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# Pressure-induced Shift of the Plasma in a Helical System with Ideally Conducting Wall

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## Abstract

The global plasma shift is calculated analytically for a helical system with an ideal wall. The derived expression for the plasma shift, incorporating both the finite- $\beta$  plasma expansion and the opposing reaction of the nearby ideal wall, can be used for interpreting the observable high- $\beta$  equilibrium effects in LHD and other helical devices.

## Keywords:

Stellarator, plasma equilibrium, finite- $\beta$  effects, plasma shift, ideal wall

## 1. Introduction

Analysis of free-boundary plasma equilibrium in a conventional stellarator shows [1] that the pressure-induced shift  $\Delta_\beta$  of the plasma column must be fairly large at high  $\beta$ :

$$\frac{\beta}{2\beta_{eq}^0} \leq \frac{\Delta_\beta}{b} \leq \frac{\beta_0}{2\beta_{eq}^0}. \quad (1)$$

Here  $\beta_{eq}^0 = \mu_b^2 b / R$ ,  $\mu_b$  is the rotational transform at the plasma edge,  $b$  is the averaged minor radius of the plasma,  $R$  is the major radius,  $\beta$  is the volume-averaged ratio  $2p/B_0^2$  with  $p$  being the plasma pressure and  $B_0$  the toroidal magnetic field at  $r=R$ , and  $\beta_0 = 2p(0)/B_0^2$  is the  $\beta$  value at the magnetic axis.

For Large Helical Device (LHD) with  $R=3.9$  m,  $b=0.6$  m,  $\mu_b=1$  [2] we have  $\beta_{eq}^0=0.15$ , and the lower bound in (1) is 0.1 for  $\beta=3\%$ , which is equivalent to 6 cm outward shift.

If such a large shift would appear in LHD, it certainly could be observed, for example, when the Shafranov shift was measured with soft X-ray CCD camera [3]. However, there is

no mentioning of observations of large ‘global’ plasma shift in [3]. Therefore a question arises why the plasma column shift in LHD is actually smaller than the above estimate.

The pressure-induced plasma shift can be suppressed by the vertical field  $B_{\perp}$  as described by the formula (see [1, 4] and references therein)

$$\Delta_b = \Delta_{\beta} + \Delta_{\perp}, \quad (2)$$

where  $\Delta_b$  is the observed shift of the plasma boundary or ‘global’ plasma shift, the expressions for both  $\Delta_{\beta}$  and  $\Delta_{\perp}$  are given below. The field  $B_{\perp}$  can be produced by the currents in the poloidal windings, which can be controlled, and by the currents induced in the conducting structures around the plasma. The induction field appears when the plasma column tends to expand toroidally with increasing  $\beta$ , as described by  $\Delta_{\beta}$ . The outward expansion would result in the change of the magnetic flux between the plasma and the conducting wall, and the induced currents oppose this change on the time scale determined by the magnetic field penetration through the wall. If this time is longer than the discharge duration, the wall may be considered as ideal.

Theory of current-carrying plasma equilibrium in a tokamak with an ideally conducting casing is described in [5, 6]. The studies of this subject have been stimulated long ago by the early experimental results demonstrating the urgent need of measures aimed at providing the plasma equilibrium along the major radius in a tokamak [7, 8]. However, the same problem for a stellarator has not yet been fully analyzed. This is so because until recently the stellarators operated at rather low  $\beta$ , and the plasma equilibrium in stellarators is provided by the original magnetic configuration which is not strongly distorted by the equilibrium plasma currents at low  $\beta$ . For more details see [1, 4].

LHD is a superconducting device, it operates now with high  $\beta$  [2, 9-12], so the toroidal effects in equilibrium are rather pronounced [3]. Therefore, the problem of global plasma equilibrium in a stellarator with account of the superconducting (ideal) wall becomes urgent, especially when the above-mentioned estimates for  $\Delta_{\beta}$  show such a noticeable effect.

