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K. Itoh et al.

NIFS-869 Oct. 2006
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1) National Institute for Fusion Science, Toki 509-5292, Japan
2) Research Institute for Applied Mechanics, Kyushu University, Kasuga 816-8580, Japan
3) Department of Nuclear Engineering, Kyoto University, Kyoto, 606-8501, Japan
4) Department of Physics, UCSD, San Diego CA 92093-0319, U.S.A.
e-mail contact of main author: itoh@nifs.ac.jp

Abstract. The role of zonal flows in the formation of the transport barrier in the helical plasmas is analyzed
using the transport code. A set of one-dimensional transport equations is analyzed, including the effect of zonal
flows. The turbulent transport coefficient is shown to be suppressed when the plasma state changes from the
weak negative radial electric field to the strong positive one. This bifurcation of the turbulent transport is newly
caused by the change of the damping rate of zonal flows. It is theoretically demonstrated that the damping rate of
zonal flows governs the global confinement in toroidal plasmas.

1. Introduction

The turbulence-driven transport and transport barriers are the key issues in fusion research. Main efforts have been focused on the understanding of improved confinement modes (such as the H-mode [1]). In these phenomena, the turbulent (anomalous) transport coefficient shows the steep gradient at a particular radius after the onset of the transition. One thread of thoughts to explain transport barriers is the structural transition of the profile of radial electric field $E_r$ and the suppression of turbulence by its gradient [2]. The bifurcation of the radial electric field in helical plasmas is influenced by the neoclassical ripple transport, and the resultant electric field interface (by which the radial domains with positive $E_r$ and negative $E_r$ are separated) was predicted to induce the internal transport barrier due to the shear of the radial electric field in the helical plasmas. The $E_r$-interface was found on the Compact Helical System (CHS), and the improvement of the electron confinement was found inside of the interface for $E_r$ [3] (hereafter called 'electron internal transport barrier', e-ITB). The appearance and the location of the $E_r$-interface were analyzed [4]. Observations on Wendelstein 7-AS [5], Large Helical Device (LHD) [6] and other confinement devices followed [7]. However, the essential issue of the e-ITB in helical devices has been unexplained; i.e., the turbulent transport coefficient was found to be suppressed not only near the interface (with the strong inhomogeneity of $E_r$) but also in the whole region of the strong positive $E_r$. Away from the transition radius, the gradient $dE_r/dr$ is not strong enough to suppress the turbulent transport. Fundamental problems remain unresolved. (In LHD, an internal diffusion barrier (IDB) is recently observed with the high gradient of the density in a super dense core plasma. In a core region, the high density of $4.5 \times 10^{20} \text{m}^{-3}$ and the temperature of 0.85keV are obtained. Physics of an IDB is beyond the scope of this article, and details of an IDB are discussed in [8].)

In this article, we study the role of zonal flows (ZFs) [9] in the formation of e-ITB. The turbulent transport coefficient, in which the screening influence of zonal flows is included, is shown to be reduced when the plasma state changes from the branch of weak negative $E_r$ to the strong positive $E_r$. This new transition of the turbulent transport is induced by the change of the damping rate of zonal flows, which is strongly influenced by the neoclassical ripple transport. We show the clear transport barrier in the electron temperature profile. The analytic results by use of the transport code are shown.

