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in a Dusty Plasma with Trapped Electrons**

Y.-N. Nejoh

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**RESEARCH REPORT**  
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# DYNAMICS OF THE DUST CHARGING ON ELECTROSTATIC WAVES IN A DUSTY PLASMA WITH TRAPPED ELECTRONS

**Y-N.Nejoh**

*Graduate School of Engineering and Department of Electronic Engineering,  
Hachinohe Institute of Technology, Myo-Obiraki, Hachinohe, 031, Japan*

Dynamics of the dust charging and the influence of the ion density and temperature on electrostatic nonlinear waves in a dusty plasma having trapped electrons are investigated by numerical calculation. The nonlinear structure of the dust charging is examined, and it is shown that the characteristics of the dust charge number sensitively depend on the plasma potential, Mach number, trapped electron temperature, ion density and temperature. An increase of the ion temperature decreases the dust charging rate and the propagation speed of ion waves. It turns out that a decrease of the trapped electron temperature increases the charging rate of dust grains. It is found that the existence of ion waves sensitively depends on the ion to electron density ratio. New findings of variable-charge dust grain particles, ion density and temperature in a dusty plasma with trapped electrons are predicted.

Key words: nonlinear dust charging, the existence of electrostatic nonlinear waves, trapped electrons, the effect of the ion to electron density ratio,

## I. INTRODUCTION

The increase of recent interest in plasmas containing charged, micrometer-sized dust particles has arisen not only from the increase of observations of such plasmas in space environments such as asteroid zones, cometary tails, planetary rings and magnetospheres, and the lower ionosphere of the Earth<sup>1-4</sup>, but also from their presence in laboratory devices<sup>5-7</sup>. In reality, the dust grains have variable-charge and mass due to fragmentation and coalescence. However, in studying collective effects involving charged dust grains in dusty plasmas one generally assumes that the dust particles behave like point charges. Multi-fluid descriptions for collective modes in micro-particle plasmas were presented by several authors. For low frequency nonlinear wave modes, the dust grains can be described as negative ions with large mass and large charge. Ion- and dust-acoustic wave modes in dusty plasmas have been treated by several authors<sup>8-13</sup>. We have suggested that high-speed streaming particles excite various kind of nonlinear waves in space<sup>14-17</sup>. Dust grain particles are charged due to the local electron and ion currents, and its charge varies as a result of the change of the parameters such as the potential, densities, *etc.* Therefore, since the dust charge variation affects the characteristics of the collective motion of the plasma, the effect of variable-charge dust grain particles is of crucial importance in understanding dusty plasma waves. However, not many theoretical works on the effect of variable-charge dust grain particles have been done in dusty plasmas. In particular, the effects of the ion temperature and trapped electrons have not been investigated in dusty plasmas.

The motivation of this articles is as follows. If streaming particles inject in plasmas, we often find that they evolve towards a coherent trapped particle state. This has been confirmed in experiments<sup>18</sup>. The onset of an electron trapping is also seen in the formation of double layers<sup>19</sup> and computer simulation<sup>20</sup>. When the amplitude of nonlinear waves becomes large, some electrons, in general, would be trapped in the electrostatic potential trough of the wave and be carried along with the wave. Electron trapping is essentially a nonlinear phenomenon. There is no doubt that an electron trapping exists in nonlinear wave phenomena. Thus, the inclusion of the effect of trapped

electrons is indispensable to considering nonlinear waves. However, theoretical investigations in relation to the trapped electron effect have not been considered in dusty plasmas. Hence, an aim of this article is to show this effect in order to apply this theory to more extensive nonlinear wave phenomena.

In this paper, we focus our attention on electrostatic ion waves in an unmagnetized dusty plasma having trapped electrons and positive ions with finite temperature. It is therefore instructive to examine the effects of the dust charging, ion temperature and trapped electrons in dusty plasmas, which, as pointed out earlier, are observed as components of broad regions of space plasmas, from the lower ionosphere of the Earth to asteroid zones, cometary tails, *etc.* Our plasma model consists of non-Boltzmann distributed electrons with a constant temperature, positive ions with finite temperature, and the negatively charged dust fluid obeying the nonlinear continuity and momentum equations. We derive a nonlinear equation for variable-charge dust particles and the Sagdeev potential of electrostatic nonlinear waves. We show the dependence of the dust grain charging on the electrostatic potential, ion temperature, trapped electrons and Mach number. Our results show the existence of supersonic electrostatic waves and illustrate the dependence of the dust charge number on the parameters such as the potential, Mach number, trapped electrons, ion to electron density ratio and ion temperature.

In Sec. II, we present a new nonlinear equation for variable-charge dust particles and derive the Sagdeev potential from the basic equations. In Sec. III, we show the numerical results of the nonlinear equations obtained in the preceding section. It is shown that the charging effect of the dust grain particles drastically changes due to the several parameters. Section IV is devoted to the concluding discussion.