The problem of global plasma equilibrium in a stellarator was discussed in [13-15], see also the reviews [1, 4]. The present analysis is based on the models described there. The new element introduced here is the ideal wall at some distance from the plasma. In [1, 4, 13-15], the vertical field  $B_{\perp}$  was considered as a free parameter. Now we find it under the constraint of the magnetic flux conservation due to the ideally conducting wall.

## 2. Brief introduction to the model and definitions

The model is based on the modified stellarator approximation allowing description of tokamaks and stellarators within the unified common approach [1, 4]. The approach essentially exploits the fact that, in a stellarator, the helical magnetic field is much smaller than the toroidal field. This allows us, in a linear approximation in this small parameter, to express explicitly the three-dimensional (3D) deformations of the magnetic surfaces through the axisymmetric part of the flux function,  $\psi(r, z)$ . The equilibrium problem is reduced then to solving 2D equation for this unknown function. Dealing with  $\psi(r, z)$ , we operate with characteristics describing the averaged geometry. For example, the toroidal surface formed by the ellipse rotating uniformly along the toroidal angle  $\zeta$  corresponds, in terms of  $\psi(r, z)$ , to the axisymmetric toroid with circular cross-section, ‘circular torus’. Here  $r, \zeta, z$  are the usual cylindrical coordinates associated with the main geometrical axis of the device.

We consider a ‘conventional stellarator’ with a circular planar axis and helical fields, a device like LHD or CHS. Within the model, the plasma boundary is described as a circular torus perturbed by the helical field. Large-aspect-ratio expansion is used in analytical calculations below. The shift  $\Delta_b$  of the plasma boundary is assumed to be small:  $\Delta_b / b \ll 1$ . For more details see [1, 13, 14].

Magnetic surfaces  $\psi = \text{const}$  in stellarators (or tokamaks) are described by the function

$$\psi = \psi_v + \psi_{ext} + \psi_{pl}, \quad (3)$$

where  $\psi_v$  is the poloidal flux of the helical magnetic field,  $\psi_{ext}$  is the poloidal flux due to the external axisymmetric field, and  $\psi_{pl}$  is the plasma-produced poloidal flux. At the first step of our analysis we need the function  $\psi$  outside the plasma, in the vacuum region between the plasma and the wall.

The plasma-produced poloidal magnetic flux outside the plasma in a conventional stellarator was calculated in [14]:

$$\psi_{pl} = 2\pi [f_0(l) + f_1(l)\cos\alpha + \dots], \quad (4)$$

where  $(l, \alpha)$  are the polar coordinates with origin at the center of the plasma cross-section, Fig. 1, and

$$f_0(l) = bRB_J \left( \ln \frac{8R}{l} - 2 \right), \quad (5)$$

$$f_1(l) = \frac{b}{l}C - \frac{bl}{2}B_J \left( \ln \frac{8R}{l} - 1 \right), \quad (6)$$

$B_J = J/(2\pi b)$  is the magnetic field of the net current  $J$ , and  $C$  is a constant which can be related to the plasma parameters by matching the inner and outer solutions for the equilibrium magnetic field at the plasma boundary.

The poloidal flux  $\psi_v$  of the helical magnetic field  $\tilde{\mathbf{B}}$  enclosed by the magnetic surface is calculated by the formula [1]:

$$\psi_v = \frac{\pi^2}{B_t} \left\langle \left[ \tilde{\mathbf{B}} \times \hat{\mathbf{B}} \right] \frac{\nabla \zeta}{|\nabla \zeta|} \right\rangle_\zeta = \frac{2\pi^2}{B_t} \left\langle \tilde{B}_z \hat{B}_r \right\rangle_\zeta, \quad (7)$$

where  $\hat{f}$  is the oscillating part of the integral  $\int f d\zeta$ ,  $B_t$  is the toroidal magnetic field, and brackets  $\langle \dots \rangle_\zeta$  denote the averaging over  $\zeta$ . The most simple approximation for  $\psi_v$  is obtained in coordinates  $(\rho, u)$  related to the geometrical axis, Fig. 1. At the moment, we introduce it as  $\psi_v = \psi_v(\rho)$ .

