentum derived from Eq.(1), which are  $\mathcal{O}(\delta^2)$ , involve source terms caused by  $\mathcal{I}_a$ .

In deriving the drift-kinetic and gyrokinetic equations, the perturbative expansion in the drift-ordering parameter  $\delta$  is utilized. When we apply this expansion procedure to the system with the large flow on the order of the ion thermal velocity, it is useful to observe particles' gyromotion from the moving frame with that flow velocity  $\mathbf{V}_0$ . Hereafter we consider only axisymmetric systems, for which the magnetic field is given by

$$\mathbf{B} = I(\Psi)\nabla\zeta + \nabla\zeta \times \nabla\Psi \quad (3)$$

where  $\zeta$  is the toroidal angle,  $\Psi$  represents the poloidal flux, and  $I(\Psi) = RB_T$ . Hinton and Wong<sup>17</sup> showed that, in the axisymmetric systems, the poloidal flow decays in a few transit or collision times and that the lowest-order flow velocity  $\mathbf{V}_0$  is in the toroidal direction and is derived from  $\mathbf{E}_0 + \mathbf{V}_0 \times \mathbf{B}/c = 0$  as

$$\mathbf{V}_0 = V_0 \hat{\zeta}, \quad V_0 = RV^\zeta = -Rc\Phi'_0(\Psi) \quad (4)$$

where  $\Phi_0(\Psi)$  denotes the lowest-order electrostatic potential in  $\delta$  and  $\mathbf{E}_0 \equiv -\nabla\Phi_0 \equiv -\Phi'_0\nabla\Psi$ . We should note that the toroidal angular velocity  $V^\zeta = -c\Phi'_0$  is directly given by

the radial electric field and is a flux-surface quantity. The lowest-order electrostatic potential is written as  $\Phi_{-1}$  in the paper by Hinton and Wong<sup>17</sup> although it is denoted by  $\Phi_0$  in the present work since we follow the Littlejohn's drift ordering rule<sup>21</sup> to regard the electric charge  $e$  (instead of  $\Phi$ ) as the parameter of  $\mathcal{O}(\delta^{-1})$ :  $e = e_{-1}$ . As in the work by Hinton and Wong,<sup>17</sup> let us introduce the phase space variables  $(\mathbf{x}', \varepsilon, \mu, \xi)$  which are defined in terms of the spatial coordinates  $\mathbf{x}$  in the laboratory frame and the velocity  $\mathbf{v}' \equiv \mathbf{v} - \mathbf{V}_0$  in the moving frame as

$$\mathbf{x}' = \mathbf{x}, \quad \varepsilon = \frac{1}{2}m_a(v')^2 + \Xi_a, \quad \mu = \frac{m_a(v'_\perp)^2}{2B}, \quad \frac{\mathbf{v}'_\perp}{v'_\perp} = \mathbf{e}_1 \cos \xi + \mathbf{e}_2 \sin \xi \quad (5)$$

where  $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{b} \equiv \mathbf{B}/B)$  are unit vectors which forms a right-handed orthogonal system at each point, and  $\mathbf{v}' = v'_\parallel \mathbf{b} + \mathbf{v}'_\perp$  with  $v'_\parallel = \mathbf{v}' \cdot \mathbf{b}$ . In the definition of the energy variable  $\varepsilon$ ,  $\Xi_a$  is given by

$$\Xi_a \equiv e_a \tilde{\Phi}_1 - \frac{1}{2}m_a V_0^2 \quad (6)$$

where  $\tilde{\Phi}_1 \equiv \Phi_1 - \langle \Phi_1 \rangle [= \mathcal{O}(\delta)]$  is the poloidal-angle-dependent part of the electrostatic potential. The magnetic flux surface average is denoted by  $\langle \cdot \rangle$ . It is shown that  $\varepsilon$  and  $\mu$  are conserved along the lowest-order guiding center orbit:  $\overline{(d\varepsilon/dt)}_0 = \overline{(d\mu/dt)}_0 = 0$  where  $\overline{\cdot} \equiv \oint d\xi/2\pi$  represents the gyrophase average. The guiding center velocity is defined by

$$\begin{aligned} \mathbf{v}_{da} &\equiv \overline{\frac{d}{dt} \left( \frac{\mathbf{v}' \times \mathbf{b}}{\Omega_a} \right)} \\ &= \frac{c\mu}{e_a B} (\nabla \times \mathbf{B}) \cdot \mathbf{b} \mathbf{b} + \frac{c}{e_a B} \mathbf{b} \times [\mu \nabla B + m_a (v'_\parallel)^2 \mathbf{b} \cdot \nabla \mathbf{b} + e_a \nabla \Phi_1 \\ &\quad + m_a \mathbf{V}_0 \cdot \nabla \mathbf{V}_0 + m_a v'_\parallel \mathbf{b} \cdot \nabla \mathbf{V}_0 + m_a v'_\parallel \mathbf{V}_0 \cdot \nabla \mathbf{b}] \end{aligned} \quad (7)$$

where we should note that the centrifugal force  $-m_a \mathbf{V}_0 \cdot \nabla \mathbf{V}_0 = m_a R (V^\zeta)^2 \nabla R$  and the Coriolis force  $-m_a v'_\parallel \mathbf{b} \cdot \nabla \mathbf{V}_0 - m_a v'_\parallel \mathbf{V}_0 \cdot \nabla \mathbf{b} = 2m_a v'_\parallel V^\zeta \mathbf{b} \times (\nabla R \times \hat{\zeta})$  contribute to the guiding center drift velocity.

Applying the same procedure as in Ref.8 to Eq.(1), we obtain the double-averaged kinetic equation over the statistical ensemble and the gyrophase angle  $\xi$  which is valid up to  $\mathcal{O}(\delta^2)$  and is written as

$$\overline{\mathcal{L}(\bar{f}_a + \tilde{f}_a^{(1)})} = \langle \bar{C}_a \rangle_{\text{ens}} + \bar{D}_a - \overline{\mathcal{L}\tilde{f}_a^{(2)}} \quad (8)$$

where  $\bar{\cdot}$  and  $\tilde{\cdot}$  denote the average and oscillating parts with respect to the gyrophase angle  $\xi$ , respectively, and the differential operator  $\mathcal{L}$  is defined by

$$\begin{aligned} \mathcal{L} &\equiv \frac{d}{dt} + \Omega_a \frac{\partial}{\partial \xi} \\ &\equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \dot{\varepsilon} \frac{\partial}{\partial \varepsilon} + \dot{\mu} \frac{\partial}{\partial \mu} + (\dot{\xi} + \Omega_a) \frac{\partial}{\partial \xi}. \end{aligned} \quad (9)$$

Here and hereafter, the spatial gradient operator  $\nabla$  is taken with  $(\varepsilon, \mu, \xi)$  fixed. The gyrophase-dependent parts of the first and second-order distribution functions are given by

$$\begin{aligned} \tilde{f}_a^{(1)} &= \frac{1}{\Omega_a} \int^\xi d\xi \widetilde{\mathcal{L}\bar{f}_a} \\ \tilde{f}_a^{(2)} &= \frac{1}{\Omega_a} \int^\xi d\xi [\widetilde{\mathcal{L}\tilde{f}_a^{(1)}} - C_a^L(\tilde{f}_a^{(1)}) - \tilde{D}_a] \\ &\equiv \tilde{f}_a^H + \tilde{f}_a^C + \tilde{f}_a^A \end{aligned} \quad (10)$$

where the integration constants related to  $\int^\xi d\xi$  are uniquely determined by the conditions  $\overline{\tilde{f}_a^{(1)}} = \overline{\tilde{f}_a^{(2)}} = 0$ .

The lowest-order solution of Eq.(8) is the Maxwellian distribution function which is written as

$$f_{a0} = n_a \left( \frac{m_a}{2\pi T_a} \right)^{3/2} \exp\left(-\frac{m_a(v')^2}{2T_a}\right) = N_a \left( \frac{m_a}{2\pi T_a} \right)^{3/2} \exp\left(-\frac{\varepsilon}{T_a}\right) \quad (11)$$

where the temperature  $T_a = T_a(\Psi)$  and  $N_a = N_a(\Psi)$  are flux-surface functions although generally the density  $n_a$  depends on the poloidal angle  $\theta$  through  $\Xi_a$  and is given by

$$n_a = N_a \exp\left(-\frac{\Xi_a}{T_a}\right). \quad (12)$$

The charge neutrality  $\sum_a e_a n_a = 0$  imposes the constraints on  $\tilde{\Phi}_1$  and  $N_a$ . For plasmas consisting of electrons and a single species of ions with charge  $e_i \equiv Z_i e$ , we have<sup>17</sup>

$$\begin{aligned} \frac{e}{T_e} \tilde{\Phi}_1 &= \frac{m_i (V^\zeta)^2 (R^2 - \langle R^2 \rangle)}{2(Z_i T_e + T_i)} \\ Z_i N_i(\Psi) &= N_e(\Psi) \exp\left(-\frac{m_i (V^\zeta)^2 \langle R^2 \rangle}{2T_i}\right) \end{aligned} \quad (13)$$



where  $m_e/m_i (\ll 1)$  is neglected. It is emphasized that the density  $n_a$  and the temperature  $T_a$  as well as the toroidal (angular) velocity  $V^\zeta$  should be specified for the lowest-order description of the rotating plasma. In the first-order in  $\delta$ , Eq.(8) reduces to the linearized drift-kinetic equation as shown in Sec.III, the solution of which gives the neoclassical transport. The second-order part of Eq.(8) describes behavior of the distribution function in the transport time scale  $\sim \delta^{-2}L/v_{T_a}$  in which the density, the temperature and the toroidal velocity vary due to the transport processes.

The anomalous transport fluxes are defined by the solution of the new gyrokinetic equation in Sec.IV, and the shear flow effects on the anomalous transport due to the ion temperature gradient driven modes are investigated as an example in Sec.V. In the next section, we find balance equations of the particles, energy, toroidal momentum, and entropy for the rotating plasma.

## II. BALANCE EQUATIONS OF PARTICLES, ENERGY, TOROIDAL MOMENTUM, AND ENTROPY

Here, equations describing temporal evolutions of the particle density, energy, toroidal momentum and entropy are given in the magnetic-surface-averaged forms. The collision operator  $C_a$  as well as the fluctuation-particle interaction operator  $\mathcal{D}_a$  in the ensemble-averaged kinetic equation (1) conserves the particle number. Then, taking the zero-th moment and the magnetic surface average of the kinetic equation, we obtain the particle density equation:

$$\frac{\partial \langle n_a \rangle}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \Psi} (V' \Gamma_a) = \left\langle \int d^3v \mathcal{I}_a \right\rangle \quad (14)$$

where  $V' = 2\pi \oint d\theta \sqrt{g}$  and  $\sqrt{g} \equiv (\nabla \Psi \cdot \nabla \theta \times \nabla \zeta)^{-1} = 1/B^\theta$ . The surface-averaged radial particle flux is denoted by  $\Gamma_a \equiv \langle \Gamma_a \cdot \nabla \Psi \rangle$ .

The surfaced-averaged energy balance equation is written as

$$\frac{\partial}{\partial t} \left\langle \frac{3}{2} p_a + n_a e_a \tilde{\Phi}_1 \right\rangle + \frac{1}{V'} \frac{\partial}{\partial \Psi} \left[ V' \left( q_a + \frac{5}{2} T_a \Gamma_a \right) \right]$$

$$\begin{aligned}
&= -\Pi_a \frac{\partial V^\zeta}{\partial \Psi} - e_a \Gamma_a \frac{\partial \langle \Phi_1 \rangle}{\partial \Psi} + \langle n_a e_a \mathbf{u}_{a1} \cdot \mathbf{E}^{(A)} \rangle + \left\langle n_a e_a \frac{\partial \tilde{\Phi}_1}{\partial t} \right\rangle \\
&\quad + \frac{1}{2} (V^\zeta)^2 \frac{\partial}{\partial t} \langle m_a n_a R^2 \rangle + \left\langle \int d^3v \varepsilon (C_a + \mathcal{D}_a + \mathcal{I}_a) \right\rangle
\end{aligned} \tag{15}$$

where  $p_a \equiv n_a T_a$  is the pressure,  $q_a$  the surface-averaged radial heat flux,  $\Pi_a$  the surface-averaged toroidal viscosity (or the radial flux of the toroidal angular momentum), and  $\mathbf{E}^{(A)} \equiv -c^{-1} \partial \mathbf{A} / \partial t$  the inductive electric field. The first-order flow velocity  $\mathbf{u}_{a1} = u_{\parallel a1} \mathbf{b} + \mathbf{u}_{\perp a1}$  is incompressible ( $\nabla \cdot \mathbf{u}_{a1} = 0$ ) and its perpendicular component is driven by the pressure gradient, the first-order radial electric field, and the centrifugal force as

$$\mathbf{u}_{\perp a1} = \frac{c}{e_a B} \mathbf{b} \times \left( \frac{1}{n_a} \nabla p_a + e_a \nabla \Phi_1 - m_a (V^\zeta)^2 R \nabla R \right). \tag{16}$$

