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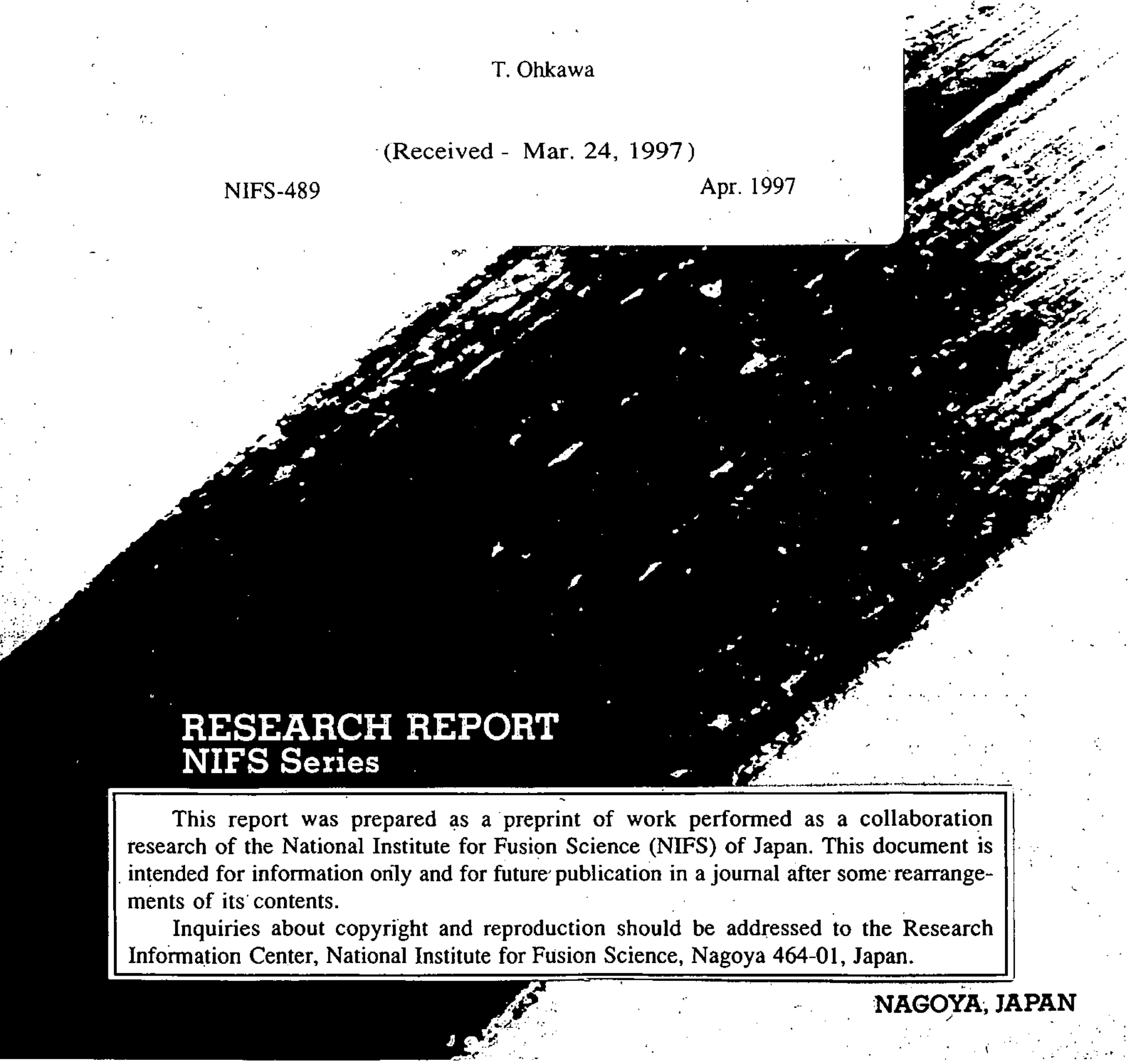