The poloidal flux  $\psi_{ext}$  of the external (vacuum) magnetic field must satisfy the equation

$$\text{div} \frac{\nabla \psi_{ext}}{r^2} = 0. \quad (8)$$

For our purposes, in the geometry assumed (circular averaged cross-sections of the plasma and the conducting wall), it is sufficient to retain here a term describing the external homogeneous vertical magnetic field:

$$\psi_{ext} = \psi_0(t) + \pi B_\perp (r^2 - R^2). \quad (9)$$

The necessary relations, relevant functions and quantities are defined now. Next we must find the constraints following from the boundary conditions for the function  $\psi$ .

### 3. Boundary conditions

By definition, the function  $\psi$  must be constant at the plasma boundary. The same is true at the wall if we assume it ideal. Thus, at these two boundaries of the vacuum region we must satisfy the conditions

$$\psi = \text{const}. \quad (10)$$

In our model, the plasma boundary is described by  $l = b$ . If  $\psi$  is known as a function of  $l$  and  $\alpha$ ,

$$\psi = A(l) + B_1(l) \cos \alpha, \quad (11)$$

the condition  $\psi = \text{const}$  at the plasma boundary is satisfied by  $B(b) = 0$ .

The plasma-generated part of  $\psi$  is already calculated in such a form, Eq. (4). We have to express two other functions in (3) in variables  $l$  and  $\alpha$ .

The poloidal flux  $\psi_{\text{ext}}$  of the external magnetic vertical field  $B_{\perp}$ , Eq. (9), can be rewritten as

$$\psi_{\text{ext}} = \pi B_{\perp} \left[ (R + \Delta_b)^2 - 2(R + \Delta_b)l \cos \alpha + l^2 \cos^2 \alpha \right]. \quad (12)$$

Here we have used the relations between the coordinates, see Fig. 1:

$$r = R - \rho \cos u = R + \Delta_b - l \cos \alpha, \quad (13)$$

$$\rho \sin u = l \sin \alpha. \quad (14)$$

It follows then that

$$\rho \approx l - \Delta_b \cos \alpha, \quad (15)$$

which is valid for  $l \gg \Delta_b$ , when the global plasma shift (shift of the plasma boundary) is a small part of the plasma radius,  $\Delta_b / b \ll 1$ . In this case, in linear approximation in  $\Delta_b / b$ ,

$$\psi_v(\rho) \approx \psi_v(l) - \psi_v'(l) \Delta_b \cos \alpha. \quad (16)$$

Combining (4), (12) and (16) and comparing the result with (11), we obtain

$$B(l) = 2\pi f_1(l) - 2\pi R l B_{\perp} - \psi_v'(l) \Delta_b, \quad (17)$$

and the boundary condition  $B(b) = 0$  gives

$$f_1(b) - R b B_{\perp} + R \Delta_b B^* = 0. \quad (18)$$

Here  $f_1$  is the function defined by (6), and

$$B^* \equiv -\frac{\psi_v'(b)}{2\pi R} = \mu_b \frac{b}{R} B_0. \quad (19)$$

In the model, the ideal wall is prescribed by  $\rho = a_c$ , which allows a shift of the plasma relative to the wall. To apply the boundary condition (10) at  $\rho = a_c$ , we must transform  $\psi$  to the form

$$\psi = D(\rho) + E(\rho) \cos u, \quad (20)$$

so that the condition  $\psi = \text{const}$  at the wall is satisfied by  $E(a_c) = 0$ .

Similar to (15), and (14) for  $\rho \gg \Delta_b$  we can express  $l$  from (13) as

$$l \approx \rho + \Delta_b \cos u. \quad (21)$$

After transformation of the functions  $\psi_{pl}$  and  $\psi_{\text{ext}}$  to variables  $(\rho, u)$  we obtain

$$E(\rho) = 2\pi [f_0'(\rho) \Delta_b + f_1(\rho)] - 2\pi R \rho B_{\perp}, \quad (22)$$

and, finally, from  $E(a_c)=0$ :

$$\frac{b}{a_c}[f_1(a_c)+f_0'(a_c)\Delta_b]-RbB_\perp=0. \quad (23)$$

Relation (18) is a part of the equilibrium conditions for a plasma. It follows directly from  $\nabla p = \mathbf{j} \times \mathbf{B}$  and must be satisfied in any case. Equation (23) is an additional constraint resulting from the requirement that the wall is ideal.