Taking the flux surface average and species summation of toroidal velocity moment of the kinetic equation, the equation, which determines the temporal evolution of the toroidal angular velocity  $V^\zeta$ , is obtained as

$$\frac{\partial}{\partial t} \left\langle \left( \sum_a m_a n_a \right) R^2 V^\zeta \right\rangle + \frac{1}{V'} \frac{\partial}{\partial \Psi} \left( V' \sum_a \Pi_a \right) = \left\langle \int d^3v m_a v_\zeta \mathcal{I}_a \right\rangle \tag{17}$$

where the first and second terms in the left-hand side represent the time derivative of the total toroidal angular momentum and the divergence of the total toroidal angular momentum flux, respectively. In the right-hand side of Eq.(17), the torque by the external sources such as neutral beam injection is given, and  $v_\zeta$  is the covariant toroidal component of the particle velocity in the laboratory frame:

$$v_\zeta \equiv R \hat{\zeta} \cdot \mathbf{v} = R^2 V^\zeta + \frac{I}{B} v'_{\parallel} + R \hat{\zeta} \cdot \mathbf{v}'_{\perp}. \tag{18}$$

The transport fluxes  $\Gamma_a$ ,  $q_a$ , and  $\Pi_a$  are written as

$$\begin{aligned}
\Gamma_a &\equiv \left\langle \int d^3v f_a \mathbf{v} \cdot \nabla \Psi \right\rangle \\
&= \Gamma_a^{\text{cl}} + \Gamma_a^{\text{ncl}} + \Gamma_a^H + \Gamma_a^{(E)} + \Gamma_a^{\text{anom}} \\
q_a &\equiv T_a \left\langle \int d^3v f_a \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla \Psi \right\rangle \\
&= q_a^{\text{cl}} + q_a^{\text{ncl}} + q_a^H + q_a^{(E)} + q_a^{\text{anom}}
\end{aligned}$$

$$\begin{aligned}\Pi_a &\equiv \left\langle \int d^3v f_a m_a v_\zeta \mathbf{v} \cdot \nabla \Psi \right\rangle \\ &= \Pi_a^{\text{cl}} + \Pi_a^{\text{ncl}} + \Pi_a^H + \Pi_a^{(E)} + \Pi_a^{\text{anom}}.\end{aligned}\quad (19)$$

Here the fluxes with the superscript  $(E)$  represent the inductive-electric-field-driven parts given by

$$\begin{aligned}\Gamma_a^{(E)} &\equiv c \left\langle n_a \frac{\mathbf{E}^{(A)} \times \mathbf{b}}{B} \cdot \nabla \Psi \right\rangle - cI \left\langle \frac{E_{\parallel}^{(A)}}{B} \left( n_a - \langle n_a \rangle \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \\ q_a^{(E)} &\equiv c \left\langle n_a \Xi_a \frac{\mathbf{E}^{(A)} \times \mathbf{b}}{B} \cdot \nabla \Psi \right\rangle - cI \left\langle \frac{E_{\parallel}^{(A)}}{B} \left( n_a \Xi_a - \langle n_a \Xi_a \rangle \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle \\ \Pi_a^{(E)} &\equiv cm_a V^\zeta \left\langle n_a R^2 \frac{\mathbf{E}^{(A)} \times \mathbf{b}}{B} \cdot \nabla \Psi \right\rangle - cm_a V^\zeta I \left\langle \frac{E_{\parallel}^{(A)}}{B} \left( n_a R^2 - \langle n_a R^2 \rangle \frac{B^2}{\langle B^2 \rangle} \right) \right\rangle.\end{aligned}\quad (20)$$

These inductive-field-driven fluxes are not dissipative in that they are not involved in the entropy production processes. The fluxes with the superscript ‘ $H$ ’, ‘ $\text{cl}$ ’, and ‘ $\text{anom}$ ’ are given by the second-order gyrophase-dependent distribution functions  $\tilde{f}_a^H$ ,  $\tilde{f}_a^C$ , and  $\tilde{f}_a^A$ , respectively, as

$$\begin{aligned}\Gamma_a^{H,\text{cl},\text{anom}} &\equiv \left\langle \int d^3v \tilde{f}_a^{H,C,A} \mathbf{v} \cdot \nabla \Psi \right\rangle \\ \frac{1}{T_a} q_a^{H,\text{cl},\text{anom}} &\equiv \left\langle \int d^3v \tilde{f}_a^{H,C,A} \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right) \mathbf{v} \cdot \nabla \Psi \right\rangle \\ \Pi_a^{H,\text{cl},\text{anom}} &\equiv \left\langle \int d^3v \tilde{f}_a^{H,C,A} m_a v_\zeta \mathbf{v} \cdot \nabla \Psi \right\rangle.\end{aligned}\quad (21)$$

The neoclassical fluxes  $\Gamma_a^{\text{ncl}}$ ,  $q_a^{\text{ncl}}$ , and  $\Pi_a^{\text{ncl}}$  are given by the first-order gyrophase-averaged distribution function, which is obtained by solving the linearized drift-kinetic equation. The definitions of the neoclassical fluxes and their properties related to the linearized drift-kinetic equation are shown in the next section.

Without knowledge of the solution of the linearized drift-kinetic equation, we can obtain the transport equations relating the fluxes  $(\Gamma_a^H, q_a^H/T_a, \Pi_a^H)$  and  $(\Gamma_a^{\text{cl}}, q_a^{\text{cl}}/T_a, \Pi_a^{\text{cl}})$  to the radial gradient thermodynamic forces  $(X_{a1}, X_{a2}, X_V)$  which are flux-surface quantities defined by

$$\begin{aligned}X_{a1} &\equiv -\frac{1}{N_a} \frac{\partial(N_a T_a)}{\partial \Psi} - e_a \frac{\partial \langle \Phi_1 \rangle}{\partial \Psi} \\ X_{a2} &\equiv -\frac{\partial T_a}{\partial \Psi} \\ X_V &\equiv -\frac{\partial V^\zeta}{\partial \Psi} = c \frac{\partial^2 \Phi_0}{\partial \Psi^2}.\end{aligned}\quad (22)$$

The transport equations for  $(\Gamma_a^H, q_a^H/T_a, \Pi_a^H)$  are given by

$$\begin{bmatrix} \Gamma_a^H \\ q_a^H/T_a \\ \Pi_a^H \end{bmatrix} = \begin{bmatrix} 0 & 0 & (L^H)_{1V}^a \\ 0 & 0 & (L^H)_{2V}^a \\ -(L^H)_{1V}^a & -(L^H)_{2V}^a & 0 \end{bmatrix} \begin{bmatrix} X_{a1} \\ X_{a2} \\ X_V \end{bmatrix} \quad (23)$$

where the transport coefficients  $(L^H)_{1V}^a$  and  $(L^H)_{2V}^a$  are given by

$$\begin{aligned} (L^H)_{1V}^a &\equiv \frac{m_a c^2 I T_a}{2e_a^2} \left\langle \frac{n_a}{B^2} \mathbf{b} \cdot \nabla \left( \frac{R^2 B_P^2}{B} \right) \right\rangle \\ (L^H)_{2V}^a &\equiv \frac{m_a c^2 I T_a}{2e_a^2} \left\langle \left( 1 + \frac{\Xi_a}{T_a} \right) \frac{n_a}{B^2} \mathbf{b} \cdot \nabla \left( \frac{R^2 B_P^2}{B} \right) \right\rangle \end{aligned} \quad (24)$$

which are independent of the collision frequency. Here  $R^2 B_P^2 = |\nabla \Psi|^2$ . The fluxes  $\Gamma_a^H$  and  $q_a^H/T_a$  are shown to be rewritten in terms of the parallel gyroviscosity as  $\Gamma_a^H = -(cI/e_a) \langle B^{-1} \mathbf{b} \cdot (\nabla \cdot \boldsymbol{\pi}_a^{\text{gyro}}) \rangle$  and  $q_a^H/T_a = -(cI/e_a) \langle (1 + \Xi_a/T_a) B^{-1} \mathbf{b} \cdot (\nabla \cdot \boldsymbol{\pi}_a^{\text{gyro}}) \rangle$ , respectively. The antisymmetry of the transport matrix in Eq.(23) shows that the transport fluxes  $(\Gamma_a^H, q_a^H/T_a, \Pi_a^H)$  are nondissipative, or that they give no entropy production:

$$T_a \langle \sigma_a^H \rangle \equiv \Gamma_a^H X_{a1} + \frac{1}{T_a} q_a^H X_{a2} + \Pi_a^H X_V = 0. \quad (25)$$

The fluxes  $(\Gamma_a^H, q_a^H/T_a, \Pi_a^H)$  are contained in the neoclassical fluxes defined by Hinton and Wong<sup>17</sup> although, in the present work, they are not included in the neoclassical fluxes, which we define from the gyrophase-averaged distribution function in Sec.III. If the system has up-down symmetry  $B(\theta) = B(-\theta)$  ( $\theta$ : a poloidal angle defined such that  $\theta = 0$  on the plane of reflection symmetry), we find from Eq.(24) that  $(L^H)_{1V}^a = (L^H)_{2V}^a = 0$  and accordingly  $\Gamma_a^H = q_a^H = \Pi_a^H = 0$ . Thus, the fluxes  $(\Gamma_a^H, q_a^H/T_a, \Pi_a^H)$  vanish in a simple approximation assuming large aspect ratio toroids with circular cross sections.

The classical fluxes  $(\Gamma_a^{\text{cl}}, q_a^{\text{cl}}/T_a, \Pi_a^{\text{cl}})$  the anomalous fluxes  $(\Gamma_a^{\text{anom}}, q_a^{\text{anom}}/T_a, \Pi_a^{\text{anom}})$  are rewritten in terms of the gyrophase-dependent part of the linearized collision operator  $C_a^L(\tilde{f}_{a1})$  and that of the fluctuation-particle interaction operator  $\tilde{D}_a$ , respectively, as

$$[\Gamma_a^{\text{cl}}, \Gamma_a^{\text{anom}}] = -\frac{m_a c}{e_a} \left\langle \int d^3 v [C_a^L(\tilde{f}_{a1}), \tilde{D}_a] \mathbf{v}'_{\perp} \cdot (R \hat{\zeta}) \right\rangle$$

$$\begin{aligned}\frac{1}{T_a}[q_a^{\text{cl}}, q_a^{\text{anom}}] &= -\frac{m_a c}{e_a} \left\langle \int d^3v [C_a^L(\tilde{f}_{a1}), \tilde{D}_a] \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right) \mathbf{v}'_{\perp} \cdot (R\hat{\zeta}) \right\rangle \\ [\Pi_a^{\text{cl}}, \Pi_a^{\text{anom}}] &= -\frac{m_a^2 c}{e_a} \left\langle \int d^3v [C_a^L(\tilde{f}_{a1}), \tilde{D}_a] \frac{1}{2} (\widetilde{v_{\zeta}^2}) \right\rangle\end{aligned}\quad (26)$$

where  $\mathbf{v}'_{\perp} \cdot (R\hat{\zeta}) = -B^{-1}(\mathbf{v}' \times \mathbf{b}) \cdot \nabla \Psi$  and

$$\begin{aligned}\frac{1}{2} (\widetilde{v_{\zeta}^2}) &\equiv \frac{1}{2} [v_{\zeta}^2 - \overline{v_{\zeta}^2}] \\ &= \left( R^2 V^{\zeta} + \frac{I}{B} v'_{\parallel} \right) \mathbf{v}'_{\perp} \cdot (R\hat{\zeta}) + \frac{1}{2} (\mathbf{v}'_{\perp} \widetilde{\mathbf{v}'_{\perp}}) : (R\hat{\zeta})(R\hat{\zeta}).\end{aligned}\quad (27)$$

In Appendix A, by rewriting the collisional frictions in Eq.(26) in terms of the perpendicular flows and the gyroviscosity, the classical transport equations relating  $(\Gamma_a^{\text{cl}}, q_a^{\text{cl}}/T_a, \Pi_a^{\text{cl}})$  to  $(X_{a1}, X_{a2}, X_V)$  is derived and the Onsager relations for the classical transport coefficients are shown. The classical entropy production defined kinetically in terms of  $\tilde{f}_{a1}$  and  $C_a^L$  is written in the thermodynamic form as the inner product of the fluxes and the forces:

$$T_a \langle \sigma_a^{\text{cl}} \rangle \equiv -T_a \left\langle \int d^3v \frac{\tilde{f}_{a1}}{f_{a0}} C_a^L(\tilde{f}_{a1}) \right\rangle = \Gamma_a^{\text{cl}} X_{a1} + \frac{1}{T_a} q_a^{\text{cl}} X_{a2} + \Pi_a^{\text{cl}} X_V. \quad (28)$$

The properties of the anomalous transport and the anomalous entropy production are shown in Sec.IV with the help of the gyrokinetic equation for the fluctuating part of the distribution function.