2. One-dimensional model for transport equations

In this section, the model equations used here are shown. The one-dimensional transport model is employed. The cylindrical coordinate is used and $r$-axis is taken in the radial cylindrical plasma in this article. The region $0 < \rho < 1$ is considered, where $a$ is the minor
radius and \( \rho = r/a \). The expression for the radial neoclassical flux associated with helical-ripple trapped particles is given in [10] which covers from the \( v_r \) regime to the \( 1/v_r \) regime, where \( v_j \) is the collision frequency for the species \( j \). The total particle flux \( \Gamma^i \) is written as \( \Gamma^i = \Gamma^{na} - D_T n' \), where \( \Gamma^{na} \) is the neoclassical flux associated with the helical-ripple trapped particles, and the prime denotes the radial derivative. Here, \( D_T \) is the turbulent (anomalous) particle diffusivity with the effect of the zonal flow. The effect of ZFs is discussed in the next section. The energy flux related with the neoclassical ripple transport, \( Q_j^{na} \), is obtained like the neoclassical particle flux. The total heat flux \( Q_j^i \) for the species \( j \) is written as \( Q_j^i = Q_j^{na} - n \chi{T'}_{j'} - 3 D_T n'T_j/2 \), where \( \chi_T \) is the anomalous heat diffusivity. A theoretical model for the anomalous heat conductivity is adopted and is explained later. The neoclassical diffusion coefficient for the electric field is expressed in [11]. The anomalous diffusion coefficient for the radial electric field is denoted by the parameter \( D_{EiT} \). The temporal equation for the density is

\[
\frac{\partial n}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma^\nu) + S_n, \tag{1}
\]

where the term \( S_n \) represents the particle source. The equation for the electron temperature is given as

\[
\frac{3}{2} \frac{\partial}{\partial t} (n T_e) = -\frac{1}{r} \frac{\partial}{\partial r} (r Q_e^i) - \frac{m_e}{m_i} \frac{n}{\tau_e} (T_e - T_i) + P_{he}, \tag{2}
\]

where the term \( \tau_e \) denotes the electron collision time and the second term in the right hand side represents the heat exchange between ions and electrons. The term \( P_{he} \) represents the absorbed power due to the ECRH heating. The temporal equation for the ion temperature is

\[
\frac{3}{2} \frac{\partial}{\partial t} (n T_i) = -\frac{1}{r} \frac{\partial}{\partial r} (r Q_i^i) + \frac{m_i}{m_e} \frac{n}{\tau_i} (T_e - T_i) + P_{hi}. \tag{3}
\]

The term \( P_{hi} \) represents the absorbed power of ions. The radial electric field equation in a nonaxisymmetric system is expressed by [11]

\[
\frac{\partial E_r}{\partial t} = -\frac{e}{\varepsilon_\perp} \sum_j Z_j \Gamma_j^{na} + \frac{1}{r} \frac{\partial}{\partial r} \left( \sum_j Z_j e (D_{Ej} + D_{EiT}) r \frac{\partial E_r}{\partial r} \right), \tag{4}
\]

where \( \varepsilon_\perp \) is the perpendicular dielectric coefficient which equals to \( \varepsilon_0 (1 + c^2 / \nu_A^2)(1 + q^2 / \sqrt{\varepsilon_\perp}) \). Here, \( \varepsilon_0 \) is the dielectric constant in vacuum, \( \nu_A \) is the velocity of the Alfvén wave, \( e_i \) is the toroidal ripple and \( q \) is a safety factor.

The source profiles are chosen here as follows. The particle source \( S_n \) is set to be \( S_n = S_0 \exp((r-a)/L_0) \), where \( L_0 \) is set to be 0.01m. This profile represents the peaking at the plasma edge of the particle source due to the ionization effect. The intensity, \( S_0 \), governs the average density, and is taken as a control parameter to specify the density in this article. The
radial profiles of the electron and ion heating terms, $P_{ne}$ and $P_{ni}$, are assumed to be proportional to $\exp(-r/(0.2a)^2)$ for the sake of the analytic insight. (Effect of the heating profile has been studied, and the assumption does not change the qualitative conclusion for the establishment of the internal transport barrier [4].)

The equations of density, temperature and electric field (1)-(4) are solved, with the prescribed source profiles, under the appropriate boundary conditions. We fix the boundary condition at the center of the plasma ($\rho = 0$) such that $n = T_e = T_i = E_r = 0$. For the diffusion equation of the radial electric field, the boundary condition at the edge ($\rho = 1$) is chosen as $\sum_j Z_j \Gamma^{m_j}_j = 0$.