**RESEARCH REPORT**
**NIFS Series**

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# Pfirsch - Schlüter Diffusion with Anisotropic and Nonuniform Superthermal Ion Pressure

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## Abstract

Pfirsch-Schlüter diffusion rate is calculated for plasma with superthermal ions. When the pressure of the superthermal ions is not uniform on a flux surface, the diffusion rate is altered. Higher pressure at the outboard side enhances the rate while higher pressure at the inboard side reduces the rate. Pfirsch-Schlüter current and diffusion change direction if the fractional increase of the pressure at the inboard side exceeds twice the inverse aspect ratio.

Keywords: Pfirsch-Schlüter diffusion, superthermal ions, poloidal asymmetry, reduced transport

One of the manifestations of the toroidal effects is the Pfirsch-Schlüter diffusion [1]. Because of the radial dependence of the toroidal magnetic field and also the geometry, the diamagnetic current cannot be divergence free by itself. The current parallel to the magnetic field is required for the divergence to vanish. If there is a finite resistivity, parallel current produces poloidal electric field which causes the plasma to convect and results in diffusion faster than the classical diffusion.

In calculating the diffusion rate, it was assumed that the plasma pressure is isotropic and uniform on a flux surface. When the plasma is heated by the injection of neutral beams or by r-f heating, the ion pressure distribution becomes anisotropic both in the velocity space and spatially. In the previous report [2], it was pointed out that the anisotropic ion pressure induces the differential toroidal rotation on a flux surface. This effect may affect the turbulent transport rate by phase mixing the turbulence. In this report, we calculate the quiescent diffusion rate and show that Pfirsch-Schlüter rate will be modified and even reversed.

We use Ohm's law and mhd equilibrium equation in $[\psi, \phi, \chi]$ coordinates, where $\psi$ is the flux function, $\phi$ is the toroidal angle and $\chi$ is the poloidal angle. Since the electron collision frequency is much greater than that of the ions, the electron temperature is assumed to be isotropic and uniform on a flux surface.

Ohm's law is given by

$$\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j} + \frac{1}{en}\left(-\nabla p_e + \vec{j} \times \vec{B}\right) \qquad (1)$$

The equilibrium equations are

$$\left(\vec{j} \times \vec{B}\right)_{\psi} = \nabla_{\psi} p + (p_{\parallel} - p_{\perp}) \nabla_{\psi} \ell n B \qquad (2)$$

$$\frac{\partial p_{\parallel}}{\partial \chi} - (p_{\parallel} - p_{\perp}) \frac{\partial \ell n B}{\partial \chi} = 0 \qquad (3)$$

where $p_\perp$ and $p_\parallel$ are the perpendicular and the parallel plasma pressure and B is the magnetic field strength.

The particle flux Γ across a flux surface is given by

$$\Gamma = 2\pi \oint v_\psi n R B_\chi J \, d\chi \tag{4}$$

where J is the Jacobian.

We calculate the flux from eq.(1) and eq.(2) by using

$$\begin{aligned}
&\oint E_\chi B_\chi J \, d\chi = 0 \\
&\frac{\partial}{\partial \chi}\left(\frac{j_\chi}{B_\chi}\right) = 0 \\
&\frac{\partial}{\partial \chi}\left(R B_\phi\right) = 0 \\
&\frac{\partial}{\partial \chi}\left(R E_\phi\right) = 0
\end{aligned} \tag{5}$$

Also we define the flux surface average by

$$\langle y \rangle \equiv \left(\oint y J \, d\chi\right)\left(\oint J \, d\chi\right)^{-1} \tag{6}$$

