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Summary: Theory of Magnetic Confinement

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Summary: Theory of Magnetic Confinement

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Abstract: This article summarizes the papers that are related to the theory of magnetic confinement plasmas.

Keywords: Transport and confinement, Direct simulation of turbulence, MHD models, Energetic particles, Heating and current drive, Edge modelling, Statistical approaches
1. Introduction

In the 19th IAEA Fusion Energy Conference (Lyon, October 2002), approximately 100 papers on the magnetic confinement theory are reported. They constitute a substantial part of the overall contributions in the conference (about 380 presentations). This conference covers majority of progress that has been made in the area of fusion theory after the previous IAEA Conference at Sorrento in year 2000.

The magnetic confinement plasmas have shown complex structures and dynamics as is illustrated in Fig. 1. Plasma parameters vary from center to surface. In addition to this gradual change of parameters, there are distinct steep variations in several circumstances, e.g., the edge transport barrier and internal transport barrier. The topology of magnetic surfaces changes at the plasma surface. It is also modified by the spontaneous appearance of magnetic islands in the core. The temporal evolution includes the slow change as well as abrupt transitions. Papers that are related to the magnetic confinement theory try to clarify the mechanisms that govern the structure and dynamics of plasmas. At the same time, these papers intend to provide methods for control of and for improvement of confined plasmas for realizing the ignited plasmas. Issues which are covered by the theory-related papers might be arranged as follows:

1. Transport and confinement
2. Direct simulation of turbulence
3. MHD models: Understanding and control
4. Energetic particles
5. Heating and current drive
6. Edge modelling
7. Statistical approaches.

This summary is prepared to highlight the new progresses that have impacts on future research. The viewpoint of this summary is chosen, putting an emphasis upon interactions between dynamics with different scale lengths and/or time scales. Such a direction of research is essential for comprehensive understanding, and drives evolution of future research. Summary and prospects are discussed in the last section.

![Fig. 1. Schematic illustration of the structures in toroidal plasmas. Poloidal cross-section is shown. Plasma parameters gradually change from center to surface, and there are distinct steep variation at the edge transport barrier (ETB) and internal transport barrier (ITB). The topology of the magnetic surfaces is also modified by the magnetic islands.](image-url)
2. Theory of Transport Phenomena

2.1 Transport-code Analysis

The understanding of the transport phenomena is one of the central issues for realizing burning plasmas. Based on the recent progress of the theory of anomalous transport in tokamaks [1-3], transport-code analyses have been extended world wide. Progress of integrated transport modelling is reported. Hoang [4] and Parail [5] discuss the property of the transport modelling by comparing the results with experimental observations. Kinsey [6] reports the test of the model with database and discusses the prediction of the performance of ITER plasmas. Transport codes have shown maturity so that they would give dependable prediction assuming that we have accurate knowledge of the transport barrier formation, of the appropriate description of the plasma boundary, and of the model for plasma transport.

2.2 Transport-barrier Mechanism

Theory of the transport barrier formation has attracted attentions. In the core, the electron barriers and ion barriers are known to behave in a different way under some circumstances. This implies the presence of at least two dynamical mechanisms that control the ITB formation. Newman [7] discusses the electron and ion barriers considering two kinds of fluctuations. Yagi [8] studied the direct nonlinear interactions between different kinds of fluctuations and reported a new turbulence bifurcation. Mutual interactions between the multiple kinds of fluctuations are the key for understanding the ITB formation.

The edge transport barrier, being associated with the H-mode, has been studied about two decades. Radial electric field bifurcation and suppression of turbulence are the central thread of theories for the transport barriers. The forefront of the ETB theories is the establishment of the model that can quantitatively explain experimental observations. The varieties of dynamical processes that may influence the barrier formation at edge [2] still prevent from completing the theoretical model, and key processes are investigated. Kasuya [9] and Daybelge [10] examined the nonlinearity of radial electric field. A testable result is presented [9] in relation with the bifurcation driven by electrode [11, 12], and the understanding of nonlinear response of radial electric field was progressed. It has also been well known that the neutral particles play essential role in the H-mode transition. Influence of neutral particles on plasma flow is discussed by Fülöp [13]. Plasma flow on a magnetic surface is composed of the flow along the magnetic field line and the rigid body rotation which is in proportion to the major radius. If neutral particles decelerate the flow at particular poloidal angle, the flow might become stronger at the different poloidal angle. This effect is predicted to become prominent for spherical tokamaks (STs). The gas-puffing at the inside of the tours was found effective in inducing the H-mode of ST.

The suppression of turbulence by inhomogeneous radial electric field is now widely accepted. (See surveys [2, 14, 15].) The turbulence-driven flux is influenced by both the fluctuation amplitude and the cross phase such as

\[ \langle \hat{n} \hat{E} \rangle \propto |\hat{E}|^2 \sin \alpha \]

where \( \alpha \) is the cross phase between the electric field and density. Terry [16] discusses that the suppression of cross-phase in sheared flow is important in determining the plasma transport.

