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Simulation of Weak and Strong Langmuir Collapse Regimes

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Abstract

In order to check the validity of the self-similar solutions and the existence of *weak* and *strong* collapse regimes, direct two dimensional simulation of the time evolution of a Langmuir soliton instability is performed. Simulation is based on the Zakharov model of strong Langmuir turbulence in a weakly magnetized plasma accounting for the full ion dynamics. For parameters considered, agreement with self-similar dynamics of the weak collapse type is found with no evidence of the strong Langmuir collapse .

Keywords: plasma turbulence, solitons , wave collapse

Introduction

Both for laboratory and space plasmas with a large energy content, it is typical that a state is reached where nonlinear wave effects compete with the dispersion [1-3] In such a physical environment spatially localized, soliton like waveforms are readily formed. These structures evolve rapidly from an arbitrary initial plasma state to determine basic features of an emerging strong plasma turbulence [2-4] In one-dimensional systems solitons are mostly stable , but in real plasmas, as a rule, solitary structures often appear to be unstable with respect to perturbations in a transverse direction [4-5]. In its nonlinear stage, this instability often leads to a soliton collapse, a unique nonlinear wave phenomenon of the formation of a singularity in a finite time. In the physical sense, wave collapse corresponds to wave breaking and particle acceleration, thus playing a role of an effective heating process in a strongly turbulent plasma [3-6]. Recent studies of the wave collapse, based on self-similar analysis, have revealed the hierarchy of collapse regimes. The basic distinction is between the *weak* collapse which formally brings the zero wave energy to the final collapse stage, and *strong* collapse where the initially trapped energy remains finite during the collapse [7]. In this paper, we present a numerical study in two spatial dimensions in order to check the existence of weak and strong Langmuir wave collapse and validity of self-similar solutions [7]. Our simulations are based on the Zakharov model of strong Langmuir turbulence for a plasma in a weak magnetic field including the full ion dynamics [3 – 7].

Nonlinear equations

The simplest example of strong plasma turbulence, thoroughly studied by a theory, simulation and experiments, is the phenomenon of strong Langmuir turbulence (SLT), where the interacting modes are of the high-frequency Langmuir and low-frequency ion sound wave. Zakharov's model of SLT in a weakly magnetized plasma is given by two time-averaged dynamical equations [3-7], which describe a nonlinear coupling between the Langmuir wave potential amplitude (φ) and the ion density variation (n) In convenient dimensionless units

$$t \to \frac{3}{2}\mu\omega_{pe}^{-1} t \quad , \qquad r \to \mu^{1/2} r_{De}\, r$$

$$\varphi \to \frac{T}{e}\mu\sqrt{12}\,\varphi, \qquad n \to \frac{4}{3}\mu\, n_0\, n, \qquad \sigma \to \frac{3}{4}\mu\left(\frac{\omega_{ce}}{\omega_{pe}}\right)^2, \qquad (1)$$

the system reads

$$\nabla^2 (i\varphi_t - \nabla^2\varphi) - \sigma\nabla_\perp^2 \varphi - \nabla(n\nabla\varphi) = 0,$$
$$n_{tt} - \nabla^2 n = \nabla^2|\nabla\varphi|^2, \qquad (2)$$

where ω_{ce} and ω_{pe} are the electron cyclotron and the electron plasma frequency, respectively, μ is the ion to electron mass ratio, r_{De} is the Debye radius and T is the electron temperature in energetic units. The system (1) is derived under an assumption that $\sigma \ll \mu$, corresponding to the physical condition of a weak magnetic field (in x-direction) $\omega_{pe} \gg \omega_{ce}$ We note that for $\sigma=0$, the system reduces to the original set of curl-free Zakharov equations [4] The vectorial form of (1) readily simplifies to a scalar model by replacing ($-\text{grad}\varphi$) by a scalar electric field E(r). In the small amplitude (static ions) limit, the set (2) further reduces to a single equation of the nonlinear Schroedinger type (NLS) However, for large Langmuir fields the inclusion of the full ion inertia is essential [3-8]