#### 4. Equation for $\Delta_b$

Without the wall, we would have Eq. (18) which gives  $\Delta_b$  as a function of  $B_\perp$  and  $f_1$ , with  $B_\perp$  being a parameter. With ideal wall, we have two equations: (18) and (23). Subtracting one from another to eliminate  $B_\perp$ , we obtain

$$f_1(b) - \frac{b}{a_c} f_1(a_c) = \frac{b}{a_c} f_0'(a_c) \Delta_b - R \Delta_b B^*, \quad (24)$$

which, by using (5) and (6), can be transformed into

$$C \left( \frac{b^2}{a_c^2} - 1 \right) + \frac{b^2}{2} B_J \ln \frac{a_c}{b} = R \Delta_b \left( B^* + \frac{b^2}{a_c^2} B_J \right). \quad (25)$$

This equation relates the plasma column shift to the geometric and magnetic parameters of the equilibrium configuration. To analyze it, we must express the constant  $C$  in similar terms.

The constant  $C$  appears in (6) as a parameter used to describe  $\psi_{pl}$  outside the plasma, Eq. (4). The introduced magnetic fluxes are related to the axisymmetric component of the poloidal magnetic field,  $\bar{\mathbf{B}}_p$ , by [1, 4]

$$\bar{\mathbf{B}}_p = \frac{1}{2\pi} \nabla(\psi - \psi_v) \times \nabla \zeta = \frac{1}{2\pi} \nabla(\psi_{pl} + \psi_{ext}) \times \nabla \zeta, \quad (26)$$

where  $\zeta$  is the toroidal angle. In the coordinates  $(l, \alpha)$ , keeping only two first terms in the Fourier series, one obtains

$$\bar{\mathbf{B}}_p \cdot \mathbf{e}_\alpha = -\frac{f_0'}{R} - \left( \frac{b}{R} f_0' + f_1' \right) \frac{\cos \alpha}{R} + B_\perp \cos \alpha + \dots, \quad (27)$$

and, with account of expressions (5) and (6) for  $f_0$  and  $f_1$ ,

$$H_1 = B_\perp + \frac{C}{Rb} + \frac{b}{2R} B_J \ln \frac{8R}{b}, \quad (28)$$

where  $H_1$  is the amplitude of the cosine harmonic of the tangential component of  $\bar{\mathbf{B}}_p$  at the averaged plasma boundary  $\Gamma$ :

$$\bar{\mathbf{B}}_p \cdot \mathbf{e}_\alpha \Big|_r = B_J + \sum_{n=1}^{\infty} H_n \cos n\alpha. \quad (29)$$

Combining (28) with Eq. (18), which in an expanded form reads as

$$B^* \frac{\Delta_b}{b} = B_\perp - \frac{C}{Rb} + \frac{b}{2R} B_J \left( \ln \frac{8R}{b} - 1 \right), \quad (30)$$

we can express  $B_\perp$  and  $C$  through the measurable values  $H_1$ ,  $B_J$ , and  $\Delta_b$ :

$$B_\perp = \frac{H_1}{2} - \frac{b}{2R} B_J \left( \ln \frac{8R}{b} - \frac{1}{2} \right) + \frac{\Delta_b}{2b} B^*, \quad (31)$$

$$C = \frac{bR}{2} \left( H_1 - \frac{\Delta_b}{b} B^* - \frac{b}{2R} B_J \right). \quad (32)$$

Then the equality (25) yields

$$\Delta_b \left( B_J + B^* \frac{a_c^2}{b^2} \right) = \frac{a_c^2}{2R} \left\{ B_J \ln \frac{a_c}{b} + \left[ \frac{1}{2} B_J - \frac{R}{b} \left( H_1 - \frac{\Delta_b}{b} B^* \right) \right] \left( 1 - \frac{b^2}{a_c^2} \right) \right\}. \quad (33)$$

Since no restriction has been imposed on  $B_J$  and  $B^*$  in the above derivation, this expression must be valid for any ratio between  $B^*$  and  $B_J$ . In other words, it can be applied either to tokamaks,  $B^* = 0$ , or to stellarators without current,  $B_J = 0$ , which are two limiting cases. It describes, as well, any intermediate configuration (stellarators with current).