2.3 Anomalous Torque, Off-diagonal Elements, etc.

An emphasis is made on the momentum transport in relation with the radial electric field structure formation. In particular, the anomalous torque attracted attentions. One issue is the zonal flow excitation. The generation mechanism is explained and experimental evidence is discussed by Diamond [17]. A global scale transport of toroidal angular momentum has been observed experimentally. Angular momentum flux, which is induced by ion temperature
gradient (ITG) mode is discussed by Coppi [18]. Pinch of momentum is also studied from the aspects of zonal flow generation. Generalization of the momentum structure formation will be given future.

Plasma diffusion in the stochastic magnetic field is still a challenging problem. Vlad [19] studies nonlinear diffusion regimes in stochastic magnetic fields, and describes the combined effects of perpendicular transport and parallel dynamics.

2.4 Finite and Long Range Transportation

2.4.1 Finite correlation length
Diffusion model does not always suffice. Experimental observations on transient response have been surveyed [20], and a limitation of the local diffusion model of transport has been discussed. This limit of local diffusion model is considered to arise from the fact that the fluctuations have finite correlation length [14]. Deviation from local diffusion model is also caused if fluctuations are excited by subcritical excitations (hysteresis).

Nonlinear instabilities in plasmas have been widely studied. The presence of nonlinear instability is one of the reasons why the simple model formula \( D = \gamma_l k_{\perp}^2 \) (\( \gamma_l \) : the linear growth rate, \( k_{\perp} \) : the perpendicular wave number) does not fit the plasma experiments. In this conference, Moestam [21] discusses the nonlinear resistive drift instability. A finite correlation length is another key. Carreras [22] discusses that the superdiffusion, in which the separation of elements \( x(t) \) has an asymptotic dependence

\[
\langle (x(t) - x(0))^2 \rangle \propto t^\alpha \quad \text{with} \quad \alpha > 1
\]

is possible in plasma turbulence. The Lyapunov exponent at finite distance \( d \), \( \lambda(d) \), becomes smaller if \( d \) becomes large. The fast decay of \( \lambda(d) \) is originated from a finite-scale correlation of fluctuations and sticking of plasma elements to it.

2.4.2 Self-organized criticality (SOC) model
The self-organized criticality (SOC) models have been applied to toroidal plasmas. SOC models have captured features of plasma transport. The hysteresis and finite correlation length of plasma turbulence are simplified in the form of SOC models. Applications of SOC models are discussed by Tangri [23], Dendy [24] and Sánchez [25]. Nondiffusive transport is modelled and the speed of radial propagation of the transport pulse is analyzed [23]. Nonlocal transport, plausible structures, and probability density function (PDF) are discussed in [24]. Application to ELMs is discussed in [25]. SOC models, if compared with experimental observations, would be a useful method to extract dynamical correlation length from experimental data.

2.4.3 Alternative modeling based on variational principle
If nonlocal nature is extended to its extreme, for instance in the case of fully-developed and large-scale turbulence, a variational principle could be constructed. The successful examples include the Taylor’s principle for RFP [26] or the minimum enstrophy principle in fluid dynamics [14]. The former minimizes the total magnetic energy \( \int |\nabla \times A|^2 \, dx \) under the constraint of the constant magnetic helicity, and the latter minimizes the total enstrophy \( \int |\nabla \times V|^2 \, dx \) under the constraint of the flow helicity. (\( A \) : vector potential, \( V \) : fluid velocity.) An extension is proposed by Mahajan [27] where the volume integral of canonical enstrophy \( \int |\nabla \times \left( V + \frac{e}{m} A \right)|^2 \, dx \) is minimized with constraints of helicities. Access to the minimum state is experimentally tested [28].
Gedankenexperiments of other variational principles are also reported [29-31].

2.5 Large-orbit Neoclassical Transport Theory

The neoclassical transport theory has been constructed upon expansion with respect to the parameter \( \rho_b^2 L^{-2} \), where \( \rho_b \) and \( L \) are the banana width and the gradient scale length, respectively. In usual circumstances, the condition \( \rho_b^2 L^{-2} \ll 1 \) holds, and formulae of neoclassical theory have been established [32]. However, the relation

\[
\rho_b^2 L^{-2} = 1
\]

is satisfied in modern toroidal plasma experiments. In tokamaks, as is shown in Fig. 1, \( \rho_b \) can be of the order of \( L \) in various cases. In this conference, efforts are reported in this direction of research, including Shiang [33], Bergmann [34], Chang [35], Nührenberg [36], Murakami [37]. These works are tabulated in Table I.

The finite-orbit-width neoclassical theory attracts attentions, because it is relevant for the transport barriers (ETB, ITB), for the regions near axis and edge, and for the magnetic island. The elaborated theory of helical plasmas is also subject to enthusiastic research. For instance, the competition between the banana width and width of magnetic island is discussed for the Bootstrap current, ion polarization current and the drag of the flow. The polarization current and Bootstrap current are crucial for the nonlinear evolution of the neoclassical tearing mode [33, 34]. The drag effect on the flow is important [33] in the generation of strongly inhomogeneous radial electric field at the magnetic island. The bifurcation of \( E_r \) at the magnetic islands has been predicted [38] for the explanation of the SNAKE structure in tokamaks [39]. The X-point transport [35] is a key as a source of electric field bifurcation at the edge [2]. Quantitative analysis will be developed in future.