## II. THEORY

We consider a collisionless, unmagnetized three component plasma consisting of non-Boltzmann electrons with a constant temperature  $T_e$ , warm ions having a temperature  $T_i$  and negatively charged, heavy, dust particles, and assume that low frequency electrostatic waves propagate in this system. The number density of the electron fluid is assumed to

be the trapped electron distribution<sup>21</sup>,

$$n_e = n_0 K \left[ \exp\left(\frac{e\phi}{T_e}\right) \operatorname{erfc}\left(\sqrt{\frac{e\phi}{T_e}}\right) + \sqrt{\frac{T_t}{T_e}} \exp\left(\frac{e\phi}{T_t}\right) \operatorname{erf}\left(\sqrt{\frac{e\phi}{T_t}}\right) \right], \quad (1)$$

where  $n_e$ ,  $n_0$ ,  $e$  and  $\phi$  are the electron density, background electron density, the magnitude of electron charge and the electrostatic potential. Here,  $K=1$  in the isothermal process,  $T_e$  and  $T_t$  are the free and trapped electron temperature. It is significant to consider the dependence of the trapped electron density  $n_e$  on the electrostatic potential  $\phi$  and free to trapped electron temperature ratio  $T_e/T_t (= \beta)$  in the sense that electrostatic waves closely connect with the trapped electron density. The electron density (1) includes the two states of the electron distribution, namely, (i) in the limit of  $\beta \rightarrow 1$  ( $T_e=T_t$ ),  $n_e$  approaches the Boltzmann distribution, (ii)  $\beta=0$  means the state for flat-topped distribution. A figure showing  $n_e$  vs  $\phi$  for various values of  $\beta$  is shown in an Appendix.

The continuity equation and the equation of motion for ions are described by,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \quad (2.a)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{T_i}{m_i n_i} \frac{\partial n_i}{\partial x} + \frac{e}{m_i} \frac{\partial \phi}{\partial x} = 0, \quad (2.b)$$

where  $n_i$ ,  $v_i$ ,  $m_i$  and  $T_i$  denote the ion density, ion velocity, ion mass and ion temperature, respectively. Here, we express Eq. (2b) by the isothermal equation of state.

For one dimensional low frequency acoustic motions, we have the following two equations for the cold dust particles,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \quad (3.a)$$

$$\left( \frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x} \right) v_d - \frac{Q_d}{m_d} \frac{\partial \phi}{\partial x} = 0, \quad (3.b)$$

where  $n_d$ ,  $v_d$  and  $m_d$  refer to the dust grain density, dust fluid velocity and dust grain mass, respectively. Here the dust charge variable  $Q_d = eZ_d$ , where  $Z_d$  is the charge number of dust particles measured in units of  $e$ .

The Poisson's equation is given as

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\epsilon_0} (n_e - n_i + Z_d n_d), \quad (4)$$

where  $m_d(m_i)$  denotes the dust grain (ion) mass. We assume that the phase velocity of electrostatic ion waves is low in comparison with the electron thermal velocity. Charge neutrality at equilibrium requires that  $n_{i0} = n_{e0} + n_{d0} Z_d$ , where  $n_{i0}$  ( $n_{d0}$ ) denotes the equilibrium ion (dust grain) density. In this system, the ordering,  $m_d \gg m_i \gg m_e$  holds, as is obtained in laboratory plasmas. Typical laboratory plasma frequencies are;  $10^2 \text{Hz}$ :  $10^{5-6} \text{Hz}$ :  $10^{9-10} \text{Hz}$ , and have roughly the same ordering as the mass ratios. Thus, the inclusion of the mass ratios is equal to considering the motion of dust particles.

We assume that the charging of the dust grain particles arises from plasma currents due to the electrons and the ions reaching the grain surface. In this case, the dust grain charge variable  $Q_d$  is determined by the charge current balance equation<sup>22</sup>:

$$\left( \frac{\partial}{\partial t} + v_d \frac{\partial}{\partial x} \right) Q_d = I_e + I_i, \quad (5)$$

Assuming that the streaming velocities of the electrons and ions are much smaller than their thermal velocities, we have the following expressions for the electron and ion currents for spherical grains of radius  $r$ :

$$I_e = -e\pi r^2 (8T_e / \pi m_e)^{1/2} n_e(\phi, \beta) \exp\left(\frac{e\Phi}{T_e}\right), \quad (6.a)$$

and

$$I_i = e\pi r^2 (8T_i / \pi m_i)^{1/2} n_i(\phi, \tau_i, \Psi) \left(1 - \frac{e\Phi}{T_i}\right), \quad (6.b)$$

where  $\Phi = Q_d / r$  denotes the dust particle surface potential relative to the plasma potential  $\phi$ . If the ion streaming velocity  $v_0$  is much larger than the ion thermal velocity, the ion current is approximately expressed as  $I_i \approx e\pi r^2 v_0 n_i (1 - 2e\Phi / m_i v_0^2)$ . At equilibrium, equating  $I_e + I_i$  to zero we obtain the floating potential  $\Phi_0$  and the equilibrium dust charge  $Q_0 = C\Phi_0$ , where  $C$  denotes the dust grain capacitance.