A stationary, spatially localized solution of the system (2) in a form of a "standing" planar (1D) soliton, for the external magnetic field in x-direction, is given by

$$\varphi_s = \sqrt{2}\,\arctan[\sinh(\lambda x)]\exp(i\lambda^2 t),$$
$$n_s = -|\varphi_{sx}|^2. \qquad (3)$$

A problem of the stability, nonlinear dynamics and collapse of Langmuir solitons was studied in great detail [4-7]. In a linear regime, agreement between direct simulation and eigenvalue problem results was recently obtained by some of these authors [5]. In the nonlinear regime, linearly unstable solitons exhibit the wave collapse. In its developed stage, Langmuir collapse is expected to follow the self-similar evolution [4,7-8]. We note that earlier works on the Langmuir collapse scaling, were restricted to a simpler, static limit of (2). In distinction, this study treats the full set of (1) accounting for the ion dynamics, that is important for large amplitude Langmuir solitons.

General self-similar solution for the Langmuir potential was proposed in a form [7]

$$\varphi(\vec{r},t) \rightarrow \frac{1}{(t_0 - t)^{a+ip}}\, f\left\{\frac{x}{(t_0-t)^b}, \frac{r_\perp}{(t_0-t)^c}\right\}, \qquad (4)$$

Where t_0 is the collapse time and a, b, c and p are real constants

Starting from (4), we find that the electric field components and maximum field energy density scale according to

$$E_x = \nabla_x \varphi \to \frac{1}{(t_0 - t)^b} \varphi$$
$$E_- = \nabla_- \varphi \to \frac{1}{(t_0 - t)^c} \varphi \qquad (6)$$
$$|E_{\max}|^2 \approx const. \frac{1}{(t_0 - t)^{2a-2b}} \quad .$$

The characteristic dimensions of the collapsing soliton (caviton) contract like

$$l_x(t) \to (t_0 - t)^b,$$
$$l_-(t) \to (t_0 - t)^c.$$

while the caviton plasmon number (vide infra) scales as

$$N^{cav}(t) = \int |\nabla \varphi|^2 d\vec{r} \approx \int E_x^2 d\vec{r} \to (t_0 - t)^{-2a-b-2c} \quad . \qquad (8)$$

In an isolated cavity, the plasmon number is conserved or decreases in time, so the following inequality must hold

-2a –b +2c > 0

which for the two-dimensional case becomes

–2a –b + c = β > 0 .

Generally taken, the equality sign corresponds to a *strong*, while ineqality stands for a *weak* collapse regime . For more details on the collapse hiearchy and the scaling analysis we refer to earlier papers [7,8].

Simulation results and discussions

We have performed direct numerical simulation of nonlinear equations(2)based on the spectral Fourier method with respect to two spatial dimensions with an explicit time integration scheme. Initial conditions are chosen in a form of a standing planar soliton (3), perturbed in a transverse, y- direction, as given by

$$\varphi_x(t=0) = \varphi_{sx}(t=0)\,(1 + 2\varepsilon \cos ky)\,,$$

where the initial ion density is taken to satisfy the adiabatic matching, in order to shorten plasma transients We have used periodic boundary conditions (L_x, L_y) , a numerical grid 64 x 64 points (checked upon 128 x 128) and the perturbation level ε=0.01, performing regular numerical check of conserved integrals of motion in (1) : the plasmon number (N)

$$N = \int |\nabla \varphi|^2 d\vec{r} \quad , \tag{9}$$

and the Hamiltonian (H)

$$H = \int \left(|\nabla^2 \varphi|^2 + \sigma |\nabla_z \varphi|^2 + n |\nabla \varphi|^2 + \frac{|\nabla \phi|^2}{2} + \frac{n^2}{2} \right) d\vec{r}$$