<table>
<thead>
<tr>
<th>( \rho_b ) of the order of</th>
<th>Phenomena</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure gradient scale</td>
<td>Internal TB</td>
</tr>
<tr>
<td>Magnetic island width</td>
<td>NC tearing, drag, etc.</td>
</tr>
<tr>
<td>Distance from separatrix</td>
<td>Edge TB</td>
</tr>
<tr>
<td>Helical plasma</td>
<td>NC transport</td>
</tr>
</tbody>
</table>

2.6 Linear Instabilities

Linear instabilities are basics of analysis of turbulence and turbulent transport.

2.6.1 Microscopic instabilities

Microscopic instabilities, the wave lengths of which are shorter than the ion gyroradius \( \rho_i \), are studied, being motivated by the understanding of electron transport. Dong [40] discusses the integral-equation analysis of the electron temperature gradient (ETG) mode. Smolyakov [41] and Hirose [42] investigate the short-wave length ITG mode instabilities. In conventional analysis of short-wavelength modes \( \rho \kappa_L > 1 \), it is considered that the ion response is screened by the factor of \( 1/\rho \kappa_L \) and is unimportant. This is not the case, in reality, and the short wave length instability appears owing to the temperature gradients of ions and electrons.
Other effects such as that of the safety factor (Wang [43]) and that of the drift reversal (Yokoyama [44]) are discussed. Drift reversal which utilizes toroidal helical structure [44] waits further studies.

2.6.2 Ballooning modes
Re-examination of ballooning mode instability is performed to understand blobs and other phenomena at edge.

The low-toroidal-mode-number ballooning mode is studied by Hastie [45]. The influences of resistivity and of the coupling with ion sound mode are discussed. The mechanism of stabilization by inhomogeneous flow is studied in detail (Furukawa [46]).

3. Direct Simulation and Turbulent Transport

Towards combined and comprehensive analysis of turbulence and transport, intensive studies have been performed in the area of direct simulation of micro turbulence.

In toroidal plasmas, there are many dynamical activities with various scale lengths and time scales. Figure 2 summarizes the scales and trends of research. In space, there are scale lengths of global inhomogeneities (represented by the plasma radius $a$), ion gyroradius $\rho_i$, collisionless skin depth $c/\omega_p$ and electron gyroradius $\rho_e$. We call $\rho_i$, $c/\omega_p$, and $\rho_e$ microscopic lengths, and $a$ macroscopic length. In between $a$ and $\rho_i$, there appears characteristic scale length that is a hybrid mean of $a$ and $\rho_i$, e.g., $\sqrt{a\rho_i}$, which we call mesoscale. (See, e.g., [14].) In time scales, we have the transport time scale, the typical example of which is the energy confinement time $\tau_E$, the ion sound transit time $alc_e$, Alfvén transit time $a/v_A$, and electron thermal transit time $a/v_{Te}$.

New direction of the research is the analysis including different scales simultaneously. The most popular one is the investigation of ITG mode being screened by the self-generated zonal flow. (See e.g., [47-49].) The ITG mode is characterized by the ion gyroradius, and the zonal flow by the microscopic length $\rho_i$ or the mesoscale $\sqrt{a\rho_i}$. The combined analysis is further extended recently. At the forefront of the analysis, inclusion of the three scales is challenged, and the coupling with global plasma profiles is investigated.

![Fig.2. Various scale lengths and time scales in plasmas](image-url)
3.1 Progress of Codes and Benchmarking

The progress of code development is noted first. Table II summarizes the direct nonlinear simulation of turbulence and turbulent transport. More than 16 articles report the details of direct simulation of turbulent transport. (There are many other articles which treat the direct nonlinear simulation. Table II illustrates efforts for turbulence and turbulent transport.)

Analysis by the code has come to a certain established level. The analysis of ITG alone provides an over-estimated value of turbulence level, and simultaneous analysis of ITG with zonal flow is now a standard procedure of analysis. A common knowledge has been obtained by gyro-kinetic/fluid codes for the system of ITG mode and zonal flows. This issue is reported in [8, 50-59].

