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Geodesic Acoustic Mode Oscillation in the Low Frequency Range

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Abstract:

In this paper, the Geodesic Acoustic Mode theory is reconsidered with regard to the linear dispersion relation obtained previously by use of the drift kinetic equation. Specifically, kinetics effects in the low frequency range are handled more precisely, suggesting an answer to the question if GAM has only a single frequency of oscillation. It is shown, for a high-aspect-ratio tokamak with circular cross-section, there are two GAM frequencies. The existence of a new solution with a lower frequency is attributed to the unique features of the geodesic response functions defined in this paper. It is also shown that inclusion of kinetic effects renders the geodesic current smaller and narrows the range of GAM existences in the configuration- and the plasma parameter-spaces.

It is also shown that a system of two mixed helicity (consisting of helical and toroidal ripples) have two modes in a wide range of plasma parameters: The one with the lower frequency mode is a small modification from the tokamak type GAM and the one with higher frequency transits between tokamak GAM to helical GAM depending on the relative size of the curvature of magnetic lines of force. With the transitional plasma parameter, there could be four mode frequencies. [1]

Key words: geodesic acoustic mode, low frequency GAM, neo-classical polarization current

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I. Introduction

Since, plasma drifts with the velocity $\vec{E}\times\vec{B}$, the rotation speed of plasma is related to the radial electric field E_r. The rotating plasma is provided with restoring force by the geodesic curvature of the magnetic lines of forces, which exist intrinsically in tori. Therefore, the plasmas are subject to oscillatory motions, the existence of which was theoretically suggested by Winsor et al [1]. This oscillation is referred to as GAM (Geodesic Acoustic Mode) according to the origin of the restoring force. The physics of GAM oscillation has initially been studied based on the MHD equations and applied in limited way to tokamaks [2,3,4]. In recent investigations, such rotations of the plasma have also been found in many of the simulations, and are believed to be driven by the plasma turbulence [5,6,7,8,9,10,11,12,13]. The radial shear flows are shown in turn to regulate transports [14,15] and thus they are gathering more and more attention. According to the progress in diagnostics, the sheared zonal flows including GAM have been evidenced in experiments [16-26].

The progress in the physics of flow drive and regulation of the turbulence have been reviewed for instance in [27,28]. The mechanism of the generation of the radial electric field is in itself an interesting subject of research. It is believed that the zonal flow with lower frequencies than GAM, which is referred to as zero frequency branch, has lower viscosity [29-31]. Therefore, it may play a larger role in regulating the turbulence.

The extension of GAM theory to helical systems has been addressed in our previous paper [32], where it was found again that the helical geodesic curvature of the magnetic field is the cause of the geodesic acoustic mode. Therefore, the flux coordinate has been introduced with proper poloidal and toroidal coordinates and a Furrier expansion of the magnetic field intensity has to be made.

$$B^2 = B_0^2(1+\sum_{m,n}\delta_{m,n}(\psi)\cos(m\theta-n\zeta)) \tag{1}$$

Here, the major results of ref. [32] are briefly summarized, for they are fully used in the present paper: Introducing conductivity constants σ_{pol} and σ_{geo}, the polarization current, j_{pol}, and geodesic current, j_{geo}, are written in the following simple forms:

$$j_{geo}(\psi) = -\sigma_{geo}(\psi)\frac{d\phi}{d\psi} \tag{2}$$

$$j_{pol}(\psi) = -\sigma_{pol}(\psi)\frac{d\phi}{d\psi} \tag{3}$$

with

$$\sigma_{pol} \equiv -\frac{\omega^2{}_{p,i}}{\omega^2{}_{c,i}}\frac{\omega}{4\pi i}\frac{qB_\zeta+B_\theta}{2\pi B^2}\int\frac{1}{B^2}\left|\vec{\nabla}\psi\right|^2 d\theta d\zeta = -\frac{\omega}{i}\tilde{\sigma}_{pol} \tag{4}$$

$$\sigma_{geo} \equiv (\frac{e^2c^2}{2} n_0 T (\frac{1}{\sqrt{g}B^2})^2 (\frac{1}{2}\tilde{V}')4B_t^2 l_\psi^2)\sum_{m,n} \eta^2{}_{m,n}(\psi)F(\zeta_{m,n})$$
$$\equiv \tilde{\sigma}_{geo}\sum_{m,n} \eta^2{}_{m,n}(\psi)F(\zeta_{m,n}) \tag{5}$$

