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Transport Modeling of Edge Plasma in an m/n=1/1 Magnetic Island

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Recent Large Helical Device experiments show that the transport modeling based only on the fluid description is not sufficient for expressing transport phenomena in/around an $m/n = 1/1$ magnetic island formed at the edge, and that neoclassical effect should be considered in order to understand the edge transport phenomena, where m is the poloidal mode number and n the toroidal mode number. To study the edge transport phenomena, we develop a new transport simulation code, KEATS, treating the neoclassical theory in/around the island. The code is programmed by expanding a well-known Monte-Carlo particle simulation scheme based on the δf method for taking account of the neoclassical effect. The guiding center distribution function of plasma is separated into a local Maxwellian background $f_{\rm M}$ and a kinetic part δf considered as a small perturbation from $f_{\rm M}$. To treat evolution of the kinetic part δf, we solve the linearized drift kinetic equation of the first order. In the modeling, evolution of the background $f_{\rm M}$ described by fluid equations is also considered self-consistently with the kinetic part δf. In this paper, we present the modeling used in the KEATS code, and show the simulation results in a test calculation.

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1 Introduction

Recent Large Helical Device experiments show that the transport modeling based only on the fluid description is not sufficient for expressing transport phenomena in/around magnetic islands formed at the edge. For example, it is observed that an $m/n = 1/1$ magnetic island, which is formed at the edge by using external coils, is healed [1, 2, 3, 4], where m and n are the poloidal and toroidal mode numbers respectively, and in the experiments the island temperature is $\gtrsim$500 eV and the island density $\sim 10^{19}$ m^{-3}. The island width depends on plasma pressure in the edge region, i.e., the width reduces as the pressure increases. The experimental results suggest that a current depending on the pressure is expected to explain the healing [4]. We have two conventional candidates of such a current; one is the Pfirsch-Schlüter current and the other is the bootstrap current. In results of a previous simulation study based on the fluid description [5], it is shown that the healing phenomenon is not explained by the Pfirsch-Schlüter current only. Thus, the bootstrap current around the island is a possible mechanism related to the pressure. However, we have the following difficulty of this argument: once the island is shrunk, the $m/n = 1/1$ mode of the bootstrap current density disappears because the pressure gradient becomes low [4]. Thus, if the bootstrap current triggers the healing, it cannot maintain the healing. There is a possibility that the healing is explained by additionally considering other current related to ergodic behavior of field lines around the X-point of the island [6], and even if the island is shrunk the current may maintain the healing because the ergodic region always exists around the island in the stellarator plasma confinement. In this case, the current is caused by asymmetry of the distribution function in velocity space, because of the existence of the island involving the ergodic region and the diffusion caused by the Coulomb collisions. From these discussions, we see that the kinetic treatment is required for the edge plasma.

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The above presumption implies that when the island is formed at the edge region, ballistic motions in the island and ergodic regions can play important roles even in the edge transport phenomena. Moreover, the Coulomb collision causes the transition between a passing particle orbit and a trapped particle orbit in toroidal and helical ripples (localized and/or blocked particle orbits) in the three dimensional field line structure. According to the ratio of the mean free path to the connection length, the orbits of the particles which mainly contribute the particle, momentum and energy fluxes in/around the island are expected to vary [7, 8]. Here, the connection length is given as a length along a field line connecting the inside (i.e. the side facing the core) to the outside of the island separatrix. With considering the above points, we treat the transport phenomena by expanding neoclassical theory. There is no established theory describing radial transport in/around magnetic islands. Then, to study the edge transport phenomena, we need a modeling of such an edge plasma, and develop a new transport simulation code without an assumption of existence of nested flux surfaces; the code is named "KEATS" [9]. The code is programmed by expanding a well-known Monte-Carlo particle simulation scheme based on the δf method [10, 11]. In this paper, we present the modeling used in the KEATS code, and show the simulation results of the code in a test calculation.