**TABLE II. DIRECT NONLINEAR SIMULATION AND TURBULENT TRANSPORT.**

<table>
<thead>
<tr>
<th>Author</th>
<th>Mode</th>
<th>Specific subject</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yagi</td>
<td>ITG+short scale modes</td>
<td>Three-scales, finite $\beta$</td>
<td>[8]</td>
</tr>
<tr>
<td>Jenko</td>
<td>ITG/ETG</td>
<td>Multiple-scales, finite $\beta$</td>
<td>[50]</td>
</tr>
<tr>
<td>Kishimoto</td>
<td>ETG/ITG</td>
<td>Semi-three-scales, reduced $\chi_i$</td>
<td>[51]</td>
</tr>
<tr>
<td>Lin</td>
<td>ITG/TEM</td>
<td>Bohm, g-Bohm</td>
<td>[52]</td>
</tr>
<tr>
<td>Waltz</td>
<td>ITG/TEM</td>
<td>Bohm vs g-Bohm</td>
<td>[53]</td>
</tr>
<tr>
<td>Ottaviani</td>
<td>ETG/ITG</td>
<td>finite-$\beta$, Bohm vs g-Bohm</td>
<td>[54]</td>
</tr>
<tr>
<td>Villard</td>
<td>ITG/TEM</td>
<td>Nonlocal transport, helical</td>
<td>[55]</td>
</tr>
<tr>
<td>Garbet</td>
<td>ITG/TEM</td>
<td>Reversed-q and ITB</td>
<td>[56]</td>
</tr>
<tr>
<td>Parker</td>
<td>ITG/TEM</td>
<td>finite-$\beta$, kinetic electrons</td>
<td>[57]</td>
</tr>
<tr>
<td>Mikkelsen</td>
<td>ITG/TEM</td>
<td>Comparison with experiments</td>
<td>[58]</td>
</tr>
<tr>
<td>Idomura</td>
<td>ITG</td>
<td>Maxwellian canonical variable</td>
<td>[59]</td>
</tr>
<tr>
<td>Beyer</td>
<td>Resis. ball.</td>
<td>Stochastic magnetic field,</td>
<td>[60]</td>
</tr>
<tr>
<td>Krasheninnikov</td>
<td>Dissipative</td>
<td>Blob and ejection</td>
<td>[61]</td>
</tr>
<tr>
<td>Ghendrih</td>
<td>Interchange</td>
<td>Bumps and PDF</td>
<td>[62]</td>
</tr>
<tr>
<td>Nevins</td>
<td>X-point mode</td>
<td>3D nonlocal ES</td>
<td>[63]</td>
</tr>
<tr>
<td>Pastukhov</td>
<td>Interchange</td>
<td>2D, large scale convection</td>
<td>[64]</td>
</tr>
</tbody>
</table>

3.2 Multiple Scale Turbulence

Simulations that include the dynamics of multiple scale lengths are the one of the highlights of this conference. Nonlinear interplay between different types of fluctuations is studied. Yagi [8] reports the simulation that includes the three scales: zonal flow and streamer (meso scale), ITG mode (micro mode) and short wavelength ITG mode (much finer mode). The fluctuations of the three scales are simultaneously excited in direct simulation. This allows the study of nonlinear interactions of different type of turbulent fluctuations. Analytic modelling based on the renormalization is also included in [8], and possible bifurcation is predicted. This waits confirmation by direct simulations. Three-scales dynamics are also investigated by Jenko [50]. The evolution of ETG, zonal flow and streamer is analyzed in [50], and the electron thermal conductivity $\chi_e$ is studied. If it is written as

$$\chi_e = F \bar{\rho}^2 v_{th} a,$$

the form factor $F$ is shown to have a dependence $F \propto \left( qR/L_{Te} \right)^3$ ($L_{Te}$ : electron temperature gradient) in the limit of strong temperature gradient, $L_{Te} << L_{Te,c}$ ($L_{Te,c}$ : critical gradient length). Interaction between ITG and ETG via driven-zonal flows is studied by Kishimoto [51].
The impact of ETG-driven zonal flow on the ITG mode is studied. For a given amplitude of ETG-driven zonal flow, the ITG-driven $\chi_i$ is suppressed and shows intermittent burst in time.

### 3.3 Global Simulation and Bohm Diffusion vs Gyro-Bohm Diffusion

In the other direction, interactions between different scale lengths are studied by the global simulations [52-56]. Plasma profiles together with fluctuations are solved simultaneously.

The correlation length is small but finite in comparison with the global gradient scale length. One of the issues is the examination of the scale lengths. If the correlation length is much shorter than the global scale length and if fluctuations in different magnetic surfaces are independently excited, the transport is described by the diffusion with the level of $l^2 \tau_{cor}^{-1}$ ($l$: scale length, $\tau_{cor}$: correlation time). For the case of drift wave fluctuations, e.g., ITG mode turbulence, it provides $\chi \approx \chi_{GB} \equiv \rho_i^2 c_s / a = (\rho / a) T / e B$. This means that the transport coefficient is of the order of gyro-reduced Bohm diffusion. If the scale length of correlation approaches to a substantial fraction of the global size, the transport deviates from gyro-reduced Bohm diffusion. (This has been discussed for the cases when the humps appear [65], the action at distance works [66], and the avalanche occurs [67, 68].) Enthusiastic verifications are reported. Lin [52] reported the dependence of the normalized ion thermal conductivity $\chi' / \chi_{GB}$ as a function of $\rho / a$. For the parameters of simulation, gyro-Bohm dependence is confirmed in the range $a / \rho_i > 300$, but the Bohm-like dependence is found for smaller system sizes. In other simulations (Waltz [53], Ottaviani [54], Villard [55]), gyro-Bohm dependence continues to smaller system size $a / \rho_i \sim 100$. The critical system size $(a / \rho_i)_c$, where gyro-Bohm diffusion change to Bohm-like behaviour for $a / \rho_i < (a / \rho_i)_c$, if it exists, should depends on the magnitude of inhomogeneity and models of the boundary of simulation domain. Discussion is on-going, and conclusive result from numerical simulation is left for future study.