## 5. Two opposite limits: tokamak and current-free stellarator

For a tokamak,  $B^* = 0$ , relation (33) is reduced to

$$\Delta_b = \frac{a_c^2}{2R} \left[ \ln \frac{a_c}{b} + \left( \Lambda + \frac{1}{2} \right) \left( 1 - \frac{b^2}{a_c^2} \right) \right], \quad (34)$$

where

$$\Lambda \equiv -\frac{R H_1}{b B_J}. \quad (35)$$

Expression (34) is a well-known Shafranov's result [5] for current-carrying plasma, and  $\Lambda$  is a quantity introduced by Shafranov to describe the oscillating part of the poloidal magnetic field on the magnetic surfaces in a tokamak (in our case we need it at the plasma boundary):

$$\mathbf{B}_p \cdot \mathbf{e}_\alpha \Big|_r = B_J \left( 1 - \frac{b}{R} \Lambda \cos \alpha \right), \quad (36)$$

compare with (29).



The tokamak expressions (34)-(36) cannot be used for current-free plasma in stellarators (more precisely, for equilibrium configurations with small  $B_j / B^*$ ) because  $\Lambda$  introduced by (35) is not defined for  $B_j = 0$  when  $H_1$  is finite. Since  $B^*$  and  $B_j$  enter Eq. (33) in different ways, it is impossible therefore to draw a close analogy between tokamaks and stellarators in this case.

For a current-free plasma, Eq. (33) gives

$$\frac{\Delta_b}{b} B^* = -\frac{1}{2} \left( H_1 - \frac{\Delta_b}{b} B^* \right) \left( 1 - \frac{b^2}{a_c^2} \right). \quad (37)$$

The quantity  $H_1$  can be found from the magnetic measurements outside the plasma [14]. The shift  $\Delta_b$  itself can be measured by the magnetic loops and probes [14, 16]. Thus, Eq. (37) relating the measurable values allows experimental verification.

Equations (33), (34) and (37) clearly show the effect of the conducting wall on the plasma shift. In all cases, for  $a_c = b$  they give a natural result  $\Delta_b = 0$ . This illustrates a general tendency: the plasma shift  $\Delta_b$  is smaller for the wall closer to the plasma.

Equations (33), (34) and (37) contain  $H_1$ , the amplitude of the cosine harmonic of  $\mathbf{B}_p$  at the plasma boundary, which is still an unknown parameter here since up to now we have considered only the magnetic field in the vacuum region between the plasma and the wall. To find  $H_1$ , one should solve the complete equilibrium problem. That can be done in two steps. First,  $H_1$  can be expressed through the geometrical quantities describing the plasma boundary: the shift, ellipticity, etc. Since  $\bar{\mathbf{B}}_p$  is determined by  $\nabla\psi$ , not only these parameters will enter the resulting expression for  $H_1$ , but their derivatives also. Determining these derivatives must be the second step, where the equilibrium equations for the plasma must be used. This problem has been solved analytically long ago for a large-aspect-ratio tokamak with a circular plasma in a circular vacuum chamber [5]. Now we briefly describe a similar procedure for a stellarator.

## 6. Current-free stellarator with circular shifted magnetic surfaces

If the cross-sections of the surfaces  $\psi = \text{const}$  near the plasma boundary  $\Gamma$  are circular (the relative shift is allowed), the relation (26) can be written in the form

$$\bar{\mathbf{B}}_p = -B_0 [a\mu(a)\nabla a - \rho\mu_h(\rho)\nabla\rho] \times \nabla\zeta, \quad (38)$$

where

$$\psi'(a) = -2\pi a B_0 \mu(a), \quad (39)$$

$$\psi'_v(\rho) = -2\pi \rho B_0 \mu_h(\rho), \quad (40)$$

$\mu(a)$  is the rotational transform,  $\mu_h(\rho)$  is the vacuum rotational transform produced by the helical field.