We normalize all the physical quantities as follows. The densities are normalized by the background electron density  $n_0$ . The space coordinate  $x$ , time  $t$ , velocities and electrostatic potential  $\phi$  are normalized by the electron Debye length  $\lambda_d = (\epsilon_0 T_e / n_0 e^2)^{1/2}$ , the inverse ion plasma period  $\omega_i^{-1} = (\epsilon_0 m_i / n_0 e^2)^{1/2}$ , the ion sound velocity  $C_s = (T_e / m_i)^{1/2}$ , and  $T_e / e$ , respectively, where  $m_i$ ,  $\epsilon_0$  and  $e$  are the ion mass, the permittivity of vacuum and the magnitude of electron charge, respectively.

In order to solve Eqs.(1)-(5), we introduce the variable  $\xi = x - Mt$ , which is the moving frame with the velocity  $M$ . We reduce a set of basic equations (1)-(4) to

$$n_e = \exp(\phi) \operatorname{erfc}(\sqrt{\phi}) + \frac{1}{\sqrt{\beta}} \exp(\beta\phi) \operatorname{erf}(\sqrt{\beta\phi}), \quad (7)$$

$$-M \frac{\partial n_i}{\partial \xi} + \frac{\partial}{\partial \xi} (n_i v_i) = 0, \quad (8.a)$$

$$-M \frac{\partial v_i}{\partial \xi} + v_i \frac{\partial v_i}{\partial \xi} + \frac{\tau_i}{n_i} \frac{\partial n_i}{\partial \xi} + \frac{\partial \phi}{\partial \xi} = 0, \quad (8.b)$$

$$-M \frac{\partial n_d}{\partial \xi} + \frac{\partial}{\partial \xi} (n_d v_d) = 0, \quad (9.a)$$

$$-M \frac{\partial v_d}{\partial \xi} + v_d \frac{\partial v_d}{\partial \xi} - \frac{Z_d}{\mu_d} \frac{\partial \phi}{\partial \xi} = 0, \quad (9.b)$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_e - n_i + Z_d n_d. \quad (10)$$

Here,  $\tau_i = T_i/T_e$  and  $\mu_d = m_d/m_i$ .

Integrating (8.a) and (8.b) and using the boundary conditions,  $\phi \rightarrow 0$ ,  $n_d \rightarrow (\delta - 1)/Z_d$ ,  $n_i \rightarrow \delta$ ,  $v_i \rightarrow 0$ ,  $v_d \rightarrow 0$ , at  $\xi \rightarrow \infty$ , we obtain

$$n_i = \frac{\delta}{\sqrt{1 - \frac{2\phi}{M^2 - \tau_i}}}, \quad (11)$$

after the brief calculation, where  $\delta = n_{i0}/n_0$ . Integration of (9.a) and (9.b) gives rise to

$$n_d = \frac{\delta - 1}{Z_d} \frac{1}{\sqrt{1 + \frac{Z_d}{\mu_d} \frac{2\phi}{M^2}}}. \quad (12)$$

At equilibrium, Eq.(5) reduces to

$$-n_e(\phi, \beta) \exp(\Psi) + \sqrt{\frac{\tau_i}{\mu_i}} n_i(\phi, \tau_i) \left(1 - \frac{\Psi}{\tau_i}\right) = 0. \quad (13)$$

where  $\mu_i = m_i/m_e$ ,  $\Psi = e\Phi/T_e$ , and the electron (ion) current  $I_e$  ( $I_i$ ) is normalized by  $e\pi r^2 (8T_e/\pi m_e)^{1/2}$ . Equation (13) includes strongly nonlinear terms. In order to solve (13) exactly, we derive a following nonlinear equation for variable-charge of dust particles as



$$\left\{ \exp(\phi) \operatorname{erfc}(\sqrt{\phi}) + \frac{1}{\sqrt{\beta}} \exp(\beta\phi) \operatorname{erf}(\sqrt{\beta\phi}) \right\} \exp(\Psi) \\ = \frac{\delta(\tau_i - \Psi)}{\sqrt{\mu_i \tau_i \left(1 - \frac{2\phi}{M^2 - \tau_i}\right)}} \quad (14)$$

We regard (14) as a new equation for the dust particle surface potential in this system. Then, we can obtain the solution of (14) by numerical calculation in the next section.

Integration of the Poisson's equation gives *the Energy Law*,

$$\frac{1}{2} \left( \frac{\partial \phi}{\partial \xi} \right)^2 + V(\phi) = 0 \quad , \quad (15)$$

with

$$V(\phi) = 1 - \exp(\phi) \operatorname{erfc} \sqrt{\phi} - 2 \left(1 - \frac{1}{\beta}\right) \sqrt{\frac{\phi}{\pi}} - \frac{1}{\beta^{3/2}} \exp(\beta\phi) \operatorname{erf} \sqrt{\beta\phi} \\ + \delta(M^2 - \tau_i) \left(1 - \sqrt{1 - \frac{2\phi}{M^2 - \tau_i}}\right) + (\delta - 1) \frac{\mu_d M^2}{\alpha \Psi} \left\{1 - \sqrt{1 + \frac{\alpha \Psi}{\mu_d} \frac{2\phi}{M^2}}\right\} \quad (16)$$

where  $\alpha = rT_e/e^2$ .