The other key issue is the combined dynamics with structure formation in global profiles. This is the microscopic basis for transport barriers. Garbet [56] reports the transport barrier formation in the reversed shear plasmas, simulating the JET experiment. The reduction of thermal conductivity is reported to occur at the minimum of the safety factor $q$ and at the rational surfaces where $q$ takes integer numbers. In the vicinity of the integer-$q$ surface, the mode rational surface that resonates the medium mode numbers becomes less dense. An example of multiple ITBs is demonstrated.

The finite-beta effect is discussed by several authors [50, 54, 57]. A gradual change of transport coefficient is reported. Beyer [60] studies the electrostatic turbulence in the static but braided magnetic field. This is motivated by the ergodised divertor experiments. Comparison of the result of direct nonlinear simulation (ITG mode and trapped electron mode) with the experimental observation is reported by Mikkelsen [58].

It is also noticed that the advancements of numerical schemes are reported allowing much more accurate computations [55, 59]. For instance, the energy conservation is studied in detail [55]. A scheme of loading particles which are Maxwellian with respect to canonical variable [59] is shown to give accurate damping rate of zonal flows.

### 3.4 Mesoscales

Mesoscale dynamics, such as the zonal flow, streamer, blob, bump and others attract attentions. This is because they contribute to large deviation from average and yield considerable contributions. Krasheninnikov [61] and Ghendrih [62] study the blobs in the scrape-off layer (SoL) plasmas. Nevins [63] reports the simulation of turbulence at plasma boundary. Owing to the dissipation at the sheath near the divertor plate and to the steep gradient, the strong
dissipative turbulence develops the SoL plasmas. Mesoscale blobs appear as a consequence of excited instabilities. A time-averaged profile follows the exponential decay, 
\[ n \propto \exp \left( - \frac{(r-a)}{\Delta} \right) \]
, with the decay length of \( \Delta \). However, an instantaneous profile is different from the exponential form. The contribution of large scale deviation is shown to have substantial contributions to the plasma flux [61, 62]. The radial velocity of blobs is calculated, and the upper bound is given as 
\[ V_r < V_{\text{max}} = c_s \rho_s^{2/5} R^{-2/5} q^{-1/5} \]
[61].

3.5 Modelling Helical Plasmas

Turbulent fluctuations in stellarators (helical plasmas) are also studied by direct simulations. (The name “helical systems” is used in this article with the same meaning as “stellarators.”) This is because the neoclassical transport in helical systems is reduced to a moderate level by properly choosing the magnetic field configurations or by the strong radial electric field. The historical concern of the neoclassical loss of ripple-trapped particle in helical systems is resolved, and the turbulence-driven transport is the main obstacle that limits the energy confinement time in helical systems. In the present level of analysis, the three-dimensional natures of the plasma parameters are considered to be not essential. With this simplification, the direct nonlinear simulation is performed for W7-X model [50, 55].

4. MHD Models: Understanding and Control

4.1 Global Understanding Including Plasma Shapes

Progress of studies of MHD stability and dynamical evolution is noticeable. In the nonlinear MHD simulation, the real plasma geometry is kept and the rapid evolution of large scale plasma profile is clarified. Representative work, e.g., Park [69] and Hayashi [70], describes the nonlinear MHD simulations of spherical tokamak and helical plasmas. Nonlinear simulation of ST’s attracts attentions, because the very high-\( \beta \) plasmas are realized in ST’s. In addition, a relatively weak magnetic field (in comparison with conventional tokamaks) makes the ion gyroradius larger. The finite-gyroradius effect on MHD mode could be more prominent in ST’s, stimulating the careful consideration of the physics resolution of the finite gyroradius effect and other effects. Park [69] reports the nonlinear simulation studies of tokamaks and ST’s. The change of magnetic surfaces at the crash is shown, illuminating the evolution of \( m = 1 \) reconnection. The shift of the peak and flattening in the profile of toroidal velocity are shown for the first time. Hayashi [70] stimulates that the medium \( m \) (\( m = 10 \sim 15 \) ) mode activity simultaneously occurs at the onset. The simultaneous occurrence of higher-\( m \) mode is also emphasized in [69] for the case of high-\( \beta \) plasmas. Whether the higher-\( m \) modes precede the \( m = 1 \) mode or not depends on the choice of the initial condition. The evolution of the higher-\( m \) modes which is induced by the \( m = 1 \) mode has been reported [71]. The nonlinear evolution, in particular, whether the rapidly growing higher-\( m \) modes saturate at lower level or not, critically depends on the presence of the \( m = 1 \) mode activity. MHD phenomena and transport of energetic ions in spherical tori are investigated by Kolesnichenko [72]. A large scale stochastization is discussed.

4.2 Fast Events, Nonlinear Instabilities

Although the evolution of the large scale MHD perturbation after the onset of the instability has become more and more quantitative, the mechanism that leads to the onset of MHD instabilities still requires theoretical and computational investigations.

The neoclassical tearing mode has been known to be a nonlinear instability and is subject to the subcritical excitation. (Kruger [73] and Procaccia [74].) The evolution in the nonlinear growth phase is controlled by the resistance against the parallel electron motions. In the resistive
plasmas, the resistive time scale applies, but the electron inertia and other kinetic effects influence the growth. Coupling with sound wave is favourable in larger devices when the analysis is extended for the future burning plasmas [74]. Better understanding has emerged, but quantitative prediction for the growth and decay rates needs future studies.