Here, the geometric factors $\eta_{m,n}^2$, l_ψ^2, and $\hat{V}'$ are defined as follows:

$$\eta^2{}_{m,n} \equiv \frac{(mB_\zeta + nB_\theta)^2 \delta^2{}_{m,n}(\psi)}{4B_t^2 l_\psi^2} \tag{6}$$

$$l_\psi^2 \equiv q^2 \int \frac{1}{B^2} \left|\nabla \psi\right|^2 d\theta d\zeta \tag{7}$$

$$(\frac{1}{2}\tilde{V}') \equiv \int \sin^2(m\theta - n\zeta)\sqrt{g}\, d\theta d\zeta \tag{8}$$

Equating the two currents, we obtain

$$j_{total} = \tilde{\sigma}_{pol}(\frac{\omega}{i} - \tilde{\tilde{\omega}}_G^2 \sum_{m,n} \eta_{m,n}^2 F_{m,n})\frac{\partial \phi}{\partial \psi} = 0 \tag{9}$$

where,

$$\tilde{\tilde{\omega}}_G^2 = \frac{\tilde{\sigma}_{geo}}{\tilde{\sigma}_{pol}} = \frac{T_i}{M_i}\frac{1}{R^2} \tag{10}$$

$F_{m,n} = F_i + F_e = F_i(\zeta_{m,n}) + F_e(\zeta_{m,n})$ is the sum of the ion and electron contributions, which we obtained in the previous work by use of the drift kinetic equation:

$$F_i = \int \left(\frac{1}{i(\omega - k_{\parallel,m,n}\upsilon_\parallel)} + \frac{1}{i(\omega + k_{\parallel,m,n}\upsilon_\parallel)}\right)\left(\frac{(m\upsilon_\parallel^2 + \frac{1}{2}m\upsilon_\perp^2)}{2T_i}\right)^2 f_{i,0}\, d\vec{\upsilon} \tag{11}$$

and

$$
\begin{aligned}
F_e &= \frac{q_i}{q_e}\frac{n_i}{n_{e,0}}T_e\int \Big(\frac{(m\upsilon_{\parallel,e}^2+\frac{1}{2}m\upsilon_{\perp,e}^2)}{2T_e}\Big) f_{e,0}d\vec{\upsilon} \\
&= T_e\frac{q_i}{q_e}\frac{n_i}{n_{e,0}}\frac{T_e}{T_i}\int \Big(\frac{1}{i\omega - ik_{\parallel,m,n}\upsilon_{\parallel}}+\frac{1}{i\omega + ik_{\parallel,m,n}\upsilon_{\parallel}}\Big)\Big(\frac{(m\upsilon_{\parallel}^2+\frac{1}{2}m\upsilon_{\perp}^2)}{2T_i}\Big) f_{i,0}d\vec{\upsilon}
\end{aligned}
\tag{12}
$$

The matching parameter, $\zeta_{m,n}$ is defined by use of the wave number $k_{\parallel,m,n} = (mB^{\theta} - nB^{\zeta})/B_0$ as follows:

$$
\zeta_{m,n} = \omega / k_{\parallel,m,n}\upsilon_T \tag{13}
$$

II. Asymptotic and Series Expansions of the Geodesic Dispersion Functions

In order to examine the properties of the response function further, we deform Eqs. (11) and (12) and define two dispersion functions $Z_{geo,1}(\zeta)$ and $Z_{geo,2}(\zeta)$