The entropy per unit volume for species  $a$  is defined by

$$S_a \equiv - \int d^3v f_a \ln f_a \quad (29)$$

of which the lowest-order expression is given by  $S_{a0} = - \int d^3v f_{a0} \ln f_{a0} = -n_a \{ \ln [n_a (m_a / 2\pi T_a)^{3/2}] - 3/2 \}$ . The temporal variation of the surface-averaged entropy density  $\langle S_a \rangle$  is described by

$$\frac{\partial \langle S_a \rangle}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \Psi} (V' J_{S_a}^{\text{tot}}) = \langle \sigma_a^{\text{tot}} \rangle + \langle \dot{S}_a^{\text{ext}} \rangle \quad (30)$$

where  $\langle \dot{S}_a^{\text{ext}} \rangle \equiv - \langle \int d^3v (\ln f_{a0} + 1) \mathcal{I}_a \rangle = \langle \int d^3v (S_{a0}/n_a - \Xi_a/T_a + \varepsilon/T_a - 5/2) \mathcal{I}_a \rangle$  represents the external entropy source. Here the surface-averaged radial entropy flux  $J_{S_a}^{\text{tot}}$  consists of the convection and conduction parts as

$$\begin{aligned}
J_{S_a}^{\text{tot}} &= \left( \frac{S_a}{n_a} - \frac{\Xi_a}{T_a} \right) \left( \Gamma_a^{\text{cl}} + \Gamma_a^{\text{ncI}} + \Gamma_a^H + \Gamma_a^{(E)} + \Gamma_a^A \right) \\
&\quad + \frac{1}{T_a} \left( q_a^{\text{cl}} + q_a^{\text{ncI}} + q_a^H + q_a^{(E)} + q_a^A \right)
\end{aligned} \tag{31}$$

where  $(S_a/n_a - \Xi_a/T_a)$  is a flux surface quantity in the lowest order in  $\delta$ . We have redefined the anomalous fluxes by  $\Gamma_a^A \equiv \Gamma_a^{\text{anom}}$ ,  $\Pi_a^A \equiv \Pi_a^{\text{anom}}$ , and

$$q_a^A \equiv q_a^{\text{anom}} + e_a \left\langle \left\langle \int d^3v \hat{f}_a \hat{\phi} \mathbf{v} \cdot \nabla \Psi \right\rangle \right\rangle \tag{32}$$

where the second term in the right-hand side represents the turbulent transport of the fluctuation-particle interaction energy and  $\langle\langle \cdot \rangle\rangle$  denotes a double average over the magnetic surface and the ensemble. The surface-averaged entropy production rate  $\langle \sigma_a^{\text{tot}} \rangle$  is given by

$$\langle \sigma_a^{\text{tot}} \rangle = \langle \sigma_a^{\text{cl}} \rangle + \langle \sigma_a^{\text{ncI}} \rangle + \langle \sigma_a^A \rangle + \frac{1}{T_a} \langle Q_a \rangle + \frac{1}{T_a} \langle \mathbf{u}_{a1} \cdot \mathbf{F}_{a1} \rangle \tag{33}$$

where  $Q_a \equiv \int d^3v \frac{1}{2} m_a (\mathbf{v}' - \mathbf{u}_{a1})^2 C_a(f_a)$  and  $\mathbf{F}_{a1} \equiv \int d^3v m_a \mathbf{v}' C_a(f_a)$  represent the collisional heat and momentum generation rates, respectively. The surface-averaged entropy production rates  $\langle \sigma_a^{\text{ncI}} \rangle$  and  $\langle \sigma_a^A \rangle$  due to the neoclassical and anomalous transport processes are given in Sec.III and Sec.IV, respectively. The second law of thermodynamics is written by

$$\sum_a T_a \langle \sigma_a^{\text{tot}} \rangle = \sum_a T_a (\langle \sigma_a^{\text{cl}} \rangle + \langle \sigma_a^{\text{ncI}} \rangle + \langle \sigma_a^A \rangle) \geq 0. \tag{34}$$

It is shown that the classical, neoclassical, and neoclassical contributions in Eq.(34) are separately positive definite.

### III. NEOCLASSICAL TRANSPORT IN TOROIDALLY ROTATING PLASMAS

In toroidally rotating axisymmetric systems, the drift-kinetic equation derived by Hazeltine and Ware<sup>16</sup> reduces to

$$v_{\parallel}' \cdot \nabla \bar{g}_a - C_a^L(\bar{g}_a) = \frac{1}{T_a} f_{a0} (W_{a1} X_{a1} + W_{a2} X_{a2} + W_{aV} X_V + W_{aE} X_E) \tag{35}$$

where  $\bar{g}_a$  is defined in terms of the first-order gyrophase-averaged distribution function  $\bar{f}_{a1}$  as

$$\bar{g}_a \equiv \bar{f}_{a1} - f_{a0} \frac{e_a}{T_a} \int^l \frac{dl}{B} \left( B E_{\parallel}^{(2)} - \frac{B^2}{\langle B^2 \rangle} \langle B E_{\parallel}^{(2)} \rangle \right). \quad (36)$$

Here  $\int^l dl$  denotes the integral along the magnetic field line, and  $E_{\parallel}^{(2)} \equiv \mathbf{b} \cdot (-\nabla \Phi^{(2)} - c^{-1} \partial \mathbf{A} / \partial t)$  is the second-order parallel electric field. The thermodynamic forces  $(X_{a1}, X_{a2}, X_V, X_E)$  are defined by Eq.(22) and

$$X_E \equiv \frac{\langle B E_{\parallel}^{(A)} \rangle}{\langle B^2 \rangle^{1/2}}. \quad (37)$$

The parallel inductive electric field  $E_{\parallel}^{(A)}$  is not included in the work by Hinton and Wong<sup>17</sup> but it is retained by Catto *et al.*<sup>18</sup> The functions  $(W_{a1}, W_{a2}, W_{aV}, W_{aE})$  are defined by

$$\begin{aligned} W_{a1} &\equiv \frac{m_a c}{e_a} v'_{\parallel} \mathbf{b} \cdot \nabla \left( R^2 V^{\zeta} + \frac{I}{B} v'_{\parallel} \right) \\ W_{a2} &\equiv W_{a1} \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right) \\ W_{aV} &\equiv \frac{m_a c}{2e_a} v'_{\parallel} \mathbf{b} \cdot \nabla \left[ m_a \left( R^2 V^{\zeta} + \frac{I}{B} v'_{\parallel} \right)^2 + \mu \frac{R^2 B_P^2}{B} \right] \\ W_{aE} &\equiv \frac{e_a v'_{\parallel} B}{\langle B^2 \rangle^{1/2}}. \end{aligned} \quad (38)$$

In the same way as in Ref.22, the neoclassical entropy production is kinetically defined in terms of  $\bar{f}_{a1}$  and  $C_a^L$  and is rewritten in the thermodynamic form by using Eq.(35). The surface-averaged total neoclassical entropy production is given by

$$\sum_a T_a \langle \sigma_a^{\text{nc1}} \rangle \equiv - \sum_a T_a \left\langle \int d^3 v \frac{\bar{f}_{a1}}{f_{a0}} C_a^L(\bar{f}_{a1}) \right\rangle = \sum_a \left( \Gamma_a^{\text{nc1}} X_{a1} + \frac{1}{T_a} q_a^{\text{nc1}} X_{a2} + \Pi_a^{\text{nc1}} X_V \right) + J_E X_E \quad (39)$$

where the fluxes  $(\Gamma_a^{\text{nc1}}, q_a^{\text{nc1}}/T_a, \Pi_a^{\text{nc1}}, J_E)$  are defined by

$$\begin{aligned} \Gamma_a^{\text{nc1}} &\equiv \left\langle \int d^3 v \bar{g}_a W_{a1} \right\rangle, & \frac{1}{T_a} q_a^{\text{nc1}} &\equiv \left\langle \int d^3 v \bar{g}_a W_{a2} \right\rangle \\ \Pi_a^{\text{nc1}} &\equiv \left\langle \int d^3 v \bar{g}_a W_{aV} \right\rangle, & J_E &\equiv \frac{\langle B J_{\parallel} \rangle}{\langle B^2 \rangle^{1/2}} \equiv \sum_a \left\langle \int d^3 v \bar{g}_a W_{aE} \right\rangle. \end{aligned} \quad (40)$$

The neoclassical transport equations are written as

$$\begin{aligned}
\Gamma_a^{\text{ncI}} &= \sum_b (L_{11}^{ab} X_{b1} + L_{12}^{ab} X_{b2}) + L_{1V}^a X_V + L_{1E}^a X_E \\
\frac{1}{T_a} q_a^{\text{ncI}} &= \sum_b (L_{21}^{ab} X_{b1} + L_{22}^{ab} X_{b2}) + L_{2V}^a X_V + L_{2E}^a X_E \\
\sum_a \Pi_a^{\text{ncI}} &= \sum_b (L_{V1}^b X_{b1} + L_{V2}^b X_{b2}) + L_{VV} X_V + L_{VE} X_E \\
J_E &= \sum_b (L_{E1}^b X_{b1} + L_{E2}^b X_{b2}) + L_{EV} X_V + L_{EE} X_E
\end{aligned} \tag{41}$$

where the transport coefficients are dependent on the radial electric field through the toroidal angular velocity  $V^\zeta = -c\Phi'_0$ . By using the self-adjointness of the linearized collision operator and the formal solution of Eq.(35), we can prove that the neoclassical transport coefficients satisfy the Onsager symmetry which is given by

$$\begin{aligned}
L_{mn}^{ab}(V^\zeta) &= L_{nm}^{ba}(-V^\zeta) \quad (m, n = 1, 2) \\
L_{MN}(V^\zeta) &= L_{NM}(-V^\zeta) \quad (M, N = V, E) \\
L_{mM}^a(V^\zeta) &= -L_{Mm}^a(-V^\zeta) \quad (m = 1, 2; M = V, E).
\end{aligned} \tag{42}$$

It is noted that the transport coefficients given in Eqs.(23) and (24) satisfy the same Onsager symmetry as Eq.(42). If the system has up-down symmetry, the neoclassical transport coefficients are shown to be more restricted by the relations:

$$\begin{aligned}
L_{mn}^{ab}(V^\zeta) &= L_{mn}^{ab}(-V^\zeta) = L_{nm}^{ba}(V^\zeta) \quad (m, n = 1, 2) \\
L_{mV}^a(V^\zeta) &= -L_{mV}^a(-V^\zeta) = L_{Vm}^a(V^\zeta) \quad (m = 1, 2) \\
L_{mE}^a(V^\zeta) &= L_{mE}^a(-V^\zeta) = -L_{Em}^a(V^\zeta) \quad (m = 1, 2) \\
L_{VE}(V^\zeta) &= -L_{VE}(-V^\zeta) = -L_{EV}(V^\zeta) \\
L_{VV}(V^\zeta) &= L_{VV}(-V^\zeta), \quad L_{EE}(V^\zeta) = L_{EE}(-V^\zeta).
\end{aligned} \tag{43}$$

In this case of up-down symmetry, we have  $\Gamma_a^H = q_a^H = \Pi_a^H = 0$  as mentioned in Sec.II.

The sum of  $\Gamma_a^{\text{ncI}}$  and  $\Gamma_a^H$  is written as

$$\Gamma_a^{\text{ncI}} + \Gamma_a^H = -cI \langle n_a \rangle \frac{\langle B E_{\parallel}^{(A)} \rangle}{\langle B^2 \rangle} - \frac{cI}{e_a} \left\langle \frac{F_{\parallel a1}}{B} \right\rangle \tag{44}$$

from which we obtain the ambipolarity



$$\sum_a e_a (\Gamma_a^{\text{ncI}} + \Gamma_a^H) = 0 \quad (45)$$

where the charge neutrality  $\sum_a e_a n_a = 0$  and the momentum conservation by collisions  $\sum_a \mathbf{F}_{a1} = 0$  are used. Neither  $\sum_a e_a \Gamma_a^{\text{ncI}} = 0$  nor  $\sum_a e_a \Gamma_a^H = 0$  is valid generally without up-down symmetry. Using the ambipolarity condition, the number of the thermodynamic forces in the transport equations for  $[\Gamma_a^{\text{ncI}} + \Gamma_a^H, (q_a^{\text{ncI}} + q_a^H)/T_a, \Pi_a^{\text{ncI}} + \Pi_a^H, J_E]$  is reduced by one, although the transport coefficients in the reduced transport equations retain the Onsager symmetry as shown in Ref.22.

Hinton and Wong<sup>17</sup>, and Catto *et al.*<sup>18</sup> derived the detailed expressions of the ion neo-classical transport coefficients. We have derived the detailed expressions of the full transport matrix including the electron-ion cross coefficients, which will be reported elsewhere. We find that the coefficient  $L_{VE}$  describes the inward flux of the toroidal momentum caused by the parallel electric field, which is associated with the Ware pinch effect<sup>23</sup> corresponding to the coefficient  $L_{1E}^e$ .