The sawtooth crash is one of the typical examples of collapse events in toroidal plasmas. The interaction between the $m = 1$ mode and semi-micro modes has been known important in understanding the onset of sawtooth collapse (see, e.g., [71], [75]-[77]). The excitation of the Kelvin-Helmholtz (K-H) instability has been observed associated with the $m = 1$ mode [76]. Naitou [78] showed that the induced K-H instability (in the range of $m \approx 10$ and beyond) causes the saturation of the $m = 1$ mode. The disappearance of the K-H fluctuations then leads to the further growth of $m = 1$ helical perturbations.

The high beta collapse is subject to the intensive studies. Ishii [79] studied the nonlinear evolution of the $m = 3$ double tearing mode in the reversed-shear tokamaks. Due to the geometrical constraints of the pair of $m = 3$ islands, the inclination angle of magnetic surfaces at the x-point becomes larger, leading to nonlinear destabilization. The growth rate of the nonlinear instability is found to be insensitive to the resistivity.

### 4.3 Helical Systems and Others

The plasma in the large helical device (LHD) has been found more stable than predictions of linear stability theory. [80] This has stimulated the MHD stability theory in helical plasmas. (Ichiguchi [81], Nakamura [82], Strauss [83], Garcia [84].) The nonlinear evolution of the pressure-gradient driven MHD instability in helical plasmas has not yet been fully understood. In [81], it is shown that the global perturbations appear if the critical parameter for the low-$n$ instability is exceeded. ($n$ : toroidal mode number) However, the local flattening affects much the kinetic energy of perturbations in comparison with the evolution which is initiated from the prescribed smooth plasma profile. Analyses of profiles including the corrugations, flattening and flows are necessary in future.

Progress is also reported on the nonlinear MHD analysis of reversed field pinch (RFP) plasmas. (Mironov [85]) Equilibrium, stability and evolution of field reversal configuration (FRC) are discussed; detailed report of the stability analysis was given by Barnes [86]. Stability is investigated by use of the methods of particle simulation by Ohtani [87]. Plasmas are more stable against the tilting instability than the calculations. Coupled evolution of the profile and MHD mode in the vicinity of the stability boundary is also reported by use of the new set of reduced equations [64].

### 4.4 Control

Understanding of the MHD modes in toroidal plasmas has progressed and is promoting experimental progresses. For instance, the experimental improvement of plasma confinement in RFP by current profile control through pulsed poloidal current drive is tightly linked with the theoretical understanding like [85]. Most typical example is the resistive wall mode in tokamaks. For increasing the beta limit, efforts have been made. Feedback control is discussed, and efficient control method is clarified (Chu [88], Liu [89] and Mauer [90]). For example, the poloidal sensor is found much efficient than the radial sensor, and the improved sensor is found effective in sustaining the beta limit in the rage of 90% of the ideal wall limit. A new issue of control of resistive wall mode (RWM) is pointed out by Wakatani [91]. The RWM stability is strongly dependent on the plasma rotation. Wakatani shows that the perturbation-driven torque (divergence of the Reynolds-Maxwell stress) tends to decelerate the flow velocity at the rational surface. This would be an origin of the nonlinear instability.
Other example is the construction of three-dimensional magnetic surface in high beta stellarators. Hudson [92] demonstrated that by carefully choosing the shape of the coil, widths of low-$m$ magnetic islands are suppressed and nested magnetic surfaces could be realized nearly in the entire region of plasmas in the range of a few $\%$ beta. The sensitivity of the shape of the coil on the nested surfaces is found strong.

5. Energetic Particles and Alfven Eigenmodes

Dynamics of energetic particles and Alfven eigenmodes are subject to intensive studies, because they are considered to be important in ignited plasmas.

5.1 Role of Plasma Geometry

In studying the Alfven eigenmodes, the radial profile of the local Alfven frequency $\omega = k \| v_A$ is a key. This radial structure is influenced by the safety factor through $k \|$ and by the density profile through $v_A$. The plasma shape and geometry also affect the gap through the couplings of poloidal (and helical) harmonics. These dependencies on the shape and profile are now well understood, and varieties of Alfven eigenmodes are found and reported.

In tokamaks, the impacts of ITB and new Alfven eigenmodes are discussed by many authors (e.g., Fukuyama [93], Breizman [94], Zonca [95]). The new eigenmodes, called reversed-shear Alfven eigenmodes (RSAE) after [93], is found to appear in the condition of ITB at reversed magnetic shear configurations. This attracts attentions that the variation of $k \|$ and $v_A$ occurs in the case of reversed-shear ITB. The transition from TAE to RSAE's is reported.

In spherical tokamaks, Alfven eigenmodes are also the key issue (McClements [96], Gorelenkov [97], Cheng [98]). In particular, the compressional Alfven eigenmode can also be important in spherical tokamaks owing to it very high beta values [98]. Alfven eigenmodes in helical systems are also investigated by Lutsenko[99] and Strauss [83].

Based on these studies, one may ask how unstable ITER is against the Alfven eigenmodes. Comparison of stability between ITER and JET is reported by Cheng [98]. The linear growth rate normalized to the characteristic Alfven frequency is a few times larger in ITER.