The oscillatory solution of the nonlinear electrostatic waves exists when the following conditions are satisfied:

(i) The pseudopotential  $V(\phi)$  has the maximum value if  $d^2V(\phi)/d\phi^2 < 0$  at  $\phi = 0$ . It

should be noted that  $V(\phi)$  is real, when  $0 < \phi < (M^2 - \tau_i)/2$ . The region for existence

of  $\phi$  is characterized by these conditions. We understand that this region depend on the Mach number and the ion to electron temperature ratio.

(ii) Nonlinear waves exist only when  $V(\phi_M) \geq 0$ , where the maximum potential  $\phi_M$  is determined by  $\phi_M = (M^2 - \tau_i)/2$ . This implies that the inequality

$$1 - \exp\left(\frac{M^2 - \tau_i}{2}\right) \operatorname{erfc}\sqrt{\frac{M^2 - \tau_i}{2}} - 2\left(1 - \frac{1}{\beta}\right)\sqrt{\frac{M^2 - \tau_i}{2\pi}} - \frac{1}{\beta^{3/2}} \exp\left\{\frac{\beta(M^2 - \tau_i)}{2}\right\} \operatorname{erf}\sqrt{\frac{\beta(M^2 - \tau_i)}{2}} + \delta(M^2 - \tau_i) + (\delta - 1)\frac{\mu_d M^2}{\alpha\Psi} \left(1 - \sqrt{1 + \frac{\alpha\Psi(M^2 - \tau_i)}{\mu_d M^2}}\right) \geq 0 \quad (17)$$

holds.

We show the maximum Mach number as a function of  $\delta$  in Fig.1a, in the case of  $\beta=0.25$  (mark *a* on the curve) and  $\beta=1.4$  (mark *b* on the curve) for  $\tau_i=0.5$ , where  $r=10^{-7}$ ,  $Z_d=5 \times 10^4$  and  $\mu_d=10^{12}$ . For example a dust grain of radius  $1 \mu\text{m}$  and mass density  $2000\text{kg/m}^3$  has a mass  $\sim 5 \times 10^{-15}\text{kg}$  so that  $\mu_d \sim 10^{12}$ . Figure 1b illustrates the dependence of the maximum Mach number on the free to trapped electron temperature ratio for  $\tau_i=5 \times 10^3$  (mark *a*) and  $0.5$  (mark *b*), where  $r=10^{-7}$ ,  $\delta=50$ ,  $\mu_d=10^{12}$  and  $Z_d=10^5$ . The maximum Mach number and, correspondingly, the maximum amplitude of electrostatic ion waves significantly depends on the parameters  $\beta$ ,  $\delta$  and  $\tau_i$ . It turns out that only the supersonic electrostatic waves can propagate in this system.

### III. DYNAMICS OF THE CHARGE VARIATION OF DUST GRAINS

We examine the numerical analysis of the nonlinear equations obtained in the preceding section. For illustration purposes, we consider a dusty plasma in which most

of the background electrons are collected by negatively-charged dust grains. Such a situation, for example, is common to the F-ring of Saturn and the environment of Jupiter<sup>23</sup>. Thus, without loss of generality, we calculate the dust charging effect concerning electrostatic nonlinear ion waves, by numerical calculation. In the following discussion, we assume that  $r=10^{-7}$ ,  $T_e=1\text{eV}$ ,  $\mu_i=1836$  and  $\mu_d=10^{12}$ .

First, we study the dependence of the dust charging effect on the electrostatic potential. If  $\beta > 0$ , we show a  $\Psi - \phi$  plane in Fig. 2a, in the case of  $M=1.4$  (solid lines) and 1.2 (dashed lines), where  $\beta = 0.5$  (mark *a*) and 1.0 (mark *b*). We note that, since an equation  $Z_d = \alpha |\Psi|$  holds, the dust charge number is easily obtained by multiplying  $\Psi$  by  $6.25 \times 10^{11}$ . As an example, we find that the dust charge number  $Z_d$  lies in  $\sim 10^4 - 10^9$ , in the case of Fig.2a. The dust charge number gradually increases and subsequently decreases as the electrostatic potential increases. We understand that the charge number increases as the trapped electron temperature decreases because  $\beta=T_e/T_i$ , and increases as the Mach number increases. Figure 2b illustrates the dependence of the dust charge number on the ion to electron density ratio  $\delta$ , in the case of  $M=2.2$  (solid lines) and  $M=1.4$  (dashed lines), where  $\beta=0.5$  (mark *a*) and 1.0 (mark *b*). We find that an increase of the ion to electron density ratio decreases the charge number of dust grains, and an increase of the Mach number increases the dust charge number. Figure 3a shows the dependence of the dust charge number on the ratio of the ion to electron temperature  $\tau_i$  for  $M=1.5$  (solid lines) and  $M=1.2$  (dashed lines), where  $\beta = 0.1$  (mark *a*) and 1.0 (mark *b*). It turns out that the dust charge number increases within the range of  $\tau_i < 0.04$  for  $M=1.5$  ( $\tau_i < 0.03$  for  $M=1.2$ ) and decreases as the ion temperature increases ( $\tau_i > 0.04$  for  $M=1.5$  and  $\tau_i > 0.03$  for  $M=1.2$ ). The dust charging ratio of the higher Mach number is higher than that of the smaller one. When the ion temperature decreases, dust grain particles are highly charged. In order to show a relationship between the