2 Modeling of edge plasma in the KEATS code

In the previous section, it is pointed out that neoclassical effect can play important roles even in the edge transport phenomena, when the island is formed at the edge. We consider that the guiding center distribution function of plasma f is separated into an equilibrium-like background f_0 and a kinetic part δf of the distribution, i.e., $f = f_0 + \delta f$, where the kinetic part δf is considered as a small perturbation from f_0. Approximately, the edge plasma can be treated as fluid, thus the background is described by fluid equations. Then the Maxwellian background is allowed. Thus, the zeroth-order distribution function f_0 is given as a local Maxwellian distribution $f_0 = f_{\mathrm{M}}(t, \boldsymbol{x}, \xi, v) = n\{m/(2\pi T)\}^{3/2}\exp\{-mv^2/(2T)\}$, where $\xi = v_{\|}/v$ is the cosine of the pitch angle, $v_{\|} = \boldsymbol{v}\cdot\boldsymbol{b}$, $\boldsymbol{b} = \boldsymbol{B}/B$ the unit vector along a field line, $\boldsymbol{B}$ a magnetic field, $B = |\boldsymbol{B}|$, $v = |\boldsymbol{v}|$, $n = n(t, \boldsymbol{x})$ the density, m the particle mass, and $T = T(t, \boldsymbol{x})$ the temperature. Applying the decomposition $f = f_{\mathrm{M}} + \delta f$ to the drift kinetic equation, we have the following equation of the kinetic part δf:

$$\frac{\mathrm{D}}{\mathrm{D}t}\delta f - S(\delta f) = -\left\{\frac{\mathrm{D}}{\mathrm{D}t} f_{\mathrm{M}} - C_{\mathrm{F}} f_{\mathrm{M}} - S(f_{\mathrm{M}})\right\}, \tag{1}$$

where the operator $\mathrm{D}/\mathrm{D}t$ is defined as $\mathrm{D}/\mathrm{D}t := \partial/\partial t + (\boldsymbol{v}_{\|} + \boldsymbol{v}_{\mathrm{d}})\cdot\nabla + \boldsymbol{a}\cdot\partial/\partial\boldsymbol{v} - C_{\mathrm{T}}$, $\boldsymbol{v}_{\|} = v_{\|}\boldsymbol{b}$ is the parallel velocity, $\boldsymbol{v}_{\mathrm{d}}$ the drift velocity of guiding center motion, $\boldsymbol{a}$ the acceleration, and $S(f)$ the source/sink term assuming $S(f) = S(f_{\mathrm{M}}) + S(\delta f)$. Note that the equation (1) includes the drift kinetic equation of the zeroth-order, i.e. the equation of f_{M}, which leads to the fluid equations describing evolution of f_{M}. The test particle collision operator C_{T} is given, for simplicity, as

$$C_{\mathrm{T}} = \nu_{\mathrm{d}}\mathcal{L} = \frac{\nu_{\mathrm{d}}}{2}\frac{\partial}{\partial\xi}\left(1-\xi^2\right)\frac{\partial}{\partial\xi}, \tag{2}$$

and it can be implemented numerically by random kicks in velocity space, which represents the Coulomb scattering process, where ν_{d} is the deflection frequency. The operator C_{F} is the field particle collision term, which represents local momentum conservation;

$$C_{\mathrm{F}} = \nu_{\mathrm{d}}\,\frac{m}{T}\,\boldsymbol{v}\cdot\boldsymbol{u}_0 \;\text{ and }\; \boldsymbol{u}_0 = \int \mathrm{d}^3v\,\nu_{\mathrm{d}}\,\boldsymbol{v}\,\delta f \Big/ \int \mathrm{d}^3v\,\nu_{\mathrm{d}}\frac{mv^2}{3T}f_{\mathrm{M}}. \tag{3}$$

Note that the operator C_{F} is needed to treat accurately the parallel transport. To solve Eq. (1) by Monte-Carlo techniques, we adopt the two-weight scheme of the δf formulation [10, 11]. In evolution of the δf part, the background f_{M} is fixed under assumptions that evolution of the background is extremely slow compared to the frequent change of δf and the background is in a quasi steady-state from the viewpoint of the δf part.