5.2 Combined Analysis of Large-scale Dynamics of Waves and Particles

The Alfven eigenmodes are considered to be possibly dangerous because they could lead to a large-scale transportation of energetic particles. Figure 3 illustrates the nonlocal interactions of energetic particles and Alfven eigenmodes. One mode could be unstable in the central region, where the strong gradient of energetic ions exists due to centralized supply. Resonant fast ions move in radius, and this motion extends to a substantial fraction of radius because of the long radial correlation length of the mode. This transportation leads to the induction of steeper gradient of hot ions at distant radius, and can cause the destabilization of another Alfven eigenmodes. As a result of this successive excitation of a few Alfven eigenmodes, an avalanche of hot ions takes place.

This type of dynamical evolution is studied by many authors. In [95], it is reported that the radial location of the peak of Alfven eigenmodes moves about a quarter of plasma minor radius within 30 Alfven-transit time. With the theoretical view of the interaction of energetic ions and Alfven eigenmodes in Fig.3, comparisons with experiments are reported to be successful. They include the reversed shear plasmas [94], low aspect ratio plasma such as NSTX [97, 98], JT-60U [93, 98] and TFTR [100]. In these studies, the quasi-linear method is successfully applied. Owing to this feature, quantitative understanding is made by numerical simulations on the beam ion loss due to Alfven eigenmode bursts (e.g., Todo [100]). Stochasticization of ion orbit and asymmetry between co-injected and counter-injected particles are clarified.
5.3 Runaway Electrons and Ions

Runaway particles are also the key issue. In particular, energy distribution after the onset of disruption is a serious problem. It is pointed out by Helander [101] that electron run-away distribution is affected by the pitch-angle scattering of electrons. The pitch-angle scattering gives rise to the velocity perpendicular to the magnetic field, which is reduced by synchrotron radiation. Runaway ions are possible to occur. Variation of magnetic moments of fast ions in spherical tori is also discussed [102].

6. Heating and Current Drive

Analysis of ICRF waves in realistic and complex geometry is necessary for the quantitative understanding of the ICRF heating. Proper treatment of microscopic wave structure for accurate evaluation of absorption is discussed by Batchelor [103]. Toroidal asymmetry in helical systems is also taken into account in [103].

An improved calculation of the rf-driven and Bootstrap currents is discussed by Peysson [104]. Magnetic braiding and current injection by anomalous current diffusivity are also discussed by Cohen [105], where the braided magnetic field has a role of increasing the current diffusivity.

7. Edge Modelling

7.1 Transport Analysis with Atomic and Molecular Processes

Transport analysis, with atomic and molecular processes, of the edge plasmas has shown steady progress these years. In this conference, many authors reported the transport code analysis on this subject (Hatayama [106], Coster [107], Snyder [108] and Parail [5]). Quantitative comparisons are reported by these authors. For instance, the degree of detachment, $n_{\text{sep}}^2 \Gamma_d^{-1}$, is analyzed by Hatayama. ($n_{\text{sep}}$: density at plasma surface, $\Gamma_d$: flux in the detached state.) It is shown numerically that the degree of detachment is enhanced by the W-shaped divertor in JT-60U [106].

7.2 Other Important Phenomena

Other related issues in the edge plasmas include the stability, the magnetic braiding and current injection [105], the study on MARFE by Herrera [109], X-point transport [35], etc. A new boundary concept, which could give a large expansion volume for tokamaks, is proposed by Kotschenreuter [110].
7.3 ELMs and Pedestal

Among the various kinds of instabilities at the plasma edge, the edge localized modes (ELMs) are subject of intensive studies. (For instance, [5] and [108].) Analyses based on quasi-linear pictures are reported. The onset of linear stability condition is studied in conjunction with the transport code analysis. A model of an impact of one ELMs event is combined, and a periodic evolution is studied.

The influence of shaping of the plasma cross-section is studied in detail by Snyder [108]. Characterizing the plasma state by two parameters, i.e., the edge pressure gradient and the current density near edge, the kink-ballooning mode instability is investigated. The triangularity is shown to be strongly influential in improving the stability. A comparison with DIII-D data is made. Parail [5] studies the stability by use of the JET 1.5D and 2D transport codes. The neutral gas density is found to increase the frequency of ELMs and reduces the size of one event of ELMs. These model analyses show a semi-quantitative agreement with experimental observation. The evolution of ELMs event (e.g., the mechanism of the sudden onset, the identification of perturbation that leads to the ELMs event, avalanche of energy and the recovery to the improved confinement state) is yet unclarified, and theory of ELM events must be explored in future.

8. Statistical Approaches

As a result of the progress in toroidal plasma experiments, the importance of the tail component in the probability density function in transport problems and in onsets of events is widely recognized. It is well known that the statistical approach is needed.