This simplification is employed because the electric field bifurcation in the core plasma is the main subject of this study. The boundary conditions at the edge ($\rho = 1$), with respect to the density and temperature, are given by specifying the gradient scale lengths. We employ those expected in LHD: $-n / n' = 0.05m, -T_e / T_e' = -T_i / T_i' = 0.02m$ in this article. The machine parameters which are similar to those of LHD are set to be $R = 3.6m, a = 0.6m, B = 3T, \ell = 2$ and $m = 10$. In this case, we set the safety factor and the helical ripple coefficient as $q = 1/(0.4 + 1.2\rho^2)$ and $\epsilon_p = 2 \sqrt{1 - (2/(mq(0))) - 1} I_2(mR/R)$, respectively. Here, $q(0)$ is the value of the safety factor at $\rho = 0$ and $I_2$ is the second-order modified Bessel function.

3. Model of turbulent transport coefficients

The system of Eqs. (1)-(4) has been known to predict the structural bifurcation such that the internal transport barrier is formed when the heating power at the core is high enough in a low density regime. The improvement of the confinement was predicted in two ways: the turbulent transport is suppressed by $E_r'$ near the transition interface, and the neoclassical transport is strongly suppressed inside of the electric field interface. The suppression of the turbulent transport is limited near the interface, and its reduction in the entire core plasma ($\rho < \rho_c$) has not been explained, where the parameter $\rho_c$ shows the location of the $E_r$-interface.

In the absence of the zonal flow, we adopt the model for the turbulent heat diffusivity $\chi_{T0}$ based on the theory of the self-sustained turbulence due to the ballooning mode and the interchange mode, both driven by the current diffusivity [12,13]. The reduction of the anomalous transport due to the inhomogeneous radial electric field was reported in the toroidal helical system. The anomalous transport coefficient for the temperatures is given as $\chi_{T0} = \chi_0 / (1 + G \omega_{pe}^2), (\chi_0 = F(s, \alpha) a^2 c^2 v_A^2 / (\omega_{pe}^3 q R))$, where $\omega_{pe}$ is the electron plasma frequency. The factor $F(s, \alpha)$ is the function of the magnetic shear $s$ and the normalized pressure gradient $\alpha$, defined by $s = r q' / q$ and $\alpha = -q' R \beta'$. For the ballooning mode turbulence (in the system with a magnetic well), we employ the anomalous thermal conductivity $\chi_{T0,BM}$. The details about the coefficients $F(s, \alpha), G$, and the factor $\omega_{pe}$, which stands for the effect of the electric field shear, are given in [13] in the ballooning mode turbulence. In the case of the interchange mode turbulence for the system of the magnetic hill [12], we adopt the anomalous thermal conductivity $\chi_{T0,IM}$. The details about $F, G$, and the factor $\omega_{pe}$ in the case of the interchange mode were given in [12]. The greater one of these two diffusivities is adopted as $\chi_{T0} = \max(\chi_{T0,BM}, \chi_{T0,IM})$. 

The zonal flows (at nearly zero frequency) are generated by the fluctuations and strongly influence the turbulent transport. The damping rate of zonal flows, \( \nu_{\text{damp}} \), controls the turbulent transport. The damping of zonal flows is caused by the collisional process and by the self-nonlinearity of zonal flows [9]. In the toroidal helical plasmas, the collisional process remains to be important even in the regime of \( v_* < 1 \) \( (v_* = v_q R / (e v_{\text{th}i}) \)). In the case that the collisional damping rate \( \nu_{\text{damp}} \) is small, the heat diffusivity \( \chi_T \) has been derived considering the screening by zonal flows [9]. Whether the zonal flows are excited or not is judged by comparing \( \chi_{\text{T}0} \) (which is given in the absence of zonal flows) with the quantity

\[
\chi_{\text{damp}} \geq k_r^2 q_r^{-2} k_\theta^{-2} \nu_{\text{damp}},
\]