In the drift kinetic equation (1), we can consider a fluctuating electric field, i.e. the effect of $\tilde{\boldsymbol{v}}_{\mathrm{d}} = \tilde{\boldsymbol{E}}\times\boldsymbol{B}/B^2$, where $\tilde{\boldsymbol{E}} = -\nabla\tilde{\phi}$ is a fluctuating electric field, and $\tilde{\phi}$ is interpreted as a random function in the present paper. We assume that the potential of the fluctuating electric field is very small, $|e\tilde{\phi}/T| \ll 1$, and that $\tilde{\boldsymbol{E}}$ appears only in

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the drift velocity in the drift kinetic equation of the first order, where e the charge. Thus, through the terms given by the δf part in the fluid equations, the background should be influenced by the fluctuating field.

The relaxation of background is affected by complex transport in typical space-time scale of the kinetic part δf. Slow evolution of the background $f_{\rm M}$ described by the fluid equations is considered self-consistently with the kinetic part δf. For simplicity in the first step of the simulation studies, we treat only the heat balance equation, and assume that 1) the magnetic field is fixed as $\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{x})$ in the relaxation, 2) the density profile is set as a given function (a constant in the present paper), and 3) the mean velocity $\boldsymbol{V}$ is negligibly small in the Maxwellian background. Then we have

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_\alpha T_\alpha\right) + \nabla\cdot\boldsymbol{Q}_\alpha = S_\alpha(t,\boldsymbol{x}), \tag{4}$$

where $\alpha = {\rm i},{\rm e}$ means a particle species and S_α the source/sink of the heat. For simplicity, $\boldsymbol{J}\cdot\boldsymbol{E}$ is assumed to be neglected in Eq. (4). The energy flux $\boldsymbol{Q}$ is given as

$$\boldsymbol{Q}(t,\boldsymbol{x}) = \overline{\int {\rm d}^3v\,\frac{1}{2}mv^2(\boldsymbol{v}_\parallel + \boldsymbol{v}_{\rm d} + \tilde{\boldsymbol{v}}_{\rm d})(f_{\rm M}+\delta f)} = \overline{\int {\rm d}^3v\,\frac{1}{2}mv^2(\boldsymbol{v}_\parallel + \boldsymbol{v}_{\rm d} + \tilde{\boldsymbol{v}}_{\rm d})\delta f}, \tag{5}$$