We obtain

$$\frac{j_\chi}{B_\chi} = \frac{1}{\langle B^2 \rangle}\left(\eta^{-1} R^2 E_\phi B_\phi \langle R^{-2} \rangle - R B_\phi \langle g \rangle\right) \tag{7}$$

and

$$j_\phi = R\left(g - \frac{\langle g \rangle B_\phi^2}{\langle B^2 \rangle}\right) + \frac{R^2 E_\phi B_\phi^2 \langle R^{-2} \rangle}{\langle B^2 \rangle \eta} \tag{8}$$

where

$$g \equiv \frac{\partial p_\perp}{\partial\psi} + (p_\parallel - p_\perp)\frac{\partial\, \ell nB}{\partial\psi} \tag{9}$$

Since the parallel current density $j_\parallel$ is approximately equal to $j_\phi$, the first term in eq.(8) represents Pfirsch-Schlüter current. When the modulation of the pressure in $\chi$ -direction becomes of the order of the inverse aspect ratio $\varepsilon$, Pfirsch-Schlüter current is significantly modified or even reverses the direction.

The particle flux is given by

$$\Gamma = 2\pi\oint J d\chi \left\{ \langle n\rangle RE_\phi \left( 1 - \frac{R^2 B_\phi^2 \langle R^{-2}\rangle}{\langle B^2\rangle} \right) - \eta\left( \langle nR^2 g\rangle - \frac{R^2 B_\phi^2 \langle n\rangle\langle g\rangle}{\langle B^2\rangle} \right) \right\} \tag{10}$$

By using

$$\langle B^2\rangle = R^2 B_\phi^2 \langle R^{-2}\rangle + \langle B_\chi^2\rangle \tag{11}$$

we obtain

$$\Gamma = 2\pi\oint J d\chi\, \frac{1}{\langle B^2\rangle}\left\{ \langle n\rangle RE_\phi \langle B_\chi^2\rangle - \eta R^2 B_\phi^2 \left( \langle nR^2 g\rangle\langle R^{-2}\rangle - \langle n\rangle\langle g\rangle \right) - \eta\langle nR^2 g\rangle\langle B_\chi^2\rangle \right\} \tag{12}$$

If the anisotropic and non-uniform pressure is created by superthermal ions resulting from neutral beam injection or ICRF heating, the contribution to the pressure from the superthermal ions can be significant but the contribution to the density is negligibly small. With this assumption, the density is approximately uniform on a flux surface and we obtain

$$\Gamma = 2\pi\oint J d\chi\, \frac{\langle n\rangle}{\langle B^2\rangle}\left\{ RE_\phi \langle B_\chi^2\rangle - \eta R^2 B_\phi^2 \left( \langle R^2 g\rangle\langle R^{-2}\rangle - \langle g\rangle \right) - \eta\langle R^2 g\rangle\langle B_\chi^2\rangle \right\} \tag{13}$$

The first and the last terms are small because of $B_\chi^2/B^2$ factor and can be neglected. We obtain

$$\Gamma \simeq -2\pi\oint Jd\chi\,\frac{\langle n\rangle\eta}{\langle B^2\rangle}R^2B_\phi^2\left(\langle R^2g\rangle\langle R^{-2}\rangle-\langle g\rangle\right) \tag{14}$$

We evaluate the particle flux for tokamak plasma with circular cross-section. We use

$$Jd\chi=\left(\frac{r}{B_\theta}\right)d\theta,$$

$$\frac{\partial(RB_\theta)}{\partial\theta}=0,$$

$$R=R_0(1+\varepsilon\cos\theta)$$

and

$$g=\bar{g}(r)+\tilde{g}(r)\cos\theta\ .$$

To the second order of ε, the particle flux is given by

$$\Gamma=-(2\pi)^2\frac{1}{B_\theta}\eta rR^2n\left(2\varepsilon^2\bar{g}+\varepsilon\tilde{g}\right) \tag{15}$$