$$with, \quad n_t = \nabla^2 \phi \tag{10}$$

To study the space-time dynamics of the soliton instability we have performed runs with different values of k, λ and σ The simulations have confirmed that all linearly unstable solitons [5-6] in the nonlinear stage enter the collapse phase. We illustrate the typical space-time evolution of the soliton collapse in figure 1 Further, we show temporal evolution of the soliton amplitude in figure 2. Initial, linearly unstable phase is followed by an explosive growth; entering a self-similar stage of a Langmuir collapse.
Further in order to check if the self-similar character of the collapse is consistent with (4), we vary t_0 to find the best fit with simulation data for the maximum electric field and corresponding soliton dimensions l_x and l_y (figure 3) We measure the values of the scaling parameters a, b and c to find β We show the calculated parameters a, b, c and β , for different values of the soliton strength λ and magnetic field σ in Table 1 In all considered cases we have found good agreement with the self-similar solution (4), as indicated by the power law dependence in figure 3 In our simulations of the inertial phase of the Langmuir collapse, for various values of k, λ and σ, weak collapse (β >0) is regularly observed, with no evidence of the strong Langmuir collapse Increase of the magnetic field speeds up the amplitude growth and the transverse contraction rate; however these effects are suppressed for a larger soliton strength
Apart from contributing to a general theory of Langmuir turbulence, above results can be applied to recent studies of SLT in ionospheric, auroral and solar plasmas [9-10].

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Figure captions

Figure 1 Space-time evolution of the amplitude of the electric field (E(x,y))during the Langmuir collapse The soliton strength is $\lambda = 5$ and magnetic field $\sigma = 10$

Figure 2 The maximum of the soliton amplitude in time for different transverse perturbation wavenumbers. The soliton strength is $\lambda = 5$ and $\sigma = 3$.

Figure 3 Characteristic soliton spatial scales (l_x, l_y) and maximum energy as a function of time (t_0-t) Numerical fits (lines) of simulation data (points)

Table 1 Scaling parameters a, b, c and β calculated for different values of the soliton strength λ and magnetic field σ.