#### IV. GYROKINETIC EQUATION AND ANOMALOUS TRANSPORT IN TOROIDALLY ROTATING PLASMAS

We assume that any fluctuating field  $\hat{F}$  is written in the WKB (or eikonal) form:

$$\hat{F}(t, \mathbf{x}', \varepsilon, \mu, \xi) = \hat{F}(t, \mathbf{x}', \varepsilon, \mu, \xi; \mathbf{k}_\perp) \exp\left(i \int^{\mathbf{x}'} \mathbf{k}_\perp \cdot d\mathbf{x}'\right) \quad (46)$$

where the eikonal  $\int^{\mathbf{x}'} \mathbf{k}_\perp \cdot d\mathbf{x}'$  represents the rapid variation in the directions perpendicular to the magnetic field lines with characteristic scale lengths  $k_\perp^{-1} \sim \rho_i$ . The gyrokinetic ordering employed here for the turbulent fluctuations is written in terms of  $\delta \equiv \rho_i/L$  as

$$\frac{\hat{f}_a}{f_a} \sim \frac{e_a \hat{\phi}}{T_a} \sim \frac{k_\parallel}{k_\perp} \sim \frac{\omega_0}{\Omega_a} \sim \delta \quad (47)$$

where the characteristic parallel wavelength is given by  $k_\parallel \sim L^{-1}$ . When a frequency  $\omega$  of the fluctuation with the perpendicular wavenumber  $k_\perp \sim \rho_i^{-1}$  is observed in the laboratory frame, it contains the rapid component due to the high plasma rotation, which is written as  $\omega_{-1} \equiv \mathbf{k}_\perp \cdot \mathbf{V}_0 \sim \delta^{-1} v_{Ti}/L$ . The frequency  $\omega_0$  in Eq.(47) is defined by  $\omega_0 \equiv (\omega - \omega_{-1}) \sim v_{Ti}/L$ .

The fluctuating part of the distribution function is divided into the adiabatic and nonadiabatic parts as

$$\hat{f}_a(\mathbf{k}_\perp) = -\frac{e_a \hat{\phi}(\mathbf{k}_\perp)}{T_a} f_{a0} + \hat{h}_a(\mathbf{k}_\perp) e^{iL_a(\mathbf{k}_\perp)} \quad (48)$$

where  $L_a(\mathbf{k}_\perp) \equiv \mathbf{k}_\perp \cdot (\mathbf{v}' \times \mathbf{b}) / \Omega_a$ . Appendix B shows that the nonadiabatic part of the distribution function satisfies the following nonlinear gyrokinetic equation:

$$\begin{aligned} & \left[ \frac{\partial}{\partial t_0} + (\mathbf{V}_0 + v'_\parallel \mathbf{b}) \cdot \nabla + i\mathbf{k}_\perp \cdot \mathbf{v}_{da} \right] \hat{h}_a(\mathbf{k}_\perp) \\ &= f_{a0} \left[ \hat{w}_{a1}(\mathbf{k}_\perp) X_{a1}^A + \hat{w}_{a2}(\mathbf{k}_\perp) X_{a2}^A + \hat{w}_{aV}(\mathbf{k}_\perp) X_{aV}^A + \hat{w}_{aT}(\mathbf{k}_\perp) X_{aT}^A \right] \\ &+ \frac{c}{B} \sum_{\mathbf{k}'_\perp + \mathbf{k}''_\perp = \mathbf{k}_\perp} [\mathbf{b} \cdot (\mathbf{k}'_\perp \times \mathbf{k}''_\perp)] J_0(\gamma') \hat{\phi}_a(\mathbf{k}'_\perp) \hat{h}_a(\mathbf{k}''_\perp) \\ &+ \oint \frac{d\xi}{2\pi} e^{-iL_a(\mathbf{k}_\perp)} C_a^L [\hat{f}_a(\mathbf{k}_\perp)] \end{aligned} \quad (49)$$

where  $\partial/\partial t_0 \equiv e^{-\omega_{-1}t}(\partial/\partial t)e^{\omega_{-1}t} = \partial/\partial t + i\omega_{-1}$ , and  $J_0(\gamma')$  is the zeroth-order Bessel function of  $\gamma' \equiv k'_\perp v'_\perp / \Omega_a$ . In the right-hand side of Eq.(49), we have defined the forces  $(X_{a1}^A, X_{a2}^A, X_{aV}^A, X_{aT}^A)$  as

$$X_{a1}^A \equiv \frac{X_{a1}}{T_a}, \quad X_{a2}^A \equiv \frac{X_{a2}}{T_a}, \quad X_{aV}^A \equiv \frac{X_{aV}}{T_a}, \quad X_{aT}^A \equiv \frac{1}{T_a} \quad (50)$$

and the fluctuating functions  $(\hat{w}_{a1}, \hat{w}_{a2}, \hat{w}_{aV}, \hat{w}_{aT})$  as

$$\begin{aligned} \hat{w}_{a1}(\mathbf{k}_\perp) &= i\mathbf{k}_\perp \cdot (R\hat{\zeta}) J_0(\gamma) \hat{\phi}(\mathbf{k}_\perp) \\ \hat{w}_{a2}(\mathbf{k}_\perp) &= i\mathbf{k}_\perp \cdot (R\hat{\zeta}) J_0(\gamma) \hat{\phi}(\mathbf{k}_\perp) \left( \frac{\varepsilon}{T_a} - \frac{5}{2} \right) \\ \hat{w}_{aV}(\mathbf{k}_\perp) &= \hat{\phi}(\mathbf{k}_\perp) \left[ J_0(\gamma) m_a c \left( R^2 V^\zeta + \frac{I}{B} v'_\parallel \right) i\mathbf{k}_\perp \cdot (R\hat{\zeta}) - \gamma J_1(\gamma) e_a \frac{1}{k_\perp^2} (\mathbf{k}_\perp \mathbf{k}_\perp) : (R\hat{\zeta})(\nabla\Psi) \right] \\ \hat{w}_{aT}(\mathbf{k}_\perp) &= e_a J_0(\gamma) \left( \frac{\partial}{\partial t_0} + \mathbf{V}_0 \cdot \nabla \right) \hat{\phi}(\mathbf{k}_\perp). \end{aligned} \quad (51)$$

Here, the last term in the definition of  $\hat{w}_{aV}$  is not contained in the gyrokinetic equation by Artun and Tang<sup>19,20</sup> but needed to derive the anomalous viscosity which reduces to the Reynolds stress in the fluid limit [see Eqs.(58) and (59)].

In the same way as in Ref.8, the contribution from the turbulent fluctuations to the entropy balance is represented by

$$\begin{aligned}
\langle \dot{S}_a^A \rangle &\equiv - \left\langle \int d^3v (\ln \bar{f}_a + 1) \left( \bar{\mathcal{D}}_a - \overline{\mathcal{L} \tilde{f}_a^A} \right) \right\rangle \\
&= - \frac{1}{V'} \frac{\partial}{\partial \Psi} \left( V' J_{S_a}^A \right) + \langle \sigma_a^A \rangle
\end{aligned} \tag{52}$$

where the surface-averaged radial anomalous entropy flux is given by

$$J_{S_a}^A = \left( \frac{S_a}{n_a} - \frac{\Xi_a}{T_a} \right) \Gamma_a^A + \frac{q_a^A}{T_a} \tag{53}$$

and the surface-averaged anomalous entropy production rate is written in the thermodynamic form as

$$\langle \sigma_a^A \rangle = \Gamma_a^A X_{a1}^A + \frac{1}{T_a} q_a^A X_{a2}^A + \Pi_a^A X_{aV}^A + Q_a^A X_{aT}^A. \tag{54}$$

The anomalous fluxes  $(\Gamma_a^A, q_a^A/T_a, \Pi_a^A, Q_a^A)$  conjugate to the forces  $(X_{a1}^A, X_{a2}^A, X_{aV}^A, X_{aT}^A)$  are given by the correlations between  $\hat{h}_a$  and  $(\hat{w}_{a1}, \hat{w}_{a2}, \hat{w}_{aV}, \hat{w}_{aT})$  as

$$\begin{aligned}
\Gamma_a^A &\equiv \left\langle \left\langle \int d^3v \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) \hat{w}_{a1}(\mathbf{k}_\perp) \right\rangle \right\rangle, & \frac{q_a^A}{T_a} &\equiv \left\langle \left\langle \int d^3v \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) \hat{w}_{a2}(\mathbf{k}_\perp) \right\rangle \right\rangle \\
\Pi_a^A &\equiv \left\langle \left\langle \int d^3v \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) \hat{w}_{aV}(\mathbf{k}_\perp) \right\rangle \right\rangle, & Q_a^A &\equiv \left\langle \left\langle \int d^3v \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) \hat{w}_{aT}(\mathbf{k}_\perp) \right\rangle \right\rangle.
\end{aligned} \tag{55}$$

The heating term due to the fluctuation-particle interaction operator  $\mathcal{D}_a$  in Eq.(15) is rewritten in the stationary turbulent states by

$$\left\langle \int d^3v \varepsilon \mathcal{D}_a \right\rangle = - \frac{1}{V'} \frac{\partial}{\partial \Psi} \left[ V' (q_a^A - q_a^{\text{anom}}) \right] + Q_a^A. \tag{56}$$

We find from Eqs.(15), (19), (32) and (56) that not  $q_a^{\text{anom}}$  but  $q_a^A$  should be used as a radial anomalous heat flux in the energy balance equation, and that the fluctuations transfer the energy to the particles of species  $a$  through the two terms  $-e_a \Gamma_a^{\text{anom}} (\partial \langle \Phi_1 \rangle / \partial \Psi)$  and  $Q_a^A$ . Using the charge neutrality condition  $\sum_a e_a \hat{n}_a = 0$  ( $\hat{n}_a \equiv \int d^3v \hat{f}_a$ ) or the Poisson's equation  $\nabla \cdot \hat{\mathbf{E}} = 4\pi \sum_a e_a \hat{n}_a$  for the self-consistent fluctuations, we have the ambipolarity of the anomalous particle fluxes  $\sum_a e_a \Gamma_a^A = 0$  and the cancellation of the total anomalous heat transfer  $\sum_a Q_a^A = 0$ , which shows that the self-consistent fluctuations cause no net heating of the total particles but result in the anomalous heat exchange between different species of particles. We obtain the balance equation for the fluctuation amplitude as

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\langle \left\langle \int d^3v \frac{1}{2f_{a0}} \sum_{\mathbf{k}_\perp} |\hat{h}_a(\mathbf{k}_\perp)|^2 \right\rangle \right\rangle \\
&= \langle \sigma_a^A \rangle + \left\langle \left\langle \int d^3v \frac{1}{f_{a0}} \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) e^{-iL_a(\mathbf{k}_\perp)} C_a^L [\hat{h}_a(\mathbf{k}_\perp) e^{iL_a(\mathbf{k}_\perp)}] \right\rangle \right\rangle. \quad (57)
\end{aligned}$$

Thus, in the stationary turbulent states, the anomalous entropy production driven by the turbulent transport equals the collisional dissipation of the fluctuating distribution function, which results in the positive definiteness of the total anomalous entropy production:  $\sum_a T_a \langle \sigma_a^A \rangle = -\sum_a \left\langle \left\langle \int d^3v \frac{1}{f_{a0}} \sum_{\mathbf{k}_\perp} \hat{h}_a^*(\mathbf{k}_\perp) e^{-iL_a(\mathbf{k}_\perp)} C_a^L [\hat{h}_a(\mathbf{k}_\perp) e^{iL_a(\mathbf{k}_\perp)}] \right\rangle \right\rangle \geq 0$ . The Onsager symmetry for the quasilinear anomalous transport equations is described in Appendix C.

Compared to the conventional gyrokinetic equation<sup>9-15</sup>, our gyrokinetic equation (49) contains the flow shear  $X_{aV}^A$  as an additional thermodynamic force and the function  $\hat{w}_{aV}$  related to the anomalous viscosity. It is instructive to derive the Hasegawa-Mima equation<sup>24</sup> from Eq.(49) and examine how the flow shear and the anomalous viscosity enter it. Considering that a collisionless plasma consists of adiabatic electrons and a single species of ions with charge  $e_i \equiv Z_i e$  and low temperature  $T_i \ll T_e$  ( $k_\perp \rho_i < 1$ ), and assuming that the ion nonadiabatic distribution function has the form  $\hat{h}_i \simeq f_{i0} \hat{n}_i^{\text{nad}}/n_i$ , we obtain from Eq.(49) with the charge neutrality condition the generalized Hasegawa-Mima equation:

$$\begin{aligned}
& \left[ (1 + k_\perp^2 \rho_s^2) \left( \frac{\partial}{\partial t_0} + \mathbf{V}_0 \cdot \nabla \right) + i\omega_* + i\mathbf{k}_\perp \cdot \frac{cT_e}{e} \nabla \times \left( \frac{\mathbf{b}}{\mathbf{B}} \right) + \rho_s^2(\mathbf{k}_\perp \mathbf{k}_\perp) : (R\hat{\zeta})(\nabla\Psi)X_V \right] \frac{e\hat{\phi}(\mathbf{k}_\perp)}{T_e} \\
&= \frac{1}{2} c_s \rho_s^3 \sum_{\mathbf{k}'_\perp + \mathbf{k}''_\perp = \mathbf{k}_\perp} [\mathbf{b} \cdot (\mathbf{k}'_\perp \times \mathbf{k}''_\perp)] [(k''_\perp)^2 - (k'_\perp)^2] \frac{e\hat{\phi}_a(\mathbf{k}'_\perp)}{T_e} \frac{e\hat{\phi}_a(\mathbf{k}''_\perp)}{T_e} \quad (58)
\end{aligned}$$