$$
Z_{geo,1} = \frac{1}{\sqrt{\pi}}\int \frac{1}{x-\zeta}((x)^4 + (x)^2 + \frac{1}{2})\exp(-x^2)dx \tag{14}
$$

$$
Z_{geo,2} = \frac{1}{\sqrt{\pi}}\int \frac{1}{x-\zeta}(x^2 + \frac{1}{2})\exp(-x^2)dx \tag{15}
$$

besides the well-known plasma dispersion function, Z_p

$$
Z_p = \frac{1}{\sqrt{\pi}}\int \frac{1}{x-\zeta}\exp(-x^2)dx \tag{16}
$$

The subscripts $_{geo,1}$ and $_{geo,2}$ denote that the quantities are related to ion and electron contributions in the presence of geodesic curvature.

Thus, we find that $Z_{geo,1}$ and $Z_{geo,2}$ functions characterize the GAM oscillations.

$$
F_{m,n} = F_i(\zeta_{m,n}) + F_e(\zeta_{m,n}) = -2\frac{1}{ik_{\parallel,m,n}\upsilon_T}(Z_{geo,1}(\zeta_{m,n}) + \frac{T_e}{T_i}Z_{geo,2}(\zeta_{m,n})) \tag{17}
$$

A. Asymptotic expansion.

Asymptotic expansions of $Z_{geo,1}$ and $Z_{geo,2}$ are easily obtained applying the established procedure in investigating the conventional plasma dispersion function Z_p, which is written in the bottom line for reference.

$$
\begin{aligned}
Z_{geo,1} &\approx -(\frac{7}{4}\frac{1}{\zeta}+\frac{23}{8}\frac{1}{\zeta^3})+i\sqrt{\pi}((\zeta)^4+(\zeta)^2+\frac{1}{2})\exp(-\zeta^2) \\
Z_{geo,2} &= -(\frac{1}{\zeta}+(\frac{1}{\zeta})^3)+i\sqrt{\pi}(\zeta^2+\frac{1}{2})\exp(-\zeta^2) \\
Z_p &\approx -(\frac{1}{\zeta}+\frac{1}{2}\zeta^3)+i\sqrt{\pi}\exp(-x^2)
\end{aligned}
\tag{18}
$$

B. Series Expansions

Using the established procedures similarly, their series expansions are obtained as:

$$
\begin{aligned}
Z_{geo,1} &\approx \frac{1}{2}\zeta-\frac{1}{3}\zeta^3+i\sqrt{\pi}\left(\frac{1}{2}+\zeta^2+\zeta^4\right)\exp(-\zeta^2) \\
Z_{geo,2} &\approx -\frac{4}{3}\zeta^3+i\sqrt{\pi}(\frac{1}{2}+\zeta^2)\exp(-\zeta^2) \\
Z_p &\approx -(2\zeta-\frac{4}{3}\frac{1}{\zeta^3})+i\sqrt{\pi}\exp(-x^2)
\end{aligned}
\tag{19}
$$

These three dispersion functions are calculated and shown in Fig.1, versus the ζ. The most notable feature is that the $Z_{geo,1}$, the ion current caused by the geodesic curvature, increases in proportion to ζ, i.e., in proportion to ω. This is quite different from the case of Z_p, where the current varies with the opposite coefficient of proportionality. It is also noted that $Z_{geo,1}$ does not have a term proportional to ζ. This unique feature of the geodesic response function originates from the multiplication factors in Eqs. (11) and (12).