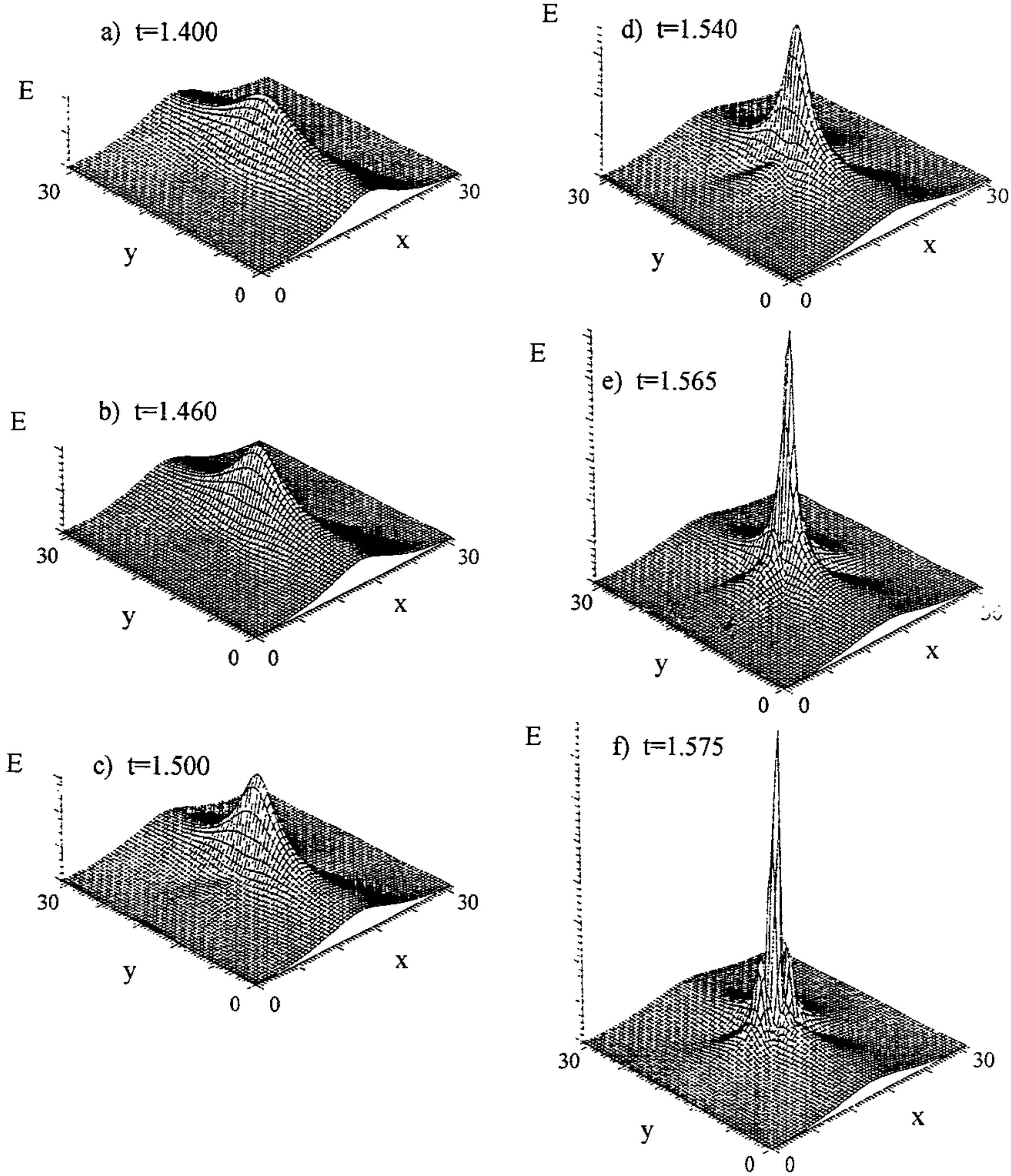

Figure 1

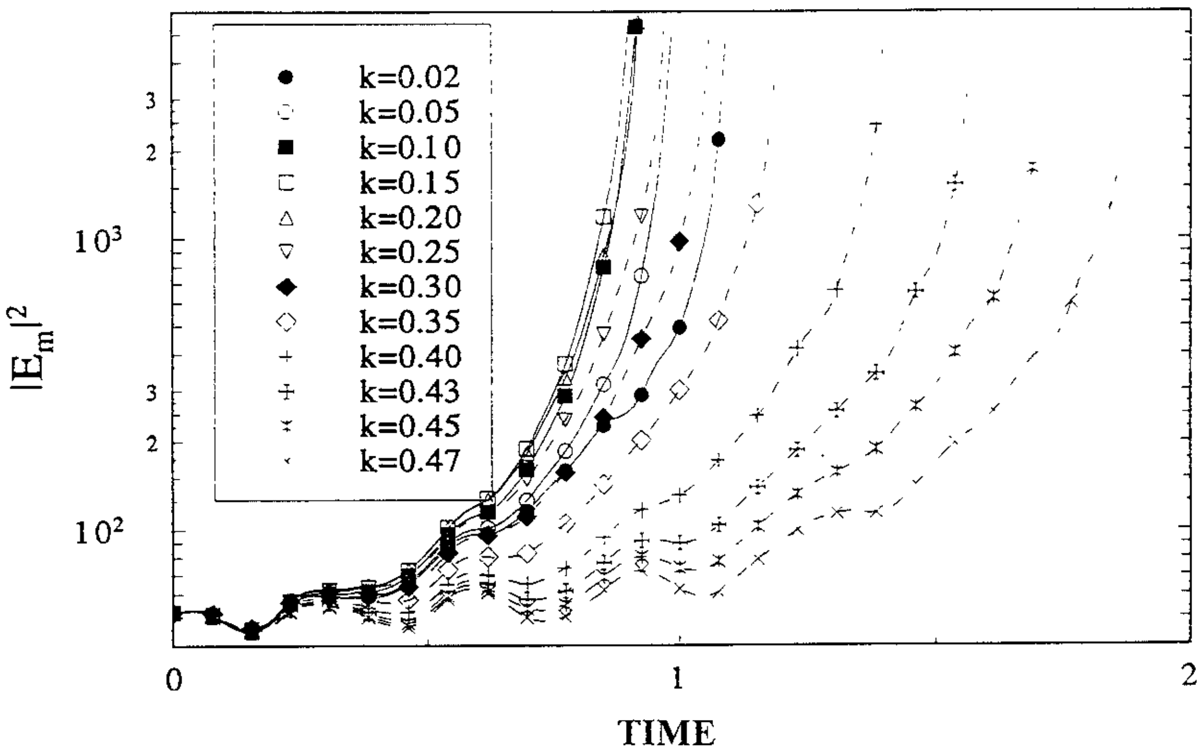

Figure 2

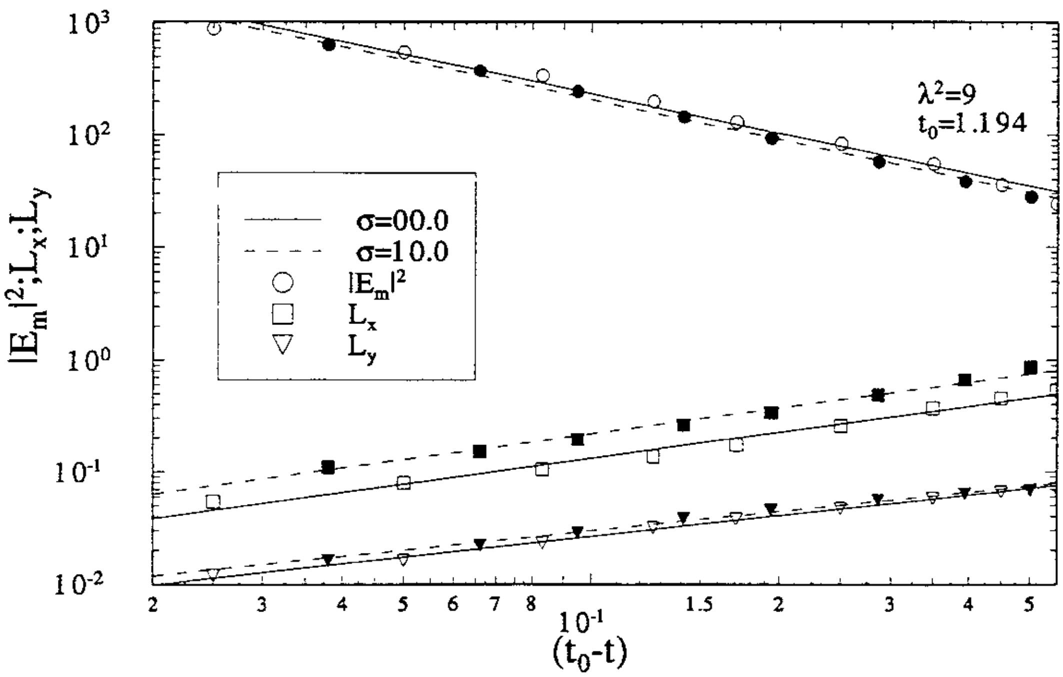

Figure 3

	$\lambda^2=1$		$\lambda^2=3$		$\lambda^2=5$		$\lambda^2=9$		$\lambda^2=25$	
	$\sigma=0$	$\sigma=10$	$\sigma=0$	$\sigma=10$	$\sigma=0$	$\sigma=10$	$\sigma=0$	$\sigma=10$	$\sigma=0$	$\sigma=10$
α	1.42	1.50	1.39	1.46	1.38	1.45	1.38	1.45	1.38	1.43
b	0 65	0 67	0.70	0.70	0.69	0.69	0.65	0.65	0.63	0.63
c	0.90	0 99	0.80	0.90	0.84	0.87	0.88	0.90	0 90	0.91
β	0.13	0.16	0.15	0.14	0.15	0.11	0.15	0.12	0.15	0.11

Table 4

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