normalized floating potential and the Mach number, we illustrate a  $\Psi - M$  plane in Fig.3b for the ion to electron density ratio  $\delta=100$  (solid lines) and 50 (dashed lines), and the ion to electron temperature ratio  $\tau_i=0.5$  (mark *a*) and 1.0 (mark *b*), where  $\beta=5.0$  and  $\phi=0.5$ . In this case, the floating potential does not vary as the ratio of the free to trapped electron temperature changes. It is found that the floating potential of dust grains decreases as the Mach number increases, and becomes higher for the lower ion to electron temperature ratio. The floating potential grows, in the same way, as the Mach number increases, and becomes higher for the lower contamination ratio of the ion density.

Second, we show a 3-dimensional (3-D) Sagdeev potential in Fig.4a as a typical case where  $Z_d=5 \times 10^4$ ,  $M=1.8$ ,  $\beta=5.0$  and  $\tau_i=0.01$ , and illustrate a 2-dimensional (2-D) Sagdeev potential in Fig.4b in the case where  $\delta=100$  (mark *a*) and 200 (mark *b*). We understand that the existence of nonlinear electrostatic waves drastically changes due to the ion to electron density ratio.

Thus, we understand that the dust charge number sensitively depends on the parameters such as the floating potential of dust particles, electrostatic potential of the plasma, trapped electron temperature, Mach number, ion density and temperature.

## IV. DISCUSSION

In this article, we have shown the effect of the dust charging of electrostatic waves in a dusty plasma whose constituents are trapped electrons, ions with the finite temperature and a cold dust fluid consisting of negatively-charged, micrometer-sized dust particles. Such plasmas may exist in both space environments and laboratory. We find the remarkable properties of the charging effect of dust grain particles obtained here as follows.

- (1) Supersonic ion waves can propagate in this system.      Dependence of the Mach

- number on the ion to electron density ratio drastically changes due to the trapped electron temperature and ion temperature. An increase of the Mach number increases the dust charge number.
- (2) Dependence of the charge variation of dust particles on the trapped electron temperature, ion density and temperature is found for the first time in dusty plasmas. The effect of the ion temperature decreases the dust charge number. Since the effect of the ion temperature affects the characteristics of the collective motion of the plasma, this effect is important in understanding nonlinear waves propagating in dusty plasmas.
  - (3) The dust charging effect is of crucial importance in the sense that the dust charge number drastically changes due to the parameters such as the floating potential of dust particles, electrostatic potential of the plasma, ion to electron density ratio dust to ion mass ratio, trapped electron temperature, ion temperature and Mach number. The region for existence of nonlinear waves varies due to the ion to electron density ratio and floating potential of dust grain particles.

In a recent plasma etch experiment<sup>24</sup>, it has been suggested that electrostatic waves may be responsible for the trapping of micrometer- and submicrometer-sized contamination particles within the plasma<sup>13</sup>. As is mentioned in this article, the inclusion of trapped electrons is required to consider nonlinear dust waves. If such waves are present in dusty plasmas, which are observed in planetary, cometary, other space environments and technological-aided plasmas, the investigation of their peculiar features will contribute to the future development in dusty plasmas. As a possible example, they may serve as a source for the intense jovian dust streams observed in the environment of Jupiter<sup>23</sup>. In this situation, our results are important in understanding the nonlinear charging mechanism of the streaming of dust grain particles and confirming the existence of arbitrary amplitude electrostatic waves in dusty plasmas.

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## APPENDIX: THE PROFILE OF THE TRAPPED ELECTRON DENSITY

It is significant to consider the dependence of the trapped electron density  $n_e$  on the electrostatic potential  $\phi$  and free to trapped electron temperature ratio  $\beta$  in the sense that electrostatic ion waves closely connect with the trapped electron density. In order to investigate the relationship, we show the trapped electron density  $n_e$  as a function of  $\phi$  and  $\beta$  associated with Eq.(7). We illustrate a bird's eye view in Fig.5a, and a  $n_e$ - $\phi$  plane in Fig.5b, for  $\beta=0.75$  (mark *a*), 2.0 (mark *b*) and 5.0 (mark *c*). As is seen in Figs. 5a and 5b, we find the following. In the case where  $\beta < 1$ , the trapped electron density  $n_e$  lies in the small range even if the electrostatic potential  $\phi$  grows. If  $\beta > 1$ ,  $n_e$  remains a small value for  $\phi < 1$  but grows for  $\phi > 1$ . The higher  $\beta$  becomes, the grower  $n_e$  becomes exponentially. It turns out that, when  $\beta$  becomes larger, this tendency becomes remarkable.