III. High and Low frequency GAMs.

A. High frequency GAMs: $k_{\parallel,m,n}\upsilon_T < \omega$

Now, it is easy to calculate $F_{m,n}$ by use of the dispersion functions defined in the previous section. In the asymptotic expansion, we have:

$$Z_{geo,1}(\zeta_{m,n})+\frac{T_e}{T_i}Z_{geo,2}(\zeta_{m,n})=\left\{\ -(\frac{7}{4}+\frac{T_e}{T_i})\frac{1}{\zeta}-(\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})\frac{1}{\zeta^3}\ \right\} \\ +i\sqrt{\pi}\left\{\ (\zeta^4_{m,n}+\zeta^2_{m,n}+\frac{1}{2})+\frac{T_e}{T_i}(\zeta^2_{m,n}+\frac{1}{2})\ \right\}\exp(-\zeta^2_{m,n}) \tag{20}$$

It is noted that this asymptotic expansions was applied in the previous work [32]. It is worthwhile to reproduce the results of the previous paper from this non-dimensional formalism before we proceed to examine the properties of GAM in the lower frequencies.
For a single helicity magnetic configuration, Eq.(9) assumes the form.

$$\zeta_{m,n}=-2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}\left[Z_{GAM,1}\left(\zeta_{m,n}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{m,n}\right)\right]\right) \tag{21}$$

The dispersion relation given in (21) is readily converted to the following form by use of Eq.(20).

$$\zeta_{m,n}=-2\frac{\tilde{\tilde{\omega}}_G^2}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}\left\{\ -(\frac{7}{4}+\frac{T_e}{T_i})\frac{1}{\zeta_{m,n}}-(\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})\frac{1}{\zeta_{m,n}{}^3}\ \right\}\right) \tag{22}$$

finding the solution given by

$$\zeta^2{}_{m,n}=2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right) \tag{23}$$

where the parameter ξ in the parenthesis is a parameter of order unity and defined by:

$$\xi=(\frac{23}{8}+\frac{5}{4}\frac{T_e}{T_i})/(\frac{7}{4}+\frac{T_e}{T_i}) \tag{24}$$

For a simple tokamak, only the toroidal ripple, (m,n)=(1,0), is dominant and we may assume that $\eta^2_{m,n}=1$, i.e. the geometric factor is unity. Thus we obtain:

$$\zeta^2{}_{m,n}=2\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(2(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right) \tag{25}$$

i.e.,

$$\omega^2=\frac{2}{M}\frac{1}{R^2}\left((\frac{7}{4}T_i+T_e)(1+\xi\frac{1}{\zeta_0{}^2})\right) \tag{26}$$

For a straight helical system, only single helicity (m,n)=(M,N) is dominant. The amplitude of the helicity $\eta^2_{M,N}$ is obtained from Eq.(6)

$$\zeta^2{}_{M,N} = 2\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,M,N}\upsilon_T\right)^2}\left(\eta^2{}_{M,N}(\frac{7}{4}+\frac{T_e}{T_i})(1+\xi\frac{1}{\zeta^2})\right) \quad (27)$$

The only differences of the GAMs in the helical and tokamak systems are in the geometric factor, $\eta^2_{M,N}$, which is usually larger for the former than the latter.

B. Low frequency GAMs: $\omega < k_{\parallel,m,n}\upsilon_T$

In series expansion, we have:

$$\begin{aligned} &Z_{geo,1}(\zeta_{m,n}) + \frac{T_e}{T_i}Z_{geo,2}(\zeta_{m,n}) = \left\{ \ (\frac{1}{2}\zeta_{m,n} - (\frac{1}{3}+\frac{4}{3}\frac{T_e}{T_i})\zeta_{m,n}{}^3) \ \right\} \\ &+i\sqrt{\pi}\left\{ \ \frac{T_e}{T_i}(\frac{1}{2}+\zeta_{m,n}{}^2) + \left(\frac{1}{2}+\zeta_{m,n}{}^2+\zeta_{m,n}{}^4\right) \ \right\}\exp(-\zeta_{m,n}{}^2) \end{aligned} \quad (28)$$

In this frequency range, the following dispersion relation is obtained by substituting Eq.(28) into Eq.(9).