where $\overline{\cdots}$ means the time-average, and the averaging time is longer than the typical time-scale of δf. In the treatment of the background evolution, the modeling of a fluctuating field is important because the fluctuating field strongly affects the transport phenomena. The modeling should be based on stochastic approach to the flow caused by the fluctuating field; e.g. see Ref. [12]. A fluid equation in a steady-state corresponds to a stochastic differential equation described as ${\rm d}X_t^i = \gamma U^i(\boldsymbol{X}_t){\rm d}t + c_j^i(\boldsymbol{X}_t){\rm d}W_t^j$ and $i,j = 1,2,3$ [13, 14], where γ is a constant (e.g. $\gamma = 5n/2$ for the heat balance equation), $\boldsymbol{U} = (U^1,U^2,U^3)$ is a flow in a steady-state, $D^{ij} = c_k^i\delta^{k\ell}c_\ell^j$ is a diffusion coefficient, $\boldsymbol{X}_t = (X_t^1,X_t^2,X_t^3)$ is a diffusion process, and $\boldsymbol{W}_t = (W_t^1,W_t^2,W_t^3)$ is a Brownian process. It is assumed that a fluctuating flow is represented as $\tilde{\boldsymbol{U}}(t,\omega) = \boldsymbol{U}(\boldsymbol{Y}_t) + $ noise, and that a fluid particle motion is described by an Itô process ${\rm d}Y_t^i = \gamma\tilde{U}^i(t,\omega){\rm d}t + c_j^i(\boldsymbol{Y}_t){\rm d}W_t^j$ instead of the process $\boldsymbol{X}_t$, where ω is a label of a fluid particle, "noise" is a random function having zero mean, and the definition of an Itô process is given in Ref. [15]. It is known that an Itô process $\boldsymbol{Y}_t$ coincides in law with a diffusion process $\boldsymbol{X}_t$ if and only if ${\rm E}^{\boldsymbol{x}}[\tilde{\boldsymbol{U}}(t,\omega)|\mathcal{P}_t^Y] = \boldsymbol{U}(\boldsymbol{Y}_t)$ [15], where $\boldsymbol{X}_0 = \boldsymbol{Y}_0 = \boldsymbol{x}$ is a starting point of a random walker (i.e. a fluid particle), $\mathcal{P}_t^Y$ is the σ-algebra generated by the set $\{\boldsymbol{Y}_s; 0 \le s \le t\}$, and ${\rm E}^{\boldsymbol{x}}[\cdots|\mathcal{P}_t^Y]$ denotes the conditional expectation with respect to $\mathcal{P}_t^Y$. From this viewpoint, the flux should be evaluated as the average; e.g. we can neglect the term relating with $\tilde{\boldsymbol{v}}_{\rm d}f_{\rm M}$ in Eq. (4).

The Monte-Carlo simulation code, KEATS, based on Eqs. (1)-(5) is programmed in an Eulerian coordinate system, i.e. so-called helical coordinates [5], thus the code does not need magnetic flux coordinates. The code solves iteratively the kinetic part of Eq. (1) and the fluid part of Eq. (4). In the calculation of obtaining a quasi steady-state of δf in Eq. (1), the background $f_{\rm M}$ is fixed. On the other hand, the fluid equation (4) is solved to obtain slight evolution of the background $f_{\rm M}$ under an assumption that contributions of the δf part, e.g. $\boldsymbol{Q}$, are fixed because the contributions are insensible to small change of $f_{\rm M}$. Solving iteratively the kinetic and fluid parts, we can find steady-state transport of the edge plasma.

3 Simulation results of the KEATS code in a test calculation

For a test calculation of the KEATS code, we use a magnetic configuration which is formed by adding an $m/n = 1/1$ island component into a simple tokamak field, where the major radius of the magnetic axis $R_{\rm ax} = 3.6$ m, the minor radius of the plasma $a = 1.0$ m and the magnetic field strength on the axis $B_{\rm ax} = 3.0$ T. Hereafter, it is called the "test configuration." The Poincaré plots of magnetic field lines on a poloidal cross section are shown in Fig.1. In the test calculation, the effect of neutrals and the effect of electric field are neglected for simplicity. In the KEATS code, the number of test particles is $N_{\rm TP} = 16{,}000{,}000$.

To investigate effect of the existence of the island on the transport phenomena, first of all, we evaluate the ion energy flux $\boldsymbol{Q}_{\rm i}$ in a fixed temperature profile given as $T_{\rm i} = T_{\rm ax}\{0.02 + 0.98\exp[-4(r/a)^{2.5}]\}$ with $T_{\rm ax} = 2$ keV and $r = \sqrt{(R-R_{\rm ax})^2 + Z^2}$, which neglects the existence of the island; the background is fixed in this calculation. The density profile is set homogeneous, $n_{\rm i} = {\rm const.} = 1\times 10^{19}$ 1/m^3. We calculate the energy