where \( q_r \) is the wave number of zonal flows, \( k_\theta \) is the poloidal wavenumber and \( k_\perp \) are the perpendicular wavenumber of the microscopic fluctuations, respectively. When the turbulence is weak and \( \chi_{\text{T}0} \) is smaller than \( \chi_{\text{damp}} \), zonal flows are not excited and one has \( \chi_T = \chi_{\text{T}0} \). If the condition \( \chi_{\text{damp}} < \chi_{\text{T}0} \) is satisfied, zonal flows are excited and the fluctuation level of the electric field \( \bar{E} \) is controlled as

\[
|\bar{E}|^2 \propto \nu_{\text{damp}} / \omega, \quad \text{where} \quad \omega \text{ is the drift frequency}.
\]

Then the turbulent diffusivity \( \chi_T \) is reduced as

\[
\chi_T = \sqrt{\chi_{\text{T}0} \chi_{\text{damp}}}.
\]

A fitting formula is often employed as

\[
\chi_T = \sqrt{\chi_{\text{T}0}} \min(\sqrt{\chi_{\text{T}0}}, \sqrt{\chi_{\text{damp}}})
\]

to include the effect of zonal flows in the transport codes.

This dependence of \( \chi_T \) on the damping rate of zonal flows, in Eq. (6), explains the improved confinement in the e-ITB region of toroidal helical plasmas. The neoclassical part of the radial current \( J_r^{\text{NEO}} \) in helical plasmas is induced by the ripple transport and has a nonlinear dependence on \( E_r \), where the superscript NEO stands for the neoclassical ripple transport. The damping rate of zonal flows due to the dependence of the radial current on the electric field is given by the neoclassical ripple transport as

\[
\nu^{\text{NEO}} = (\partial J_r^{\text{NEO}} / \partial E_r) / \epsilon_\perp.
\]

We obtain the damping rate \( \nu_{\text{damp}} \) as

\[
\nu_{\text{damp}} = \min(1, v_*) \frac{v_{\text{th}i}}{q R} + \nu^{\text{NEO}},
\]

where \( v_{\text{th}i} \) is the thermal velocity of ions. The first term \( \min(1, v_*) \) takes the smaller value between the unity and \( v_* \), which represents the transports in the plateau and banana regimes, respectively. The second term comes form the ripple transport with the dependence of the radial electric field. Owing to the dependence of \( \nu_{\text{damp}} \) on \( E_r \), \( \nu_{\text{damp}} \) of Eq. (7) is large in the ion root branch (i.e., the weak negative solution of \( E_r \) that satisfies \( J_r^{\text{NEO}}(E_r) = 0 \)), while it is small in the electron root branch of e-ITB (i.e., the strong positive \( E_r \) solution of \( J_r^{\text{NEO}}(E_r) = 0 \)). The turbulent transport coefficient becomes smaller when the strong positive radial electric field is established in e-ITB [15], when we consider the role of zonal flows in the e-ITB formation in helical plasmas. The bifurcation of \( E_r \) itself induces the transition of
turbulent transport in the bulk of the plasma column as well as at the interface of the electric field.

For simplicity, the value for the anomalous diffusivities of the particle is set as \(D_T = 10 \text{m}^2/\text{s}\). The essence of the results shown later does not change due to the value of \(D_T\). Therefore, the value of \(D_T\) is set to be constant spatially and temporally. We also set \(D_{TE} = \chi_T\) in order to examine the variation of the typical length for the electric field shear at the transition point.