$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\eta^2{}_{m,n}(\frac{1}{2}\zeta_{m,n} - (\frac{1}{3}+\frac{4}{3}\frac{T_e}{T_i})\zeta_{m,n}{}^3)\right) \quad (29)$$

Equation (29) has two solutions:

$$\zeta^2{}_{m,n} = (1+\eta^2{}_{m,n}\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2})\left(\eta^2{}_{m,n}\frac{\tilde{\tilde{\omega}}^2{}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}(\frac{2}{3}+\frac{8}{3}\frac{T_e}{T_i})\right)^{-1} \quad (30)$$

and

$$\zeta_{m,n} = 0\,. \quad (31)$$

The latter solution has been known in the neoclassical transport and has been realized as one of the zero frequency branches.

The former (Eq. (29)), reduces to a simpler form in the case of simple tokamak.

$$\omega^2 = \left\{ \ 1+(q^2/2) \ \right\}\left\{ \ (q^2/2)(\frac{2}{3}+\frac{8}{3}\frac{T_e}{T_i}) \ \right\}^{-1}\frac{1}{q^2}(\frac{2T/M}{R^2}) \quad (\ 32)$$

For large values of q^2 , Eq. (32) is further reduced to

$$\omega^2 \sim \frac{1}{q^2}(\frac{2T/M}{R^2}) \quad (33)$$

It means that there will be a branch of GAM oscillation in the frequency range lower than

GAM, $\omega \sim \frac{1}{q}\omega_{GAM}$. In Figure 2, shown versus ζ are j_{pol} and j_{geo} calculated by use of the full kinetic equation; the frequencies where they cross each other are GAM frequencies. Among the two such frequencies, the one with higher frequency is well approximated by Eq.(23) and the one with lower frequency is approximated by Eq.(32). The two terms in the first bracket in Eq.(32) are classical and neoclassical polarization currents. It is noted that incorporation of the velocity change of the particles along the magnetic lines of forces will change the ratio of the geodesic current and the polarization current and therefore will cause a small modification in the frequency of the low frequency GAM. Therefore, $\omega \sim \frac{1}{q}\omega_{GAM}$ may not be exact, though the presence of the low frequency GAM will be unchanged. The detail of the formalism including the velocity change will be presented in a separate publication [33]

IV. Validity of the approximations and criticism to the previous results

In the pervious paper [32], we presented an extended formalism by using the drift kinetic equation. However, in the practical applications to specific problems we used an approximation, which correspond to the asymmetric expansions in the present paper. Therefore, the rather general formalism given there was not fully utilized. Typically, in the application to helical systems, the matching parameter, $\zeta_{m,n}$, tends to be small due to the large number of toroidal mode number n.

$$\zeta_{m,n} = \omega / k_{\parallel,m,n}\upsilon_T \quad \text{with} \quad k_{\parallel,m,n} = \frac{1}{B_0}(mB^{\theta} - nB^{\zeta}).$$

In order for the asymptotic expansion to be relevant, the solution to the dispersion relation has to satisfy $\zeta_{m,n}$>1 for consistency. In the previous sections, we just compared the results obtained with asymptotic and series expansions. Here, it is instructive to consider the two-helicity problem using full kinetic dispersion functions in order to clarify the applicability of these approximations.

For standard helical tori like CHS and LHD, the dispersion relation is obtained keeping dominant helical and toroidal ripples.

$$\zeta_{m,n} = -2\frac{\tilde{\tilde{\omega}}_G}{\left(k_{//,m,n}\upsilon_T\right)^2}\left(\begin{array}{l}\eta^2{}_{m,n}\left[Z_{GAM,1}\left(\zeta_{m,n}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{m,n}\right)\right] \\ +\eta^2{}_{M,N}\,\frac{k_{//,m,n}}{k_{//,M,N}}\left[Z_{GAM,1}\left(\zeta_{M,N}\right)+\frac{T_e}{T_i}Z_{GAM,2}\left(\zeta_{M,N}\right)\right]\end{array}\right) \tag{34}$$

where (m,n)=(1,0) stands for the tokamak type ripple and (M,N)=(2,8) stands for the dominant helical ripple; for example N=8 in CHS device.