For low β plasmas the second term in eq.(2), the logarithmic derivative of B, is much smaller than the first term. If the pressure distribution is given by

$$p_\perp=\bar{p}_\perp(r)+\tilde{p}_\perp(r)\cos\theta \tag{16}$$

we obtain

$$\bar{g} \simeq \frac{1}{RB_\theta}\bar{p}_\perp(r) \qquad (17a)$$

and

$$\tilde{g} \simeq \frac{1}{RB_\theta}\tilde{p}_\perp(r) \qquad (17b)$$

The flux becomes

$$\Gamma = -(2\pi)^2\frac{1}{B_\theta^2}\eta rRn\left(2\varepsilon^2\bar{p}_\perp' + \varepsilon\tilde{p}_\perp'\right) \qquad (18)$$

or by using the safety factor q,

$$\Gamma = -(2\pi)^2\frac{q^2}{B^2}\eta rRn\left(2\bar{p}_\perp' + \varepsilon^{-1}\tilde{p}_\perp'\right) \qquad (19)$$

The first term represent the familiar Pfirsch-Schlüter diffusion [1]. The second term is of the lower order in $\varepsilon$. This indicates that the pressure modulation of order $\varepsilon$ modifies the diffusion rate significantly. In particular, if $\tilde{p}$ is negative, namely the pressure is higher at the inboard side, the diffusion is reduced. Furthermore if

$$\tilde{p}_\perp < -2\,\varepsilon\bar{p}_\perp \qquad (20)$$

the particle flux is inward. As discussed before it is because the direction of Pfrisch-Schluter current reverses.

The anisotropic and non-uniform pressure can be created by neutral beam injection. However it is nontrivial to alter the injection geometry for varying the pressure distribution in practice. More straightforward way is to use ICRF with minority ion resonance. The minority ions, orbits of which intersect the resonance radius gain the perpendicular energy from r-f. If the resonance radius is at the outboard side, the accelerated minority ions will be in the trapped orbits because of the gain in the perpendicular energy. Thus the pressure will be higher at the outboard side and $\tilde{p}_\perp$ will be positive. On the other hand, if the resonance radius is at the inboard side, the minority ions in either the barely trapped or the barely untrapped orbits are accelerated most. Also the acceleration does not alter the orbits very much because the parallel velocity at the resonance radius is small to begin with. As a result, the pressure becomes higher at the inboard side and $\tilde{p}_\perp$ is negative.

The reduction of the turbulent diffusion due to the differential toroidal rotation on a flux surface described in the previous report [2] depends on the magnitude of $\tilde{p}_\perp$ not its sign. Therefore if the sign of $\tilde{p}_\perp$ is negative, both Pfirsch-Schluter diffusion and the turbulent diffusion are reduced.

By varying the toroidal magnetic field, the resonance radius can readily be changed. It would be interesting to compare the transport rates of the plasmas heated by minority ion ICRF, one with the resonance radius at the inboard side [lower toroidal field] and the other [higher toroidal field].

There are more subtle effects when $\alpha$ -particles are produced by fusion reactions. The superthermal ions have larger fusion cross-sections and more $\alpha$-particles are produced where the pressure of the superthermal ions are higher. The angular distribution of $\alpha$-particles at birth is isotropic. When extra $\alpha$ -particles are produced at the outboard side, their density will be uniform on a flux surface, because they are born at the minimum of the magnetic field. On the other hand if they are created at the inboard side, they cannot be in the deeply trapped orbits. Therefore the pressure of the extra $\alpha$ -particles will be higher at the inboard side. The pressure of the extra $\alpha$ -particles will augment the superthermal ion pressure already higher at the inboard side before the fusion reactions take place.

Pfirsch-Schluter diffusion is the collisional limit of the neoclassical diffusion. One suspects that the effects of the superthermal ions similar to the effects described here may appear in the neoclassical regime. More elaborate kinetic calculations are necessary to confirm the suspicion.

Productive discussions with Dr. K. Itoh of NIFS and Dr. H. Kimura of JAERI are appreciated.

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