4. Results of the Analysis

The reduction of the turbulent transport in the entire region of the strong positive \(E_r\) is quantitatively demonstrated by use of the transport code analysis. The one-dimensional transport analysis for LHD-like plasma has been performed and the mean profiles of \(E_r\), \(T_e\), \(T_i\) and \(n\) are solved using the Eqs. (1), (2), (3) and (4) and adapting Eqs. (5) and (6) as the heat diffusivities. In this analysis, the thermal diffusivity is given as the sum of \(\chi_T\) and neoclassical transport. (In this calculation, \(\chi_{T0}\) is given based on the nonlinear current-diffusive interchange mode which was found relevant in preceding analysis [4]. Here, the screening of \(\chi_T\) near the edge is not taken into account, because the study is focused in the core.) An example is taken from the plasma which is sustained by electron cyclotron resonance (ECR) heating. In order to set the line-averaged temperature of electrons to be around \(T_e = 1.6\text{keV}\) (\(T_e\) at the center, \(T_e(0) = 4.4\text{keV}\) ) and the line-averaged density to be around \(n = 2 \times 10^{19} \text{m}^{-3}\), the absorbed power of electrons is set to be 1MW and the coefficient \(S_0\) is taken as \(7 \times 10^{-22} \text{m}^{-3} \text{s}^{-1}\), for the choice of the above mentioned anomalous transport coefficients. The line-averaged ion temperature \(T_i\) is chosen to be about \(T_i = 0.9\text{keV}\) (\(T_i\) at the center, \(T_i(0) = 2.5\text{keV}\) ), where the absorbed power of ions is taken as 500kW. When we evaluate \(\chi_{\text{damp}}\), we employ an estimate \(k_\perp q_r^{-2} k_\theta^{-2} \rho_i^{-2} \sim 50\). [14]

The profile of the stationary radial electric field is demonstrated in FIG. 1 by the solid line, including the effect of zonal flows (ZFs). To show the reference, the radial profile of \(E_r\) without the effect of the zonal flow is also shown in FIG. 1 by the dashed line. The parameter
\( \rho_T \) represents the location of the transition for the strong positive \( E_r \) to the negative \( E_r \). In both cases, the \( E_r \)-interface is established at \( \rho_T = 0.3 \) and the steep gradient of \( E_r \) can be obtained. Therefore, the improvement near the transition point can be obtained in a narrow region even in the case without the effect of the zonal flows. Maxwell’s construction chooses the consistent solution for \( E_r \) from the multiple local solutions (circle marks in FIG. 1 with the effect of zonal flows) for a radial point. The local solution for \( E_r \) satisfies the ambipolar condition which is derived from the obtained density and the temperature profiles. It is found that \( E_r \) is strongly positive for \( \rho < \rho_T \). Including the effect of zonal flows, the stronger positive \( E_r \) (owing to the increment of the temperature gradient) is shown in the region \( \rho < \rho_T \) in FIG. 1. Profiles of the electron temperature with and without the zonal flow effects are shown in FIG. 2. We obtain the steeper gradient in the \( T_e \) profile in the case with the effect of zonal flows, in comparison with the case without the effect of zonal flows. We also show the profiles of the ion temperature and the density with and without the effect of zonal flows in FIG. 3 and FIG.4, respectively. The clear change of the gradient for the ion temperature can be obtained in the case with the effect of zonal flows. In the density profile, the significant effect of zonal flows cannot be found, because the particle diffusivity is set to be constant as \( D_T = 10 \text{m}^2/\text{s} \) in both cases with and without the effect of zonal flows. A weak hollowness of the density \( (dn/dr > 0) \) is generated. This is due to the outward neoclassical particle flux, which is driven by temperature gradient and radial electric field.