In Fig.3, the j_{pol} and j_{geo}, (the left hand side and the right hand side of Eq.(34)) are plotted versus $\zeta_{m,n}$; $\zeta_{m,n}$ is mutually related to $\zeta_{M,N}$ with the following equation:

$$\zeta_{m,n} = \frac{k_{\parallel,M,N}}{k_{\parallel,m,n}} \zeta_{M,N} \sim \frac{N}{m} q \zeta_{M,N} \tag{35}$$

The parameters employed for Fig.3 (a),(b) and (c) are $(T_e / T_i, \eta^2, N) = (10, 2, 8)$, $(T_e / T_i, \eta^2, N) = (10, 3, 8)$, and $(T_e / T_i, \eta^2, N) = (10, 2.44, 8)$, respectively. In general, the geodesic current due to the helical ripple is small in amplitude and gives small contributions unless T_e / T_i is large reaching 3~10 (see Eq.(34)). This occurs, in practice, in the case of CHS [30]. In Fig 3(a), where the geometric factor η^2 is relatively small ($\eta^2 \sim 2$), the contribution of helical ripple is still trivial. In Fig 3(b), where the geometric factor η^2 is relatively large ($\eta^2 \sim 3$), the contribution of helical ripple plays an important role. Here, a jump of the frequency occurs from tokamak like GAM to helical GAM. Thus the GAM frequency does not always show monotonous shift for the changes of the various parameters. Fig.3(c) indicates that there could be 4 solutions of GAM for an intermediate special value of η^2 (η^2~2.44).

Conclusions:

Geodesic acoustic mode theory was extended to helical systems in the previous paper [32], where the drift kinetic equation was used to evaluate the kinetic effects properly. However, in the applications to practical problems to study the nature of the GAMs, an approximation was used which we find to limit the range of validity. In the present paper, we attempted to get rid of this approximation and use, in stead, full kinetic calculations. Two kinds of dispersion functions were introduced and their properties were investigated: In the low frequency regime, the direction of the geodesic current is reversed; the reversed current is regarded as neoclassical polarization current. A new low frequency GAM was found in a simple tokamak in the low frequency regime due to the presence of this mechanism.

The geodesic current is estimated to be smaller with the inclusion of the kinetic effects, and it turned out that the helical GAM occurs only under limited conditions. In the systems of mixed helicity, the properties of GAM jump from that of the tokamak type to that of the helical type as the helical geodesic curvature is increased. The helical geodesic curvature increases in CHS towards the edge of the plasma, the nature of GAM is therefore expected to transform spatially from tokamak type to helical type. Under certain conditions four GAM frequencies can appear, composed of one low frequency GAM and three tokamak-helical hybrids.