where  $c_s \equiv (Z_i T_e / m_i)^{1/2}$ ,  $\rho_s \equiv c_s / \Omega_i$ ,  $\omega_* \equiv \mathbf{k}_\perp \cdot \mathbf{u}_*$ , and  $\mathbf{u}_* \equiv (T_e / T_i)(c\mathbf{b} \times \nabla\Psi / eB)[X_{i1} + (-1 + \Xi_i / T_i)X_{i2} + m_i R^2 V^\zeta X_V]$ . The last term in the left-hand of Eq.(58) determines the exchange of energy between the fluctuating  $\hat{\mathbf{E}} \times \mathbf{B}$  flows and the background flow  $\mathbf{V}_0$ . To see this, we derive the energy balance equation from Eq.(58) as

$$\begin{aligned}
& \left( \frac{\partial}{\partial t_0} + \mathbf{V}_0 \cdot \nabla \right) \sum_{\mathbf{k}_\perp} \left( \frac{1}{2} n_i m_i \langle |\hat{\mathbf{v}}_E(\mathbf{k}_\perp)|^2 \rangle_{\text{ens}} + \frac{\langle |\hat{\phi}(\mathbf{k}_\perp)|^2 \rangle_{\text{ens}}}{8\pi\lambda_{De}^2} \right) \\
&= - \sum_{\mathbf{k}_\perp} n_i m_i \langle \hat{\mathbf{v}}_E^*(\mathbf{k}_\perp) \hat{\mathbf{v}}_E(\mathbf{k}_\perp) \rangle_{\text{ens}} : (\nabla\mathbf{V}_0) \quad (59)
\end{aligned}$$

where  $\lambda_{De} \equiv T_e / (4\pi n_e e^2)^{1/2}$  is the electron Debye length, and  $\hat{\mathbf{v}}_E(\mathbf{k}_\perp) \equiv -i(c/B)\hat{\phi}(\mathbf{k}_\perp)\mathbf{k}_\perp \times \mathbf{b}$  is the  $\hat{\mathbf{E}} \times \mathbf{B}$  drift velocity due to the electrostatic fluctuations. Here we have used

the relation  $(\mathbf{k}_\perp \mathbf{k}_\perp) : (R\hat{\zeta})(\nabla\Psi)X_V = [(\mathbf{k}_\perp \times \mathbf{b})(\mathbf{k}_\perp \times \mathbf{b})] : (\nabla\mathbf{V}_0)$ . The right-hand side of Eq.(59) represents the energy transfer from the background sheared flow to the fluctuations through the Reynolds stress multiplied by the flow shear. In this case, the anomalous ion viscosity  $\Pi_i^A$  is rewritten in terms of the Reynolds stress as  $\Pi_i^A \simeq \langle\langle \sum_{\mathbf{k}_\perp} n_i m_i [\hat{\mathbf{v}}_E^*(\mathbf{k}_\perp) \hat{\mathbf{v}}_E(\mathbf{k}_\perp)] : (R\hat{\zeta})(\nabla\Psi) \rangle\rangle$ .

## V. EFFECTS OF SHEARED TOROIDAL FLOW ON ION TEMPERATURE GRADIENT DRIVEN MODES

Here we use the sheared slab geometry to consider the ion temperature gradient (ITG) driven modes localized in the bad curvature region of the large-aspect-ratio system. Let us assume that the plasma is collisionless and consists of adiabatic electrons and a single species of ions with charge  $e_i \equiv Z_i e$ . Using the charge neutrality and the linearized version of the gyrokinetic equation (49) with the approximations  $|\mathbf{k}_\perp \cdot \mathbf{v}_{di}/\omega_0| \ll 1$ ,  $|k_\parallel v_{Ti}/\omega_0| \ll 1$  ( $\omega_0 \equiv \omega - \omega_{-1}$ ), and  $k_\perp \rho_i \ll 1$ , we obtain the following linear eigenmode equation:

$$\left[ \frac{d^2}{dx^2} - b_s + \frac{1 + \alpha - \hat{\omega}}{\hat{\omega} + K} - \frac{G}{\hat{\omega}} - \frac{\Sigma}{(\hat{\omega} + K)^2} + \frac{L_N^2}{L_s^2 \hat{\omega}^2} (x - \Delta)^2 \right] \hat{\phi} = 0 \quad (60)$$

where  $b_s \equiv k_\theta^2 \rho_s^2$  ( $k_\theta$ : the poloidal wavenumber),  $\hat{\omega} \equiv \omega_0/\omega_{*e}$ ,  $\omega_{*e} \equiv k_\theta c T_e / (e B L_N)$ , and  $L_N \equiv -(d \ln N_e / dr)^{-1}$ . The radial distance from the mode rational surface normalized by  $\rho_s$  is denoted by  $x \equiv (r - r_s) / \rho_s$  ( $r_s$ : the minor radius of the mode rational surface), and the magnetic shear length is defined by  $L_s \equiv Rq/s$  with the safety factor  $q$  and the shear parameter  $s \equiv r |d \ln q / dr|$ . Other parameters in Eq.(60) are defined by

$$\begin{aligned} \alpha &\equiv \eta_i \left( \frac{\Xi_i}{T_i} - \frac{L_N V_0^2}{R K c_s^2} \right) + \frac{L_N m_i V_0^2}{L_E T_i} \left( 1 + 2 \frac{L_N \tau}{R K} \right) \\ K &\equiv \tau \left[ 1 + \eta_i \left( 1 + \frac{\Xi_i}{T_i} \right) + \frac{L_N m_i V_0^2}{L_E T_i} \right] \\ G &\equiv 2 \frac{L_N}{R} \left[ 1 + \frac{V_0^2}{2 K c_s^2} \left\{ 1 + \eta_i \frac{\Xi_i}{T_i} + \frac{L_N}{L_E} \left( 2 + \frac{m_i V_0^2}{T_i} \right) \right\} \right] \\ \Sigma &\equiv \frac{L_N^2 V_0^2}{4 L_E^2 c_s^2} \\ \Delta &\equiv - \frac{L_s V_0}{2 L_E c_s} \frac{\hat{\omega}}{(\hat{\omega} + K)} \end{aligned} \quad (61)$$

where  $\tau \equiv T_i/(Z_i T_e)$ ,  $\eta_i \equiv L_N/L_{T_i}$ , and  $L_{T_i} \equiv -(d \ln T_i/dr)^{-1}$ . In Eq.(60), the parameters  $K$  and  $G$  destabilize the ITG modes. Especially, the parameter  $G$ , which contains the toroidal curvature term  $2L_N/R$ , causes the toroidal ITG modes. From Eq.(61), we find that  $G$  is modified by the terms resulting from the centrifugal force and the Coriolis force due to the toroidal rotation. The parameter  $\Sigma$  represents the effects of the sheared toroidal flow, and  $L_E \equiv -(d \ln V^\zeta/dr)^{-1} = -(d \ln E_r/dr)^{-1}$  denotes the gradient scale length for the toroidal flow (or for the radial electric field).

The linear eigenmode equation (60) is easily solved to give the dispersion relation:

$$-b_s + \frac{1 + \alpha - \hat{\omega}}{\hat{\omega} + K} - \frac{G}{\hat{\omega}} - \frac{\Sigma}{(\hat{\omega} + K)^2} = (2n + 1) \frac{i L_N}{\hat{\omega} L_s} \quad (n = 0, 1, 2, \dots) \quad (62)$$

and the corresponding eigenfunction:

$$\hat{\phi} = \text{const} \times H_n \left[ \left( \frac{i L_N}{\hat{\omega} L_s} \right)^{1/2} (x - \Delta) \right] \exp \left[ -\frac{i L_N}{2\hat{\omega} L_s} (x - \Delta)^2 \right] \quad (63)$$

where  $H_n$  is the Hermite function of order  $n$ . If we put  $G = \alpha = 0$ , Eq.(62) reduces to the dispersion relation obtained by Dong and Horton for the slab ITG modes in the sheared flow.<sup>26</sup>

Now, let us consider the simple cases where  $b_s$ ,  $\alpha$ , and  $(2n + 1)L_N/L_s$  are negligibly small, and  $K \gg 1 \gg G$  is satisfied. If there is no sheared flow  $\Sigma = 0$ , we have from Eq.(62) the linear growth rate of the toroidal ITG mode:  $\text{Im}(\hat{\omega}) \simeq (KG)^{1/2}$  for  $KG \gg 1$ . When the sheared flow is large enough to satisfy  $\Sigma > K^2/4$ , we obtain the linear growth rate of the sheared flow driven instability:  $\text{Im}(\hat{\omega}) \simeq \Sigma^{1/2}$ . We find that the stability condition is approximately given by  $K[1 + 2(KG)^{1/2}] < \Sigma < K^2/4$ , which is rewritten by

$$2K^{1/2}[1 + 2(KG)^{1/2}]^{1/2} < \frac{L_N V_0}{L_E c_s} < K. \quad (64)$$

From this estimation, the stability window is expected to appear more clearly when  $K \gg 1$  is well satisfied. In order to have large  $K$ , hot ion temperature  $\tau \equiv T_i/(Z_i T_e) \gg 1$  or large ion temperature gradient  $\eta_i \gg 1$  is required.

Figure 1 shows the normalized linear growth rates  $\text{Im}(\hat{\omega}) = \text{Im}(\omega)/\omega_{*e}$  obtained from Eq.(62) as functions of the flow shear parameter  $(L_N/L_E)(V_0/c_s)$  for  $K = 5, 8, 11$  with

no magnetic shear  $L_N/L_s = 0$ . Here and hereafter, we put  $G = 0.1$ ,  $k_\theta \rho_s = 0.1$ , and  $\alpha = 0$ . It is found that the increase of  $K$  broadens the width of the stability window but heightens the strength of the flow shear required for the stabilization, which is expected from the approximate stability condition (64). (Figure 1 recalls the effects of the radial electric field shear on the resistive interchange modes,<sup>25</sup> which have similar shear dependence of the linear growth rates with the stability window.) The real and imaginary parts of the normalized eigenfrequency  $\hat{\omega} \equiv \omega_0/\omega_{*e}$  for  $K = 8$  are given as functions of  $(L_N/L_E)(V_0/c_s)$  in Figs.2(a) and (b), respectively, where the cases with different values of magnetic shear  $L_N/L_s = 0, 0.01, 0.05, 0.1$  are shown. As the flow shear increases, the real frequency has the sign of the ion diamagnetic frequency and its absolute value increases. The reduction of the growth rates by the flow shear becomes weaker for stronger magnetic shear.

Using the linearized gyrokinetic equation for the response of the distribution function to the electrostatic fluctuations, we can obtain quasilinearly the anomalous ion radial heat flux and the anomalous ion toroidal viscosity, which are given by

$$\begin{aligned} q_{ir}^A &\equiv \frac{q_i^A}{RB_P} \simeq -n_i \chi_i^A \frac{dT_i}{dr} \\ \Pi_{ir\zeta}^A &\equiv \frac{\Pi_i^A}{R^2 B_P} \simeq -n_i m_i \mu_i^A \frac{dV_0}{dr} \end{aligned} \quad (65)$$

where the contributions from the non-diagonal parts of the anomalous transport coefficients are neglected by using the approximations  $|\mathbf{k}_\perp \cdot \mathbf{v}_{dt}/\omega_0| \ll 1$  and  $|k_\parallel v_{Ti}/\omega_0| \ll 1$ . The anomalous ion thermal diffusivity  $\chi_i^A$  and the anomalous ion toroidal momentum diffusivity  $\mu_i^A$  in Eq.(65) are given by

$$\chi_i^A = \frac{3}{2} \mu_i^A, \quad \mu_i^A = \frac{cT_e}{eB} \frac{e^2 |\hat{\phi}|^2}{T_e^2} |k_y| L_N \frac{\text{Im}(\hat{\omega})}{|\hat{\omega}|^2} \quad (66)$$

where  $k_y$  and  $\hat{\omega}$  denote the representative wavenumber in the energy containing range of the fluctuation spectrum and the corresponding eigenfrequency, respectively.

If we use the mixing length treatment to estimate the potential fluctuation amplitude as  $e|\hat{\phi}|/T_e \sim (k_\perp L)^{-1}$  with the characteristic wavenumber  $k_\perp \sim k_y$  and the ion pressure gradient scale length  $L \sim L_N/(1 + \eta_i)$ , we have

$$\frac{2}{3}\chi_i^A = \mu_i^A \sim \frac{cT_c}{eB} \frac{\rho_s}{L_N} \frac{(1 + \eta_i)^2 \text{Im}(\hat{\omega})}{|k_y \rho_s| |\hat{\omega}|^2}. \quad (67)$$

[In the case where the flow shear is the dominant destabilizing source, we should take account of  $V_0/L_E$  to estimate the fluctuation level as in Ref.27.] The anomalous transport coefficients given by Eq.(67) depend on the flow shear through the linear response factor  $\text{Im}(\hat{\omega})/|\hat{\omega}|^2$ , which is shown as a function of  $(L_N/L_E)(V_0/c_s)$  in Fig.3 using the same parameters as in Fig.2. We see that the decrease of the growth rate  $\text{Im}(\hat{\omega})$  together with the increase of  $|\hat{\omega}|$  results in the abrupt cutoff of the anomalous transport coefficients for the flow shear (or the radial electric field shear) greater than a critical value.