## References

- <sup>1</sup> C.K.Goertz, Rev. Geophys. **27**, 1764 (1986).
- <sup>2</sup> E.Grun, G.E.Morfill and D.A.Mendis, in *Planetary Rings* (Univ. Arizona Press, Tucson, 1984), p.275.
- <sup>3</sup> T.W.Hartquist, O.Havnes and G.E.Morfill, Fundam. Cosmic Phys. **15**, 107 (1992).
- <sup>4</sup> V.N.Tsyтович, and O.Havnes, Comm. Plasma Phys. Control. Fusion, **15**, 267 (1993).
- <sup>5</sup> R.Bingham, U.de Angelis, V.N.Tsyтович, and O.Havnes, Phys. Fluids B **3**, 811 (1991).
- <sup>6</sup> G.S.Selwyn, J.E.Heidenreich and K.L.Haller, Appl. Phys. Lett. **57**, 1876 (1990).
- <sup>7</sup> L.Boufendi, A.Plain, J.P.Blondeau, A.Bouchoule, C.Laure and M.Toogood Appl. Phys. Lett. **60**, 169 (1992).
- <sup>8</sup> C.K.Goertz and G.E.Morfill, Icarus **53**, 219 (1983).
- <sup>9</sup> N.N.Rao, P.K.Shukla and M.Y.Yu, Planet. Space Sci. **38**, 543 (1990).
- <sup>10</sup> R.K.Varma, P.K.Schukla and V.Krishan, Phys. Rev. E, **47**, 3612 (1993).
- <sup>11</sup> Y-N.Nejoh, IEEE Trans. Plasma Sci. **25**, 492 (1997).
- <sup>12</sup> A Barkan, and R.L.Merlino, Phys. Plasmas **2**, 3261 (1995).
- <sup>13</sup> A.Barkan, R.L.Merlino and N.D'Angelo, Phys. Plasmas **2**, 3563 (1995).
- <sup>14</sup> Y-N.Nejoh, Phys. Plasmas **2**, 4122 (1995).
- <sup>15</sup> Y-N.Nejoh, Austr. J. Phys. **49**, 956 (1996) .
- <sup>16</sup> Y-N.Nejoh, Phys. Plasmas **3**, 1447 (1996).
- <sup>17</sup> Y-N.Nejoh, J. Plasma Phys. **56**, 67 (1996).
- <sup>18</sup> J.P.Lynov, P.Michelson, H.L.Pecseli, J.Rasmussen, Phys. Lett., **80**, 23 (1980).
- <sup>19</sup> Y-N.Nejoh, Astrophys. Space Sci. **235**, 245 (1996); M.Raadu, Phys. Rep. **178**, 25 (1989).



<sup>20</sup>J.E.Borovsky and G.Joyce, J. Plasma Phys. **29**, 45 (1983).

<sup>21</sup>H.Schamel, J. Plasma Phys. **9**, 377 (1973).

<sup>22</sup>F.Melandso, F.T.Askalsen and O.Havnes, Planet. Space Sci. **41**, 312 (1993).

<sup>23</sup>E.Grun, E.H.Zook, M.Baguhl, A.Balogh, S.J.Bame, H.Fechtig, R.Forsyth, M.S.Hanner, M.Horanyi, J.Kissel, B.A.Lindblad, D.Linkert, G.Linkert, I.Mann, J.A.McDonnell, G.E.Morfil, J.L.Phillips, C.Polanskey, G.Schwehm, N.Siddique, P.Staubach, J.Svestka, A.Taylor, Nature **362**, 428 (1993).

<sup>24</sup>J.W.Coburn and H.F.Winters, J. Appl. Phys. **50**, 3189 (1979).

## FIGURE CAPTIONS

Fig.1a The maximum Mach number as a function of the ion to electron density ratio in the case of  $\beta=0.25$  (mark *a* on the curve) and  $\beta=1.4$  (mark *b* on the curve) for  $\tau_i=0.5$ , where  $Z_d=5 \times 10^4$ .

Fig.1b The dependence of the maximum Mach number on the ratio of the free to trapped electron temperature for  $\tau_i=5 \times 10^3$  (mark *a*) and 0.5 (mark *b*), where  $Z_d=10^5$  and  $\delta=50$ .

Fig.2a The dependence of the dust charging effect on the electrostatic potential. A  $\Psi - \phi$  plane, in the case of  $M=1.4$  (solid lines) and  $M=1.2$  (dashed lines), where  $\beta=0.5$  (mark *a*) and 1.0 (mark *b*).

Fig.2b The dependence of the dust charge number on the ion to electron density ratio in the case of  $M=2.2$  (solid lines) and  $M=1.4$  (dashed lines), where  $\beta=0.5$  (mark *a*) and 1.0 (mark *b*).

Fig.3a The dependence of the dust charge number on the ratio of the ion to electron temperature  $\tau_i$  for  $M=1.5$  (solid lines) and  $M=1.2$  (dashed lines), where  $\beta=0.1$  (mark *a*) and 1.0 (mark *b*).