A profile of the total diffusivity with the zonal flow effect (solid line) is shown in FIG. 5. The clear reduction of the total diffusivity is shown in the region \( \rho < \rho_T \) compared with that in the region \( \rho > \rho_T \). The total thermal diffusivity of electrons \( \chi^\text{total}_e \) represents the sum of the turbulent part and neoclassical part. Therefore, we can obtain the clear transport barrier in FIG.2 (\( T_e \) profile). It is found that the condition and \( \chi_T \) is reduced there (thin broken line), due to the smaller damping of zonal flows. [In the reference simulation using bare fluctuations (without the effects of zonal flows), the thermal diffusivity \( \chi_{T0} \) has a dip near the interface, but does not show the noticeable reduction for \( \rho < \rho_T \) (dashed line); \( \chi_{T0} \) increases in the core owing to the higher temperature (thin dotted line) in FIG. 6.] The screened transport coefficient is close to what has been reported from LHD experiments [16]. Outside the transition point \( \rho_T \), where the ion root branch appears, \( v_{\text{damp}} \) becomes large and the strong
zonal flows are not excited. Thus, the reduction is not expected for the region $\rho > \rho_\tau$ in this example. We also show that the neoclassical parts for the heat diffusivities of electrons $\chi_{e\,\text{NEO}}$ and ions $\chi_{i\,\text{NEO}}$ in FIGS. 7 and 8, respectively (with the dashed line) in the case with the effect of zonal flows. In the inner region ($\rho < \rho_\tau$), the turbulent transport is found to be dominant compared to the neoclassical transport in both electron and ion cases. Specially, the neoclassical transport of ions in the inner region ($\rho < \rho_\tau$) is suppressed due to the strong $E_r$.

5. Summary

We have studied the role of zonal flows in the e-ITB formation of toroidal helical plasmas. It was found that the neoclassical ripple transport can enhance the turbulent transport, through its impact on the damping of the zonal flows. The bifurcation of the radial electric field (from the state with weak and negative $E_r$ to that with strong and positive $E_r$) was found to induce the transition of the turbulent transport coefficient. The electron ITB is established by the mechanisms of (i) the bifurcation of the radial electric field via the neoclassical process and the reduction of neoclassical energy transport, (ii) the establishment of the electric field interface that quenches the turbulence, and (iii) the reduced damping of zonal flows which causes the suppression of the turbulent transport. When the electron internal transport barrier of LHD appears, the results of the transport code analysis show that the total heat diffusivity $\chi_r$ in the entire inner region ($\rho < \rho_\tau$) is reduced. The effect of zonal flows on the anomalous transport is investigated. The transport reduction is obtained in the wide region for $E_r > 0$. 

![FIG. 5. Profiles of the heat diffusivity by use of the transport code analysis of LHD-like plasma with the effect of the zonal flow.](image1)

![FIG. 6. Profiles of the heat diffusivity without the effect of the zonal flow.](image2)

![FIG. 7. Profile for the neoclassical heat diffusivity of electrons with dashed line as the result of the transport code analysis.](image3)

![FIG. 8. Profile for the neoclassical heat diffusivity of ions with dashed line. The reduction of the neoclassical transport of ions is obtained in the region $\rho < \rho_\tau$ due to the strong positive $E_r$.](image4)
(p<\rho_T) in conjunction with the internal transport barrier. This is demonstration that the change of the collisional damping of zonal flows can cause the transition in the turbulent transport. The experimental confirmation is given on CHS. It is reported that the fluctuation energy is transferred into zonal flows when the damping rate \nu_damp becomes weak. Details will be discussed in [17].

The result of this article provides further understanding of confinement in toroidal helical plasmas. The important finding is that the reduction of the effective helical ripple ratio e_h causes the reduction of \chi_damp (thus, of \chi_T) even in the branch of the ion root. This finding can explain the observations on LHD. In the collisionless plasmas of LHD (ion root branch), if the smaller the helical ripple, the lower the anomalous transport. The study of this article is not limited to toroidal helical plasmas. The control of the damping rate of zonal flows for further improvement of plasma confinement has been investigated [18]. The wide extension of the present work is possible, and is left for the future work.

Authors wish to acknowledge Dr. K. Hallatschek for useful discussions and Prof. O. Motojima for the encouragement. This work was partly supported by the Grant-in-Aid for Specially-Promoted Research (16002005) and the Grant-in-Aid for Scientific Research (15360495). This work is also partly performed with the support and the auspices of the NIFS Collaborative Research Programs, Nos. NIFS06KLDD008 and NIFS06KNXN062 and of the Research Institute for Applied Mechanics of Kyushu University.

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