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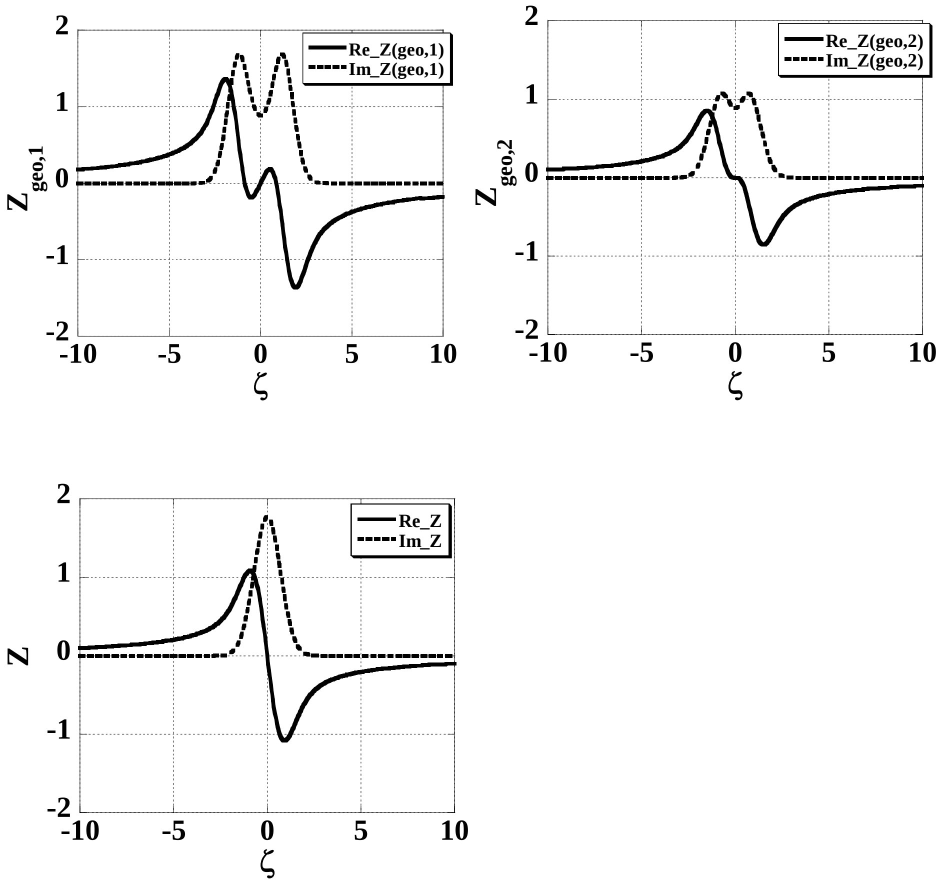


Fig.1 The comparison of the Three Z-functions Z(geo,1), Z(geo,2) and Z_p, defined by Eq.(14) Eq.(15) and Eq.(16) respectively. The Z(geo,1) is an increasing function of ζ in the visinity of $\zeta = 0$ revealing the unique response of ions due to the kinetic weighs in their drift.