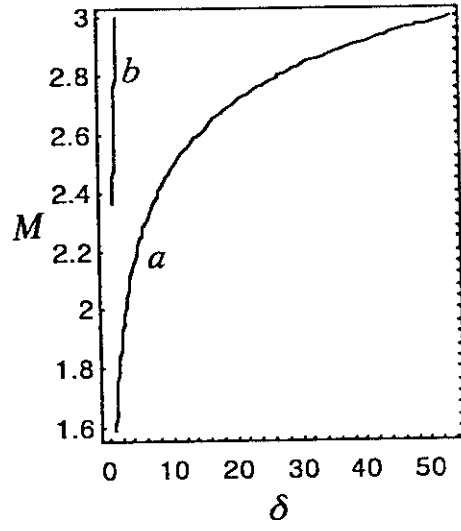
Fig.3b A  $\Psi - M$  plane for the ion to electron density ratio  $\delta=100$  (solid lines) and 50 (dashed lines), where  $\beta=5.0$  and  $\phi=0.5$ . Marks  $a$  and  $b$  on the curves denote  $\tau_i=0.5$  and 1.0, respectively.

Fig.4a A 3-D Sagdeev potential in the case where  $Z_d=5 \times 10^4$ ,  $M=1.8$ ,  $\beta=5.0$  and  $\tau_i=0.01$ .

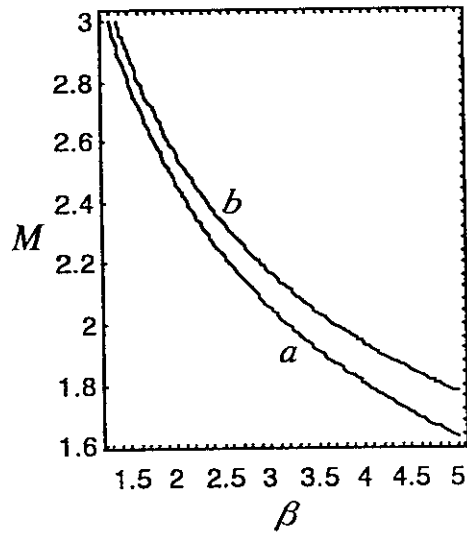
Fig.4b A 2-D Sagdeev potential as is shown with the two cases of  $\delta=100$  (mark  $a$ ) and 200 (mark  $b$ ), where  $Z_d=5 \times 10^4$ ,  $M=1.8$ ,  $\beta=5.0$  and  $\tau_i=0.01$ .

Fig.5a A bird's eye view of the trapped electron density  $n_e$  depending on the electrostatic potential  $\phi$  and free to trapped electron temperature  $\beta$ .

Fig.5b A  $n_e - \phi$  plane, for  $\beta=0.75$  (mark  $a$ ), 2.0 (mark  $b$ ) and 5.0 (mark  $c$ ).

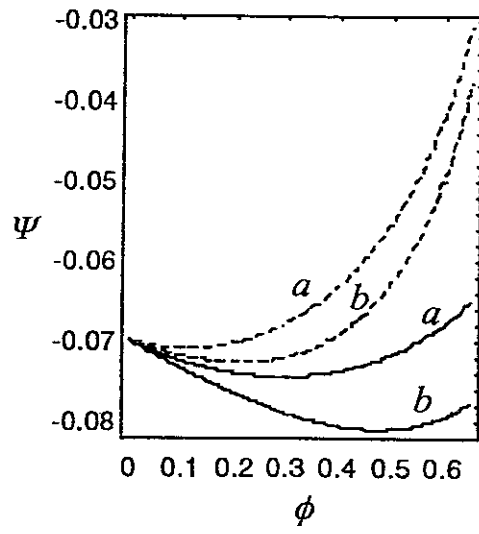


(a)

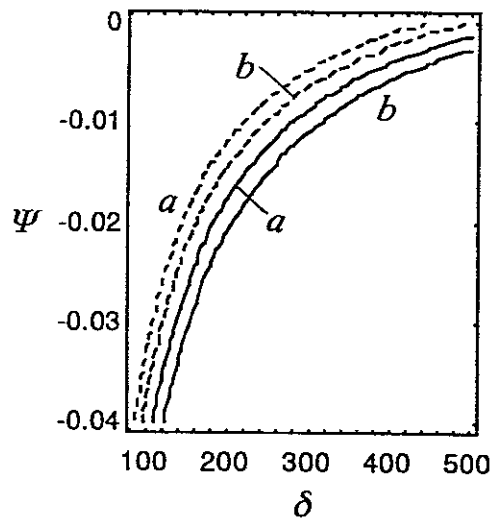


(b)

Fig.1

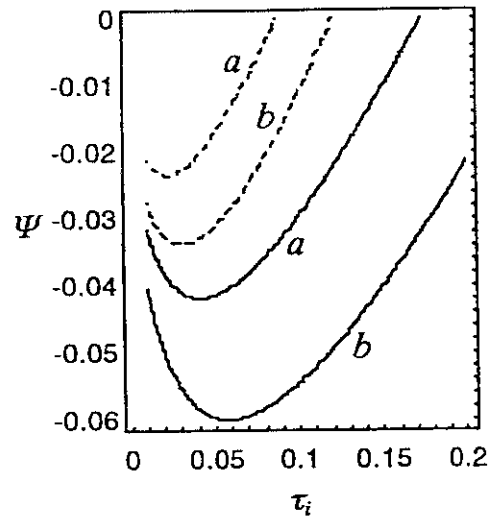


(a)