The probability density function (PDF) is considered essential in many articles. One may write a Langevin equation to study the statistical property of the quantity $X$ which is the subject of interest:

$$\frac{d}{d\tau} X + \Lambda X = \omega(\tau) g .$$

In this equation $\Lambda$ is the (nonlinear) damping rate, $g$ is the magnitude of the noise source and $\omega(\tau)$ represents the Gaussian white noise. Non-Gaussianity of the PDF is caused either by the nonlinearity in the damping rate $\Lambda$, or by the nonlinearity in the noise source $g$. The former process is investigated by Kim [111]. The formation of drift wave modon is studied by use of the reduced set of equations in the presence of a noise with fixed magnitude. Longer life-time is expected for larger-amplitude modon, so that the stretched non-Gaussian PDF is obtained. The PDF of the local Reynolds stress $\mathcal{R}$ is obtained as

$$P(\mathcal{R}) \sim \exp \left( - C \mathcal{R}^2 \right) .$$

In this case, an exponential tail is obtained. The nonlinearity in the noise source gives much more prominent tails. In the renormalization model, $g$ is given as a function of the amplitude of turbulence. Stronger nonlinear noise appears for stronger fluctuations. This gives power-law tails in the study of multiple-scale turbulence and bifurcation by Yagi [8]. The presence of non-Gaussian tail means that the statistical average is deviated from the peak of the PDF: In other words, large-scale but rare events could play a dominant role in determining the average.

Substantial tail of PDF is obtained in the analyses based on the SOC modelling. The hysteresis nature that is inherent to the SOC model leads the avalanche of transported quantity (heat, particle, etc.). Such an avalanche constitutes the tail in PDF and contributes to the considerable part of the statistical average.
The PDF of the heat flux $P(q_r(x,y))$ is observed in direct nonlinear simulations of turbulence, where $q_r(x,y)$ is an instantaneous value of the local heat flux at $(x,y)$. Abbreviating $P(q_r(x,y))$ and $q_r(x,y)$ by $P(q_r)$ and $q_r$, a power law tail $P(q_r) \propto q_r^{-\alpha}$ was found in the result of the direct nonlinear simulation. The power index $\alpha$ is an important parameter: depending on the condition whether $\alpha > 2$ or $\alpha < 2$, the statistical average $\langle q_r \rangle$ changes its nature. Consider that the power law $P(q_r) \propto q_r^{-\alpha}$ holds in the range $q_{\text{min}} < q_r < q_{\text{max}}$. If $\alpha > 2$ holds, the contribution from the tail to the average is given by $q_{\text{min}}^{2-\alpha}$. If, on the contrary, the relation $\alpha < 2$ holds, the term $q_{\text{max}}^{2-\alpha}$ is dominant in it. The rare events (i.e., $q_r \sim q_{\text{min}}$) control the average. According to the Holland's report [17], the result is fitted to $\alpha = 1.5$ for the case of weak shear flow and to $\alpha = 2.3$ for the case of strong shear flow. In the former case, the rare events (probably being associated with the larger spatial scale) play important roles in determining the average. The statistical analysis has also been performed on the bursty flux in SoL [62]. In this case, exponential tail for bursty flux is obtained. These results show that the non-Gaussianity in PDF is commonly observed and is important, and that the specific form of the tail depends on the explicit form of the nonlinearity that governs the dynamics.

Based on the progress of the statistical theory of transport, the statistical theory of L-H transition is developed by Itoh [112]. A Langevin equation for the radial electric field in the plasma edge $E_r$ is derived. The PDF of $E_r$ and the transition rates between the L-mode and the H-mode are calculated. The phase boundary is obtained by using the statistical average. The statistical average of the gradient-flux relation $q_r p'$ is derived. Although the deterministic model predicted a hysteresis in $q_r p'$, which is important in explaining the rapid jumps of transport and $E_r$ at the onset of transition, the statistical average $\langle q_r \rangle$ is a single valued function of $p'$.

9. Summary and Prospect

This conference covers majority of progress that has been made in the area of fusion theory. This summary paper is made by putting an emphasis on aspects as follows. First, the integration is the area where many efforts were made. As is illustrated in Fig.4, many models have been developed in the history of theory of magnetic confinement plasmas, based on the assumption of scale separation. Combination of dynamics that are describing various processes with different scales (path A in Fig.4) provides the new dynamics and structures. Thus new

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![Fig.4. Trends in evolution of theory of magnetic confinement plasmas.](image-url)
findings are made through integration of models. This is because the confined plasma is in the far-nonequilibrium state with strong nonlinearity and the integrated system is far away from the linear superposition of parts. In addition to this integration, the importance of the mutual interaction between different scales is also highlighted in this conference, and is emphasized in this summary. The path B in Fig. 4, i.e., the study of the mutual interactions between different scales, before closing a model, is one of the main outcome of this conference. Examples of such progress (e.g., the system of drift wave turbulence and zonal flows, the system with semi-micro and micro fluctuations, the evolution of low-n MHD mode associated with the small-scale fluctuations, the emergence of blobs from fluctuations in SOL, the evolution of Alfvén eigenmodes and hot ions, the role of large-scale events in PDF, etc.) show the direction of the future progress in the fusion theory. If integrated after renormalizing the internal interactions with different scale dynamics (path C in Fig. 4), better understanding of the plasmas and more dependable prediction will be provided. Much progress in such a direction will be reported in the next IAEA conference. It might be helpful in illuminating the progress of theory of magnetic confinement if the “Theory Overview Session” is prepared at the beginning of the Next IAEA Conference.

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