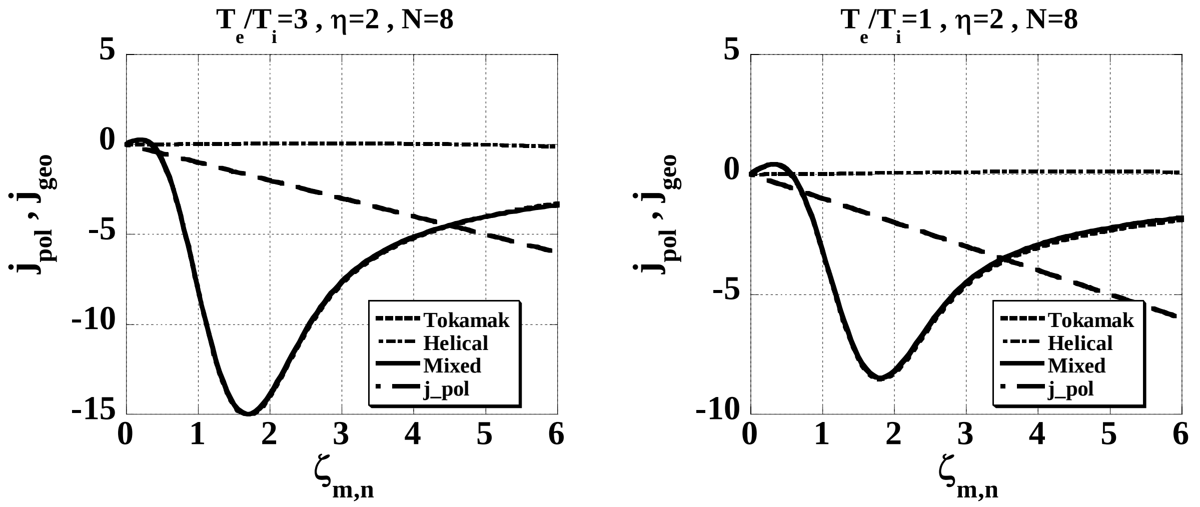
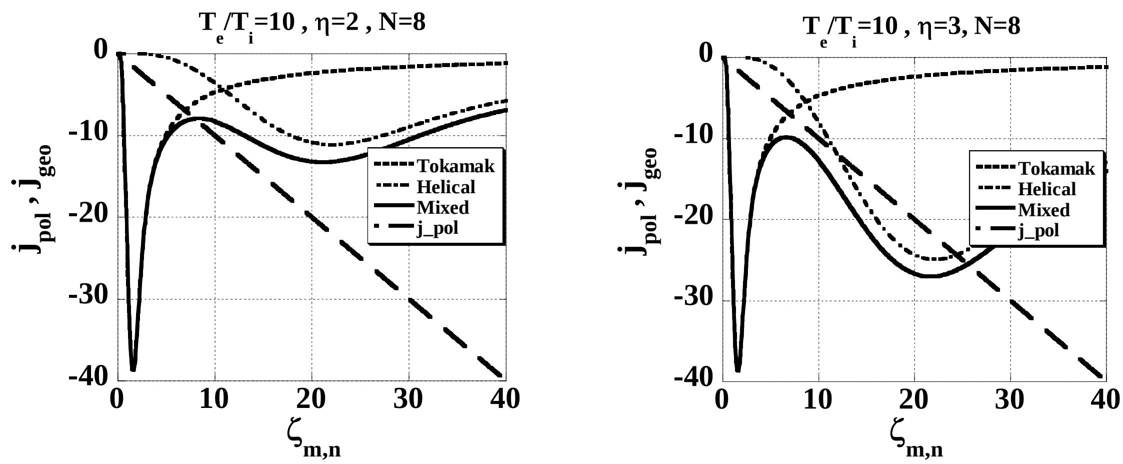
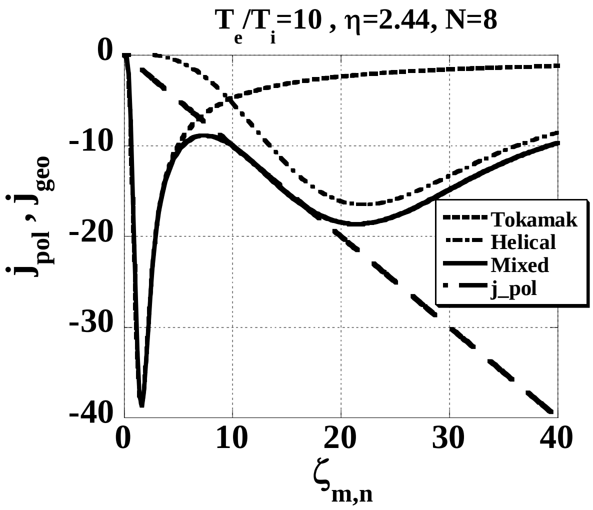


Fig.2 The polarization current versus the geodesic current versus $\zeta_{m,n}$, the oscillation frequency of GAM is given where they agree. In these Figures, two such frequencies are found: one at $\zeta_{m,n} = 0.7$ and the other at $\zeta_{m,n} = 3.5$. The larger solution is that which we found in the previous paper and the smaller solution is new. The new solution appears due to the reversed current in the low frequency range.



Figure,3 (a),(b)

The GAM frequencies are determined as the ones where j_{pol} equals j_{geo} : Parameters, $T_e / T_i = 10, \eta^2 = 2, N = 8$ were used in calculating Fig3.(a). Parameters, $T_e / T_i = 10, \eta^2 = 3, N = 8$ were used in calculating Fig3.(b). A large value of T_e / T_i magnifies the geodesic current. For the geometric factor, $\eta^2 > 3$, the GAM jumps to helical type GAM.



Figure,3(c)

The GAM frequencies are determined as the ones where j_{pol} equals j_{geo} : Parameters, $T_e / T_i = 10, \eta^2 = 2.44, N = 8$ were used in calculating Fig3(c). With these special parameters, a triple solution is found, as well as the low frequency GAM.