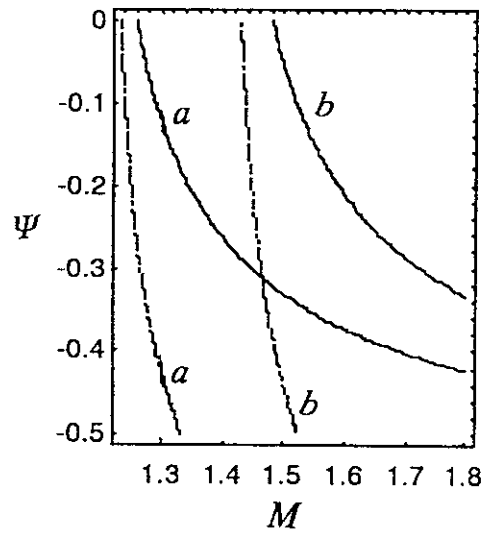


(b)

Fig.2

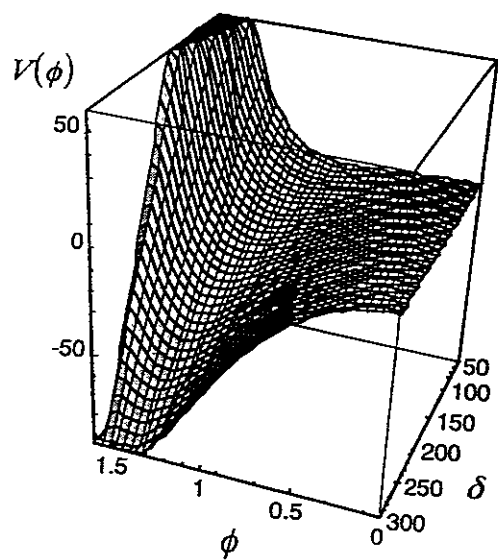


(a)

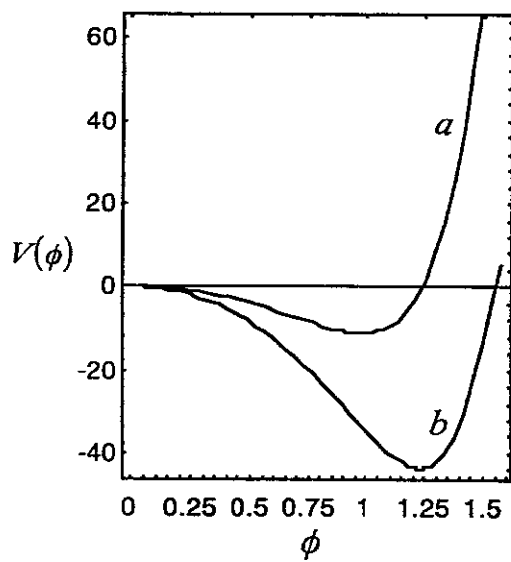


(b)

Fig.3

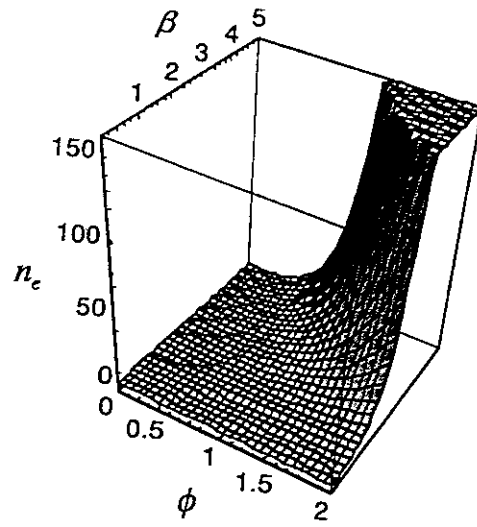


(a)

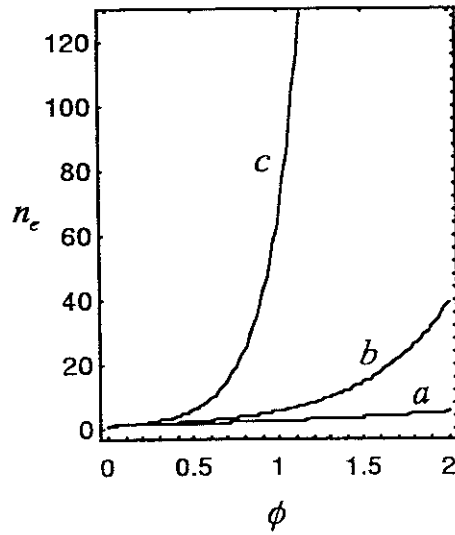


(b)

Fig.4



(a)



(b)

Fig.5



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