The dispersion relation obtained by Dong and Horton<sup>26</sup> using the kinetic integral equation predicts the critical value of the radial electric field shear for stabilization smaller than the results of our dispersion relation Eq.(62) obtained in the limit of low wavenumbers and high phase velocities. Their kinetic dispersion relation also shows that the smaller magnetic shear is favorable for the stability window as shown here. The present transport analysis demonstrates the relationship between the energy and momentum transport in the presence of drift wave fluctuations in an axisymmetric toroidal plasma.

## VI. CONCLUSIONS AND DISCUSSION

In this work, the synthesized theory of neoclassical and anomalous transport<sup>7,8</sup> is generalized to that for the rotating turbulent plasma with large flow velocities on the order of the ion thermal speed. Taking account of all transport processes, *i.e.*, classical, neoclassical and anomalous transport processes, we have obtained balance equations for the particles, energy, toroidal momentum (or radial electric field), entropy, and the fluctuation amplitude, which are given by Eqs.(14), (15), (17), (30), and (57), respectively. We also have given the rigorous expressions which define the classical, neoclassical, and anomalous transport fluxes of the particles, energy, and toroidal momentum. Nonequilibrium thermodynamic properties such as the entropy production rate, the conjugate pairs of the fluxes and forces,



and the transport equations with the Onsager symmetry are established from the basic kinetic equations as the first principle. The drift kinetic equation and the new gyrokinetic equation for the rotating plasma are used to formulate the neoclassical and anomalous fluxes which are connected to the conjugate thermodynamic forces by the corresponding transport equations.

In the presence of the high-speed toroidal flows, the shear of the toroidal flow (or the radial electric field shear) enters all the classical, neoclassical, and anomalous transport equations as an additional thermodynamic force, and accordingly influences the transport fluxes and the fluctuation level. On the other hand, through the anomalous viscosity term in the toroidal momentum balance equation, the fluctuations affect the flow or the radial electric field as a reaction to the sheared flow effect on the fluctuations. Thus, a self-consistent description of the interaction between the fluctuations and the sheared flow is given.

Our gyrokinetic equation for the rotating plasma contains the new sheared flow driving term, from which the perpendicular anomalous viscosity is defined kinetically. We have derived the generalized Hasegawa-Mima equation from the gyrokinetic equation, and found that the kinetically defined anomalous viscosity reduces to the Reynolds stress in the fluid limit.

We have examined effects of the toroidal flow shear on the toroidal ITG modes by using the approximate dispersion relation derived from the gyrokinetic equation. It is found that there exists a stability window in the flow shear parameter space when the parameter  $K$  [see Eq.(61)] is large (or when  $T_i/T_e \gg 1$  or  $\eta_i \equiv L_N/L_{Ti} \gg 1$ ). As  $K$  increases, the width of the stability window is broadened and the threshold value of the flow shear is heightened. The flow shear stabilization is relatively stronger for the case of weak magnetic shear. We have given the quasilinear anomalous diffusivities for the heat and toroidal momentum transport using the linear eigenfrequencies and the mixing length argument. The anomalous toroidal momentum diffusivity is proportional to the anomalous thermal diffusivity and they decrease as the flow shear increases. In the improved confinement of the JT-60U tokamak,<sup>28</sup>

the internal transport barrier (ITB) with the steep ion temperature gradient is formed in the region where the gradient of the toroidal flow is steep, the magnetic shear is weak, and  $T_i > T_e$ . These ITB formation conditions are in qualitative agreement with our results on the stabilization of the toroidal ITG modes by the sheared toroidal flow. In the stabilized region, there still remain the neoclassical fluxes which are presented in Sec.III.

### ACKNOWLEDGMENTS

The author (HS) thanks Professor M. Okamoto for his encouragement of this work. This work is supported in part by the Grant-in-Aid from the Japanese Ministry of Education, Science and Culture, and in part by the U.S. Department of Energy Grant DE-FG05-80ET-53088.

### APPENDIX A: CLASSICAL TRANSPORT EQUATIONS FOR TOROIDALLY ROTATING PLASMAS

The gyrophase-dependent part of the distribution function is of the order  $\delta$ , and is written as

$$\begin{aligned} \tilde{f}_{a1} = f_{a0} \frac{m_a}{T_a} \left[ \mathbf{u}_{\perp a} \cdot \mathbf{v}'_{\perp} + \frac{2}{5} \left( \frac{\mathbf{q}_{\perp a}}{p_a} - \frac{\Xi_a}{T_a} \mathbf{u}_{\perp a} \right) \cdot \mathbf{v}'_{\perp} \left( \frac{m_a (v')^2}{2T_a} - \frac{5}{2} \right) \right. \\ \left. + \frac{1}{2p_a} \pi_a^{\text{gyro}} : \left( \mathbf{v}' \mathbf{v}' - \frac{(v')^2}{3} \mathbf{l} \right) \right] \end{aligned} \quad (\text{A1})$$

where the perpendicular flows and the gyroviscosity are given by

$$\begin{aligned} n_a \mathbf{u}_{\perp a} &\equiv \int d^3v \tilde{f}_{a1} \mathbf{v}'_{\perp} = \frac{c}{e_a B} \left( X_{a1} + \frac{\Xi_a}{T_a} X_{a2} + m_a R^2 V^{\zeta} X_V \right) \nabla \Psi \times \mathbf{b} \\ \frac{\mathbf{q}_{\perp a}}{T_a} &\equiv \frac{1}{T_a} \int d^3v \tilde{f}_{a1} \varepsilon \mathbf{v}'_{\perp} = \frac{5c X_{a2}}{2e_a B} \nabla \Psi \times \mathbf{b} + \frac{\Xi_a}{T_a} n_a \mathbf{u}_{\perp a} \\ \pi_a^{\text{gyro}} &\equiv \int d^3v \tilde{f}_{a1} m_a \left( \mathbf{v}' \mathbf{v}' - \frac{(v')^2}{3} \mathbf{l} \right) \\ &= \frac{p_a X_V}{2B\Omega_a} [ -(\nabla \Psi)(\nabla \Psi) + (\nabla \Psi \times \mathbf{b})(\nabla \Psi \times \mathbf{b}) + 2I \{ \mathbf{b}(\nabla \Psi \times \mathbf{b}) + (\nabla \Psi \times \mathbf{b})\mathbf{b} \} ]. \end{aligned} \quad (\text{A2})$$

Using Eq.(A1), the collisional frictions are related to  $\mathbf{u}_{\perp a}$ ,  $\mathbf{q}_{\perp a}$ , and  $\pi_a^{\text{gyro}}$  by

$$\int d^3v C_a^L(\tilde{f}_{a1})m_a \mathbf{v}'_{\perp} \begin{bmatrix} 1 \\ \left(\frac{m_a(v')^2}{2T_a} - \frac{5}{2}\right) \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{21}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\perp b} \\ \frac{2}{5} \left( \frac{\mathbf{q}_{\perp b}}{p_b} - \frac{\Xi_b}{T_b} \mathbf{u}_{\perp b} \right) \end{bmatrix}$$

$$\int d^3v C_a^L(\tilde{f}_{a1})m_a \left( \mathbf{v}'\mathbf{v}' - \frac{(v')^2}{3}\mathbf{1} \right) = \sum_b l_V^{ab} \frac{\pi_b^{\text{gyro}}}{p_b} \quad (\text{A3})$$

where the coefficients  $l_{jk}^{ab}$  are the same ones as defined in Ref.3 and Ref.22, and  $l_V^{ab}$  are defined by

$$l_V^{ab} \equiv \delta_{ab} \frac{m_a^2}{15T_a} \sum_{a'} \int d^3v (v')^2 C_{aa'}[(v')^2 f_{a0}, f_{a'0}] + \frac{m_a m_b}{15T_b} \int d^3v (v')^2 C_{ab}[f_{a0}, (v')^2 f_{b0}]. \quad (\text{A4})$$

The self-adjointness of the linearized collision operator gives the symmetry properties of the coefficients:

$$l_{jk}^{ab} = l_{kj}^{ba}, \quad l_V^{ab} = l_V^{ba}. \quad (\text{A5})$$

From Eqs.(26) and (A2), we obtain the classical transport equations as

$$\Gamma_a^{\text{cl}} = \sum_b [(L^{\text{cl}})_{11}^{ab} X_{b1} + (L^{\text{cl}})_{12}^{ab} X_{b2}] + (L^{\text{cl}})_{1V}^a X_V$$

$$\frac{1}{T_a} \dot{q}_a^{\text{cl}} = \sum_b [(L^{\text{cl}})_{21}^{ab} X_{b1} + (L^{\text{cl}})_{22}^{ab} X_{b2}] + (L^{\text{cl}})_{2V}^a X_V$$

$$\sum_a \Pi_a^{\text{cl}} = \sum_b [(L^{\text{cl}})_{V1}^b X_{b1} + (L^{\text{cl}})_{V2}^b X_{b2}] + (L^{\text{cl}})_{VV} X_V \quad (\text{A6})$$

where the classical transport coefficients are given by

$$\begin{bmatrix} (L^{\text{cl}})_{11}^{ab} & (L^{\text{cl}})_{12}^{ab} \\ (L^{\text{cl}})_{21}^{ab} & (L^{\text{cl}})_{22}^{ab} \end{bmatrix} = \left\langle \frac{c^2 R^2 B_P^2}{e_a e_b B^2} \begin{bmatrix} 1 & 0 \\ \frac{\Xi_a}{T_a} & 1 \end{bmatrix} \begin{bmatrix} -l_{11}^{ab} & l_{12}^{ab} \\ l_{21}^{ab} & -l_{22}^{ab} \end{bmatrix} \begin{bmatrix} 1 & \frac{\Xi_b}{T_b} \\ 0 & 1 \end{bmatrix} \right\rangle$$

$$\begin{bmatrix} (L^{\text{cl}})_{1V}^a \\ (L^{\text{cl}})_{2V}^a \end{bmatrix} = \begin{bmatrix} (L^{\text{cl}})_{V1}^a \\ (L^{\text{cl}})_{V2}^a \end{bmatrix} = \sum_b \left\langle m_b R^2 V \zeta \frac{c^2 R^2 B_P^2}{e_a e_b B^2} \begin{bmatrix} 1 & 0 \\ \frac{\Xi_a}{T_a} & 1 \end{bmatrix} \begin{bmatrix} -l_{11}^{ab} \\ l_{21}^{ab} \end{bmatrix} \right\rangle$$

$$(L^{\text{cl}})_{VV} = - \sum_{a,b} \frac{m_a m_b c^2}{e_a e_b} \left\langle \frac{R^2 B_P^2}{B^2} \left( \frac{(R^2 B_P^2 + 4I^2)}{4B^2} l_V^{ab} + R^4 (V\zeta)^2 l_{11}^{ab} \right) \right\rangle. \quad (\text{A7})$$

From Eqs.(A5) and (A7), we obtain the Onsager symmetry for the classical transport coefficients which is written as

$$\begin{aligned}
(L^{\text{cl}})_{mn}^{ab}(V^\zeta) &= (L^{\text{cl}})_{mn}^{ab}(-V^\zeta) = (L^{\text{cl}})_{nm}^{ba}(V^\zeta) \quad (m, n = 1, 2) \\
(L^{\text{cl}})_{mV}^a(V^\zeta) &= -(L^{\text{cl}})_{mV}^a(-V^\zeta) = (L^{\text{cl}})_{Vm}^a(V^\zeta) \quad (m = 1, 2) \\
(L^{\text{cl}})_{VV}(V^\zeta) &= (L^{\text{cl}})_{VV}(-V^\zeta)
\end{aligned} \tag{A8}$$

which has the same form as Eq.(43). Equation (A8) is valid even without up-down symmetry since the classical transport is a spatially local process. The momentum conservation by collisions assures the intrinsic ambipolarity of the classical particle fluxes  $\sum_a e_a \Gamma_a^{\text{cl}} = 0$ , which reduces the number of the independent thermodynamic forces in the classical transport equations by one. As shown in Ref.22, the reduced classical transport equations retain the Onsager symmetry.