We need Eq. (38) for calculating  $H_1$ , the cosine harmonic of  $\bar{\mathbf{B}}_\rho \cdot \mathbf{e}_\alpha$  at the plasma boundary. For these purposes, we have to know the geometry of magnetic surfaces near the boundary only.

We parameterize the averaged magnetic surfaces near the plasma boundary as

$$\begin{cases} r = R + \Delta(a) - a \cos \theta \\ z = a \sin \theta \end{cases}, \quad (41)$$

where  $a$  is the label of a magnetic surface, and  $\theta$  is the poloidal angle. In this case

$$(1 - \Delta' \cos \theta) \nabla a = \mathbf{e}_z \sin \theta - \mathbf{e}_r \cos \theta \quad (42)$$

with  $\mathbf{e}_r = \nabla r$  and  $\mathbf{e}_z = \nabla z$  being the unit vectors.

At the plasma boundary we have  $\theta = \alpha$ ,  $a = b$ , so that

$$\mathbf{e}_l \cdot \nabla a|_\Gamma = \frac{1}{1 - \Delta'(b) \cos \alpha}, \quad (43)$$

and

$$\mathbf{e}_l \cdot \mathbf{e}_\rho|_\Gamma = \frac{b - \Delta_b \cos \alpha}{\rho}. \quad (44)$$

Here the next relations have been used, see Fig. 1:

$$\mathbf{e}_l = -\mathbf{e}_r \cos \alpha + \mathbf{e}_z \sin \alpha, \quad (45)$$

$$\rho \mathbf{e}_\rho = l \mathbf{e}_l + \Delta_b \mathbf{e}_r. \quad (46)$$

With (43) and (44) we obtain from (38)

$$\bar{\mathbf{B}}_\rho \cdot \mathbf{e}_\alpha|_\Gamma = B_0 \frac{b}{r} \left[ \frac{\mu(b)}{1 - \Delta'(b) \cos \alpha} - \mu_h(\rho) \left( 1 - \frac{\Delta_b}{b} \cos \alpha \right) \right]. \quad (47)$$

All values here must be taken at the averaged plasma boundary. According to (15),

$$\mu_h(\rho)|_\Gamma = \mu_h(b) - \mu'_h(b) \Delta_b \cos \alpha, \quad (48)$$

and the amplitude of the cosine harmonic of (47) is

$$H_1 = B_0 \frac{b}{R} \left\{ [\mu(b) - \mu_h(b)] \frac{b}{R} + \mu(b) \Delta' + \mu'_h(b) \Delta_b \right\} + B^* \frac{\Delta_b}{b}. \quad (49)$$

Inside the plasma the rotational transform profile  $\mu(a)$  can be strongly distorted at finite  $\beta$  [1, 4]. However, at the plasma boundary it can be expressed as

$$\mu = \mu_j + \mu_h(b), \quad (50)$$

where  $\mu_j = RB_j/(bB_0)$  is the rotational transform due to the net toroidal current. This allows us to rewrite (49) in the form

$$H_1 = \frac{b}{R}B_j + \frac{b}{R}B_0(\mu\Delta' + \mu_h'\Delta_b) + B^*\frac{\Delta_b}{b}. \quad (51)$$

This is valid for both the tokamaks and stellarators. Here we need  $H_1$  for the current-free stellarator,  $B_j = 0$  (actually the result is valid for  $|B_j| \ll B^*$ ). In low- $\beta$  approximation, assuming that the model of circular shifted magnetic surfaces (41) is applicable for the description of the whole plasma column, one can obtain from the equilibrium equations [1, 4]

$$(\mu_h\Delta) \Big|_r = 2\frac{R}{b}\frac{B_\beta}{B_0}, \quad (52)$$

where

$$B_\beta = \frac{1}{B_0} \int_0^b \frac{p'(a)a^2}{\mu(a)b^2} C_{PS} da, \quad (53)$$