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flux in two cases, i.e., in the configurations (a) without islands (the simple tokamak field) and (b) with the $m/n = 1/1$ island (the test configuration). The radial profile of the energy flux can be estimated from the KEATS computations and is shown in Fig.2. For simplicity, the radial energy fluxes are given neglecting the existence of the island, because we have no magnetic coordinate system including several magnetic field structures as the island and the core region. The energy flux $\boldsymbol{Q}_\mathrm{i}$ is averaged over concentric circular shell region in the whole toroidal angles as if there were nested flux surfaces. From the results, we find that the energy flux is strongly affected by the magnetic island. In the case (a), the radial energy flux has a gentle profile (solid circles). This result is agreed with "FORTEC-3D" code [16] which uses magnetic flux coordinates. On the other hand, in the case (b), we see that there is obvious difference compared with the case (a). In the island region, we obtain that the flux has a steeple peak (solid triangles); the flux is considered to include the effect of convective flow. This result suggests that the flux arises to make the temperature flat in the region.

Next, we solve iteratively the kinetic and fluid parts, and obtain a temperature profile in a steady-state. The source/sink term is represented by fixing the distribution f in the outside of the "calculation region" of $r = 0.5 \sim 0.95$ m. Figure 3 shows the temperature profile along the line of $Z = 0$ in Fig.1, where the initial profile is given as solid squares and the profile in the steady-state is given as the solid line. Around the O-point of the island, we see the shoulder of the profile. On the other hand, around the X-point, the profiles is very complicated because there is the edge of the island. In the ergodic region, the temperature becomes very low, but in the island the temperature remains higher compared to one in the ergodic region.

4 Summary

We have been developing the new neoclassical transport code, KEATS, to study the transport phenomena in the island. The KEATS code has the two parts solving 1) a steady-state of the kinetic part of distribution and 2) a steady-state of the background. We applied the code to the edge of the test configuration, and confirmed the relaxation of ion temperature profile in the island region. More detailed analysis is needed to understand the transport phenomena in the island, e.g., we should evaluate the flux across the magnetic flux surfaces in the island. Then, we may need to label magnetic flux surfaces using our labeling technique of the flux surfaces [17]. We are going on the detailed analysis. The results will be reported in near future.

In the evolution of the background, although the density n and the mean velocity $\boldsymbol{V}$ may evolve for accurate treatment, we consider only the heat balance equation in this paper. The part solving the fluid equations in the KEATS code is required to be improved. Moreover, in order to estimate net current and calculate MHD equilibrium including the current, we need further development of the code treating electron transport.

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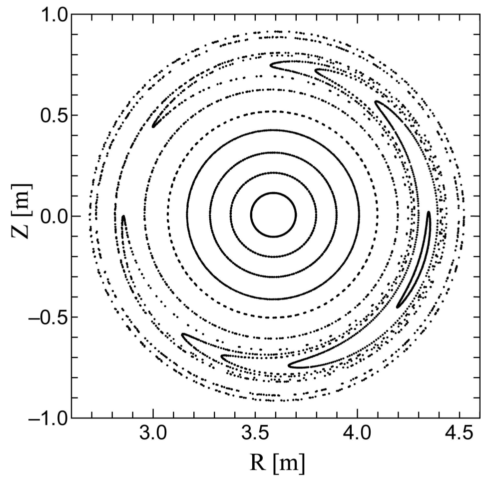

Fig. 1 Poincaré plots of magnetic field lines on a poloidal cross section in the test configuration.

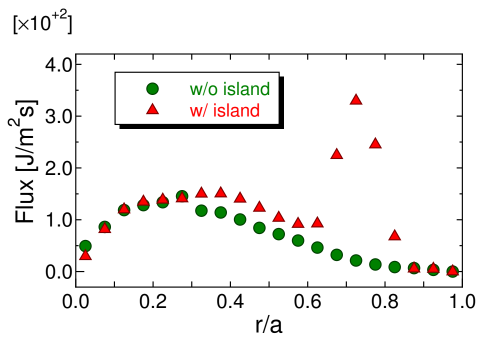

Fig. 2 Radial profiles of ion energy flux in the simple tokamak field (solid circles) and in the test configuration (solid triangles), where $r = \sqrt{(R - R_{\rm ax})^2 + Z^2}$.