## APPENDIX B: DERIVATION OF GYROKINETIC EQUATION FOR TOROIDALLY ROTATING PLASMAS

The derivation of the gyrokinetic equation (49) for rotating plasmas with the toroidal flow velocity  $\mathbf{V}_0 = \mathcal{O}(v_{T_a})$  is briefly explained in this appendix. Subtracting the ensemble-averaged kinetic equation (1) from the original non-ensemble-averaged one gives the equation for the fluctuating distribution  $\hat{f}_a$  as

$$\left[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{e_a}{m_a} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial}{\partial \mathbf{v}} \right] \hat{f}_a - C_a^L(\hat{f}_a) = -\frac{e_a}{m_a} \hat{\mathbf{E}} \cdot \frac{\partial (f_a + \hat{f}_a)}{\partial \mathbf{v}} - \mathcal{D}_a \tag{B1}$$

where the magnetic fluctuations are not considered. Let us assume that the ensemble-average part and the fluctuating part of the distribution function are expanded in terms of  $\delta \equiv \rho_a/L$  as  $f_a = f_{a0} + f_{a1} + \dots$  and  $\hat{f}_a = \hat{f}_{a1} + \hat{f}_{a2} + \dots$ . The lowest-order part of Eq.(B1) in  $\delta$  is written for the fluctuations in the WKB form of Eq.(46) as

$$\left[ \frac{\partial}{\partial t_{-1}} + i\mathbf{k}_\perp \cdot (\mathbf{V}_0 + \mathbf{v}') - \Omega_a \frac{\partial}{\partial \xi} \right] \hat{f}_{a1}(\mathbf{k}_\perp) = -i\mathbf{k}_\perp \cdot \mathbf{v}'_\perp \frac{e_a \hat{\phi}(\mathbf{k}_\perp)}{T_a} f_{a0} \tag{B2}$$

where  $\partial/\partial t_{-1}$  is used to describe the rapid temporal variation with the characteristic frequencies in the order of  $\delta^{-1} v_{T_a}/L \sim \Omega_a$ . From the average part and the oscillating part of

Eq.(B2) with respect to the gyrophase  $\xi$ , we find that  $\hat{f}_{a1}(\mathbf{k}_\perp) \propto \exp(-i\omega_{-1}t)$  ( $\omega_{-1} \equiv \mathbf{k}_\perp \cdot \mathbf{V}_0$ ) and that

$$\hat{f}_{a1}(\mathbf{k}_\perp) = -\frac{e_a \hat{\phi}(\mathbf{k}_\perp)}{T_a} f_{a0} + \hat{h}_a(\mathbf{k}_\perp) e^{iL_a(\mathbf{k}_\perp)} \quad (\text{B3})$$

which gives Eq.(48). From the  $\mathcal{O}(\delta)$  part of Eq.(B1), we have the equation for the second-order fluctuating function  $\hat{f}_{a2}$  as

$$\begin{aligned} \left( i\mathbf{k}_\perp \cdot \mathbf{v}'_\perp - \Omega_a \frac{\partial}{\partial \xi} \right) \hat{f}_{a2}(\mathbf{k}_\perp) &\equiv -\Omega_a e^{iL_a(\mathbf{k}_\perp)} \frac{\partial}{\partial \xi} \left[ e^{-iL_a(\mathbf{k}_\perp)} \hat{f}_{a2}(\mathbf{k}_\perp) \right] \\ &= R_{1a}(\hat{h}_a) + R_{2a}(\hat{\phi}_a) + R_{3a}(\hat{\phi}_a, \hat{f}_{a1}) + C_a^L(\hat{f}_{a1}) \end{aligned} \quad (\text{B4})$$

where

$$\begin{aligned} R_{1a}(\hat{h}_a) &\equiv - \left[ \frac{\partial}{\partial t_0} + (\mathbf{V}_0 + \mathbf{v}') \cdot \nabla - \frac{e_a}{m_a} \nabla \Phi_1 \cdot \frac{\partial}{\partial \mathbf{v}'} \right] \left( \hat{h}_a(\mathbf{k}_\perp) e^{iL_a(\mathbf{k}_\perp)} \right) \\ R_{2a}(\hat{\phi}_a) &\equiv \left[ \frac{\partial}{\partial t_0} + (\mathbf{V}_0 + \mathbf{v}') \cdot \nabla - \frac{e_a}{m_a} \nabla \Phi_1 \cdot \frac{\partial}{\partial \mathbf{v}} \right] \left( \frac{e_a \hat{\phi}(\mathbf{k}_\perp)}{T_a} f_{a0} \right) \\ &\quad + \frac{e_a}{m_a} \nabla \hat{\phi}(\mathbf{k}_\perp) \cdot \frac{\partial f_{a0}}{\partial \mathbf{v}} + i \frac{e_a}{m_a} \hat{\phi}(\mathbf{k}_\perp) \mathbf{k}_\perp \cdot \frac{\partial f_{a1}}{\partial \mathbf{v}} \\ R_{3a}(\hat{\phi}_a, \hat{f}_{a1}) &\equiv i \frac{e_a}{m_a} \sum_{\mathbf{k}'_\perp + \mathbf{k}''_\perp = \mathbf{k}_\perp} \hat{\phi}(\mathbf{k}'_\perp) \mathbf{k}'_\perp \cdot \frac{\partial}{\partial \mathbf{v}} \left( \hat{f}_a(\mathbf{k}''_\perp) \right). \end{aligned} \quad (\text{B5})$$

The solvability condition for Eq.(B4) is written by

$$\oint \frac{d\xi}{2\pi} e^{-iL_a(\mathbf{k}_\perp)} \left[ R_{1a}(\hat{h}_a) + R_{2a}(\hat{\phi}_a) + R_{3a}(\hat{\phi}_a, \hat{f}_{a1}) + C_a^L(\hat{f}_{a1}) \right] = 0. \quad (\text{B6})$$

After lengthy calculations, the first three terms in the left-hand side of Eq.(B6) are written as

$$\begin{aligned} \oint \frac{d\xi}{2\pi} e^{-iL_a(\mathbf{k}_\perp)} R_{1a}(\hat{h}_a) &= - \left[ \frac{\partial}{\partial t_0} + (\mathbf{V}_0 + v'_\parallel \mathbf{b}) \cdot \nabla + i\mathbf{k}_\perp \cdot \mathbf{v}_{da} \right] \hat{h}_a(\mathbf{k}_\perp) \\ \oint \frac{d\xi}{2\pi} e^{-iL_a(\mathbf{k}_\perp)} R_{2a}(\hat{\phi}_a) &= f_{a0} \left[ \hat{w}_{a1}(\mathbf{k}_\perp) X_{a1}^A + \hat{w}_{a2}(\mathbf{k}_\perp) X_{a2}^A + \hat{w}_{aV}(\mathbf{k}_\perp) X_{aV}^A + \hat{w}_{aT}(\mathbf{k}_\perp) X_{aT}^A \right] \\ \oint \frac{d\xi}{2\pi} e^{-iL_a(\mathbf{k}_\perp)} R_{3a}(\hat{\phi}_a, \hat{f}_{a1}) &= \frac{c}{B} \sum_{\mathbf{k}'_\perp + \mathbf{k}''_\perp = \mathbf{k}_\perp} [\mathbf{b} \cdot (\mathbf{k}'_\perp \times \mathbf{k}''_\perp)] J_0(\gamma') \hat{\phi}_a(\mathbf{k}'_\perp) \hat{h}_a(\mathbf{k}''_\perp) \end{aligned} \quad (\text{B7})$$

where  $\hat{h}_a$  is regarded as a function of  $(t_0, \mathbf{x}', \varepsilon, \mu; \mathbf{k}_\perp)$  [see Eqs.(5) and (46)] and the definitions in Eqs.(50) and (51) are used. Finally, the gyrokinetic equation (49) is obtained from Eqs.(B6) and (B7).

## APPENDIX C: ONSAGER SYMMETRY OF QUASILINEAR ANOMALOUS TRANSPORT EQUATIONS

Here we assume that the spectra of the electrostatic fluctuations  $\hat{\phi}(\mathbf{k}_\perp)$  are given *a priori* and that the nonlinear term in the gyrokinetic equation (49) is negligible. Then, using the definitions in Eq.(55) with the solution of the linearized gyrokinetic equation, we obtain the quasilinear anomalous transport equations:

$$\begin{bmatrix} \Gamma_a^A \\ q_a^A/T_a \\ \Pi_a^A \\ Q_a^A \end{bmatrix} = \sum_b \begin{bmatrix} (L^A)_{11}^{ab} & (L^A)_{12}^{ab} & (L^A)_{1V}^{ab} & (L^A)_{1T}^{ab} \\ (L^A)_{21}^{ab} & (L^A)_{22}^{ab} & (L^A)_{2V}^{ab} & (L^A)_{2T}^{ab} \\ (L^A)_{V1}^{ab} & (L^A)_{V2}^{ab} & (L^A)_{VV}^{ab} & (L^A)_{VT}^{ab} \\ (L^A)_{T1}^{ab} & (L^A)_{T2}^{ab} & (L^A)_{TV}^{ab} & (L^A)_{TT}^{ab} \end{bmatrix} \begin{bmatrix} X_{a1}^A \\ X_{a2}^A \\ X_{aV}^A \\ X_{aT}^A \end{bmatrix}. \quad (\text{C1})$$

Here the anomalous transport coefficients  $(L^A)_{rs}^{ab}$  ( $r, s = 1, 2, V, T$ ) are functionals of the fluctuation spectra, and they also contains the equilibrium fields  $\mathbf{B}$  and  $\mathbf{V}_0$  as parameters:

$$(L^A)_{rs}^{ab} = (L^A)_{rs}^{ab}[\mathbf{B}, \mathbf{V}_0, \{\hat{\phi}\}]. \quad (\text{C2})$$

In the same way as in Ref.8, we can show that the quasilinear anomalous transport coefficients satisfy the following Onsager symmetry:

$$\begin{aligned} T_a(L^A)_{mn}^{ab}[\mathbf{B}, \mathbf{V}_0, \{\hat{\phi}(t)\}] &= T_b(L^A)_{nm}^{ba}[-\mathbf{B}, -\mathbf{V}_0, \{\hat{\phi}(-t)\}] & (m, n = 1, 2) \\ T_a(L^A)_{MN}^{ab}[\mathbf{B}, \mathbf{V}_0, \{\hat{\phi}(t)\}] &= T_b(L^A)_{NM}^{ba}[-\mathbf{B}, -\mathbf{V}_0, \{\hat{\phi}(-t)\}] & (M, N = V, T) \\ T_a(L^A)_{mM}^{ab}[\mathbf{B}, \mathbf{V}_0, \{\hat{\phi}(t)\}] &= -T_b(L^A)_{Mm}^{ba}[-\mathbf{B}, -\mathbf{V}_0, \{\hat{\phi}(-t)\}] & (m = 1, 2; M = V, T) \end{aligned} \quad (\text{C3})$$

where  $\hat{\phi}(-t)$  represents the fluctuation spectra obtained by the time reversal of the original spectra  $\hat{\phi}(t)$ .

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## FIGURES

FIG. 1. The normalized linear growth rates  $\text{Im}(\hat{\omega}) = \text{Im}(\omega)/\omega_{*e}$  from Eq.(62) as function of the flow shear parameter  $(L_N/L_E)(V_0/c_s)$  for  $K = 5, 8, 11$  with no magnetic shear  $L_N/L_s = 0$ . Here  $G = 0.1$ ,  $k_\theta \rho_s = 0.1$ , and  $\alpha = 0$ . The increase of  $K$  broadens the width of the stability window but heightens the strength of the flow shear required for the stabilization.

FIG. 2. The real part (a) and the imaginary part (b) of the normalized eigenfrequency for  $K = 8$  are given as functions of the flow shear parameter  $(L_N/L_E)(V_0/c_s)$ . The cases with different values of magnetic shear  $L_N/L_s = 0, 0.01, 0.05, 0.1$  are shown. The values of  $G$ ,  $k_\theta \rho_s$ , and  $\alpha$  are the same as in Fig.1. As the flow shear increases, the real frequency has the sign of the ion diamagnetic frequency and its absolute value increases. The reduction of the growth rates by the flow shear becomes weaker for stronger magnetic shear.

FIG. 3. The linear response factor  $\text{Im}(\hat{\omega})/|\hat{\omega}|^2$  as a function of the flow shear parameter  $(L_N/L_E)(V_0/c_s)$ . The same parameters as in Fig.2 are used.