and  $C_{PS}$  is the coefficient [17] describing the reduction of the Pfirsch-Schlüter current when the plasma column is strongly shifted relative to the geometrical center of a stellarator. In conventional stellarators, usually  $C_{PS} \approx 1$ . For the case considered, Eq. (51) yields

$$H_1 - B^*\frac{\Delta_b}{b} = 2B_\beta. \quad (54)$$

This allows us to rewrite (37) in a compact form

$$\Delta_b = \Delta_\beta \left( 1 - \frac{b^2}{a_c^2} \right), \quad (55)$$

where

$$\Delta_\beta = -b\frac{B_\beta}{B^*}. \quad (56)$$

The latter is the free-boundary pressure-induced plasma shift discussed in the Introduction.

Expression (55) explicitly describes the ideal wall effect on the pressure-induced shift of the current-free plasma column. The effect is stronger for the wall closer to the plasma. However, the effect is noticeable even when the gap between the plasma and the ideal wall is rather large. For example, Eq. (55) shows that for  $b/a_c = 0.7$  the wall effect results in 50%

reduction of the shift. This means that in the example considered in the Introduction we would obtain 3 cm plasma shift with the ideal wall instead of 6 cm for the free-boundary case.

Note that  $\Delta_b$  can be also written in the form (2) with  $\Delta_\beta$  given by (56) and with

$$\Delta_\perp = b \frac{B_\perp}{B^*}, \quad (57)$$

which follows from (18) for the current-free plasma. Equations (55)-(57) imply that with ideal wall  $\Delta_\perp = -\Delta_\beta b^2 / a_c^2$ , so that in this case

$$B_\perp = B_\beta \frac{b^2}{a_c^2}. \quad (58)$$

The latter relation gives the value of the vertical field due to the currents induced in the ideal wall.

## 7. Conclusion

The analysis shows that the ideal wall must strongly affect the pressure-induced plasma shift in helical devices. According to (55), the effect is noticeable even for  $b/a_c = 1/2$ . The plasma shift reduction due to the currents induced in the ideal wall may be a reason why the Shafranov shift measured with soft X-ray CCD camera [3] was small.

According to (55), the plasma shift may be completely suppressed by the wall reaction in case only when the plasma-wall gap is negligible. In real devices, the gap is finite. For LHD, a rough estimate can be  $b/a_c = 0.6/0.9$  [18]. In this case, the plasma shift must be twice smaller than it would be without the wall. For  $\beta \geq 3\%$  we obtain from (55) that the shift will be several centimetres in a helical device with ideal wall similar to LHD. Such a shift is large enough to be measured, which can be used for diagnostic purposes. It must be noted that the analysis based on assumption of the ideal wall overestimates the effect of the wall in the real devices. In other words, Eq. (55) should be considered as a lower estimate for  $\Delta_b$ , and actually the pressure-induced plasma shift must be larger. Therefore, the problem deserves more attention.

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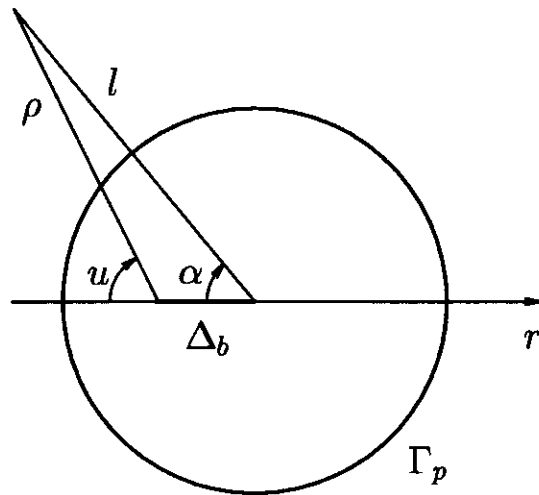


Figure 1. Transverse cross section of the plasma column shifted to a distance  $\Delta_b$  relative to the geometrical axis  $r = R$ ;  $l, \alpha$  are the quasi-cylindrical coordinates associated with the plasma column center.

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