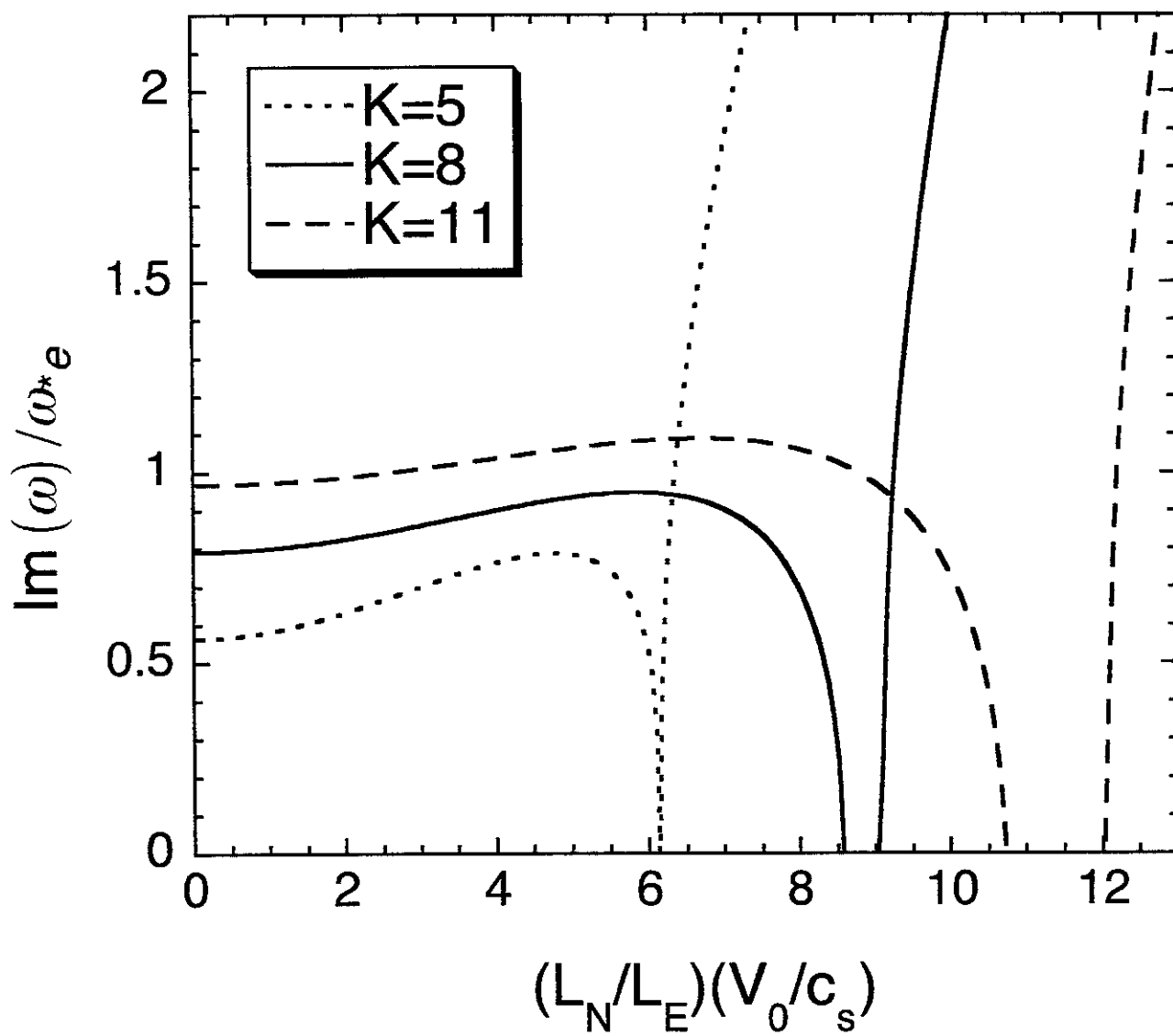


FIG.1

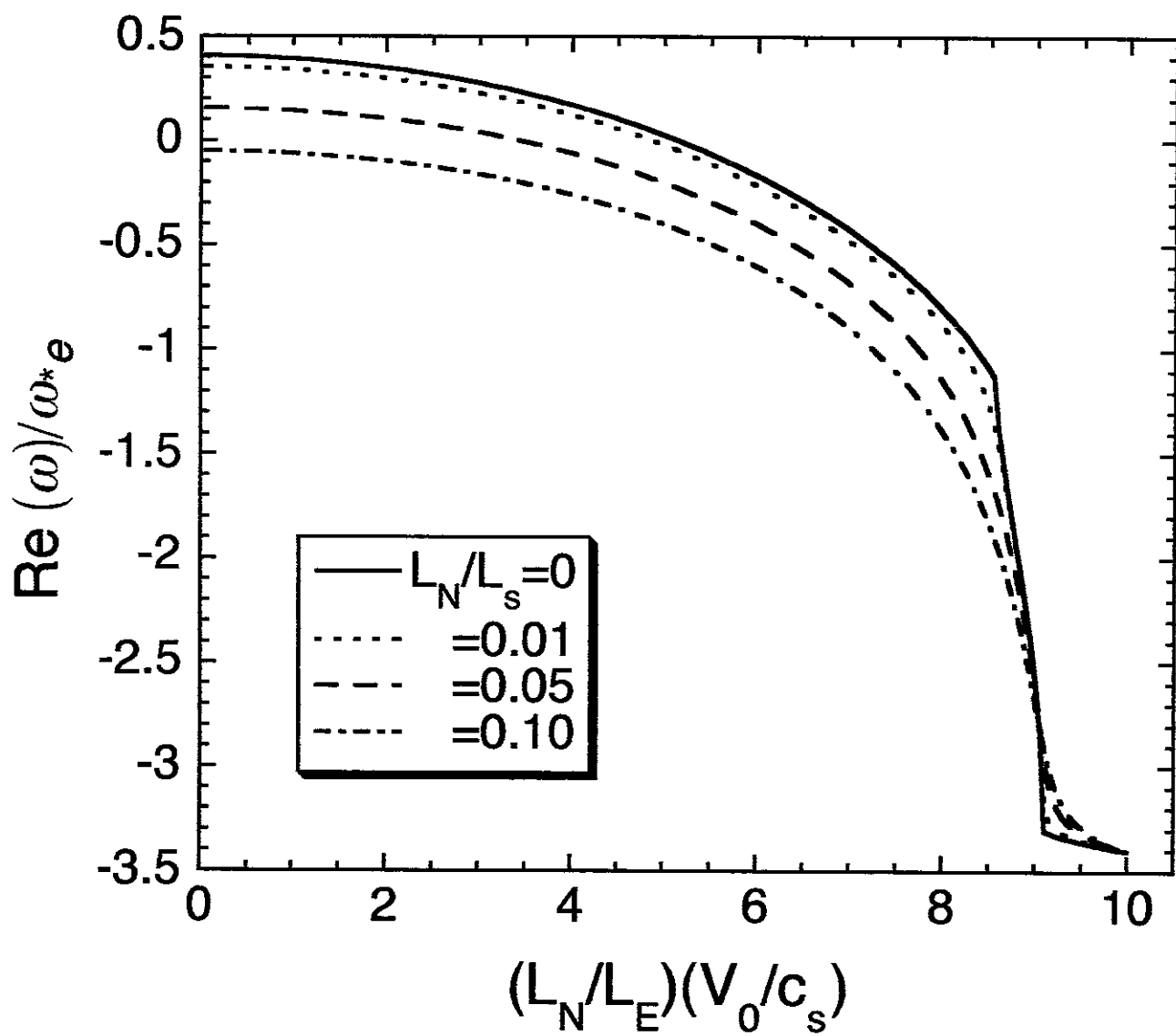


FIG.2(a)

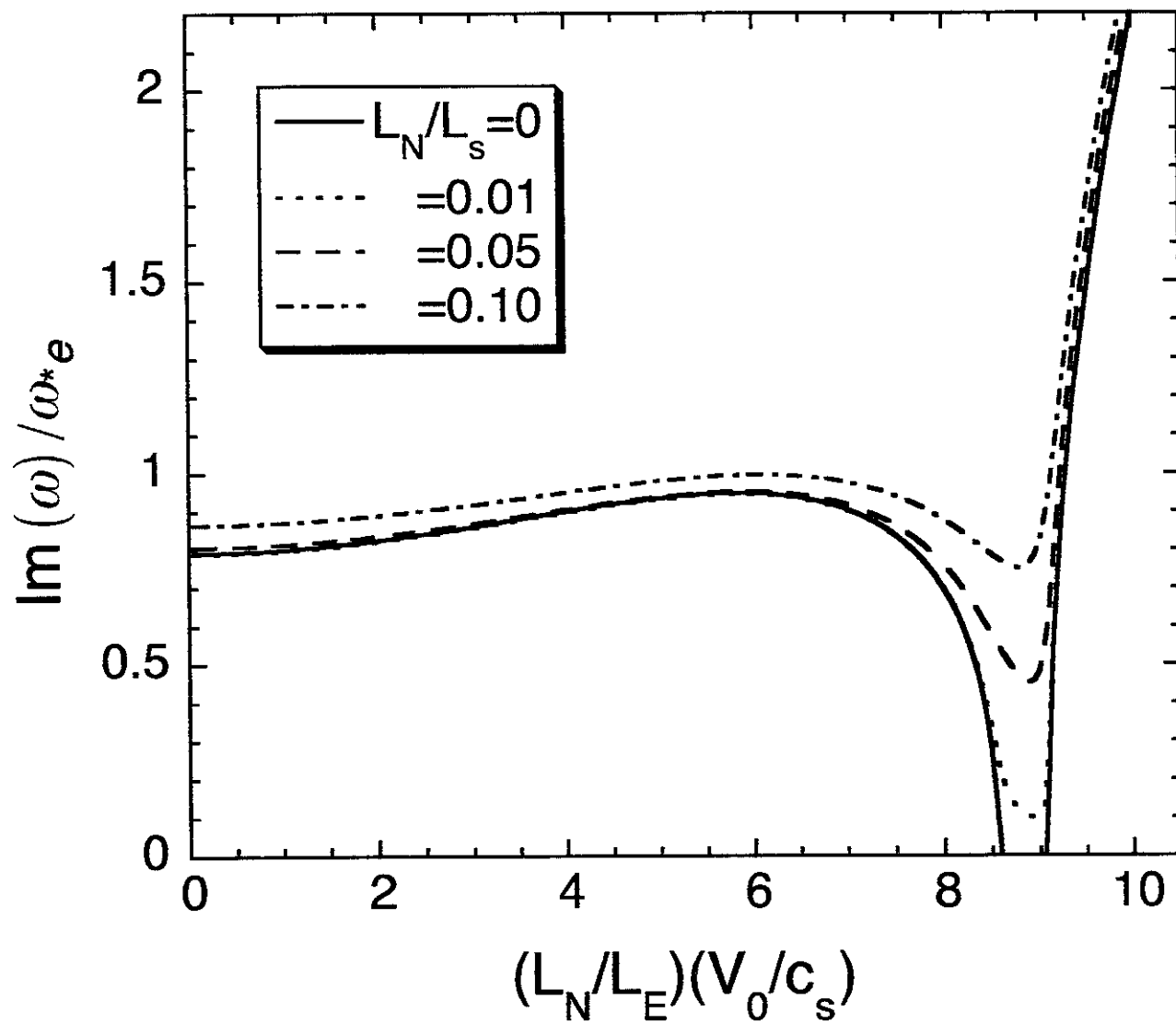


FIG.2(b)

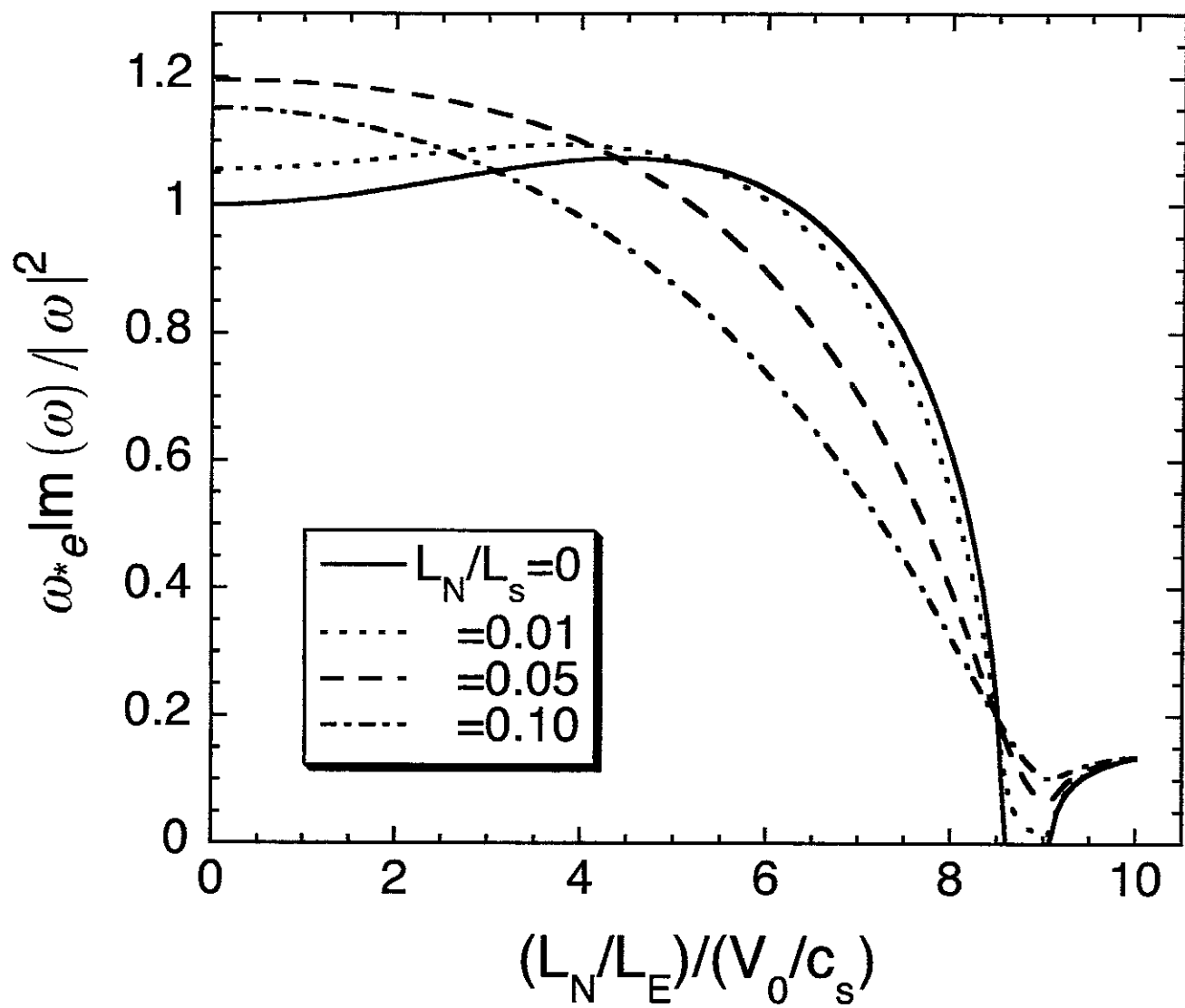


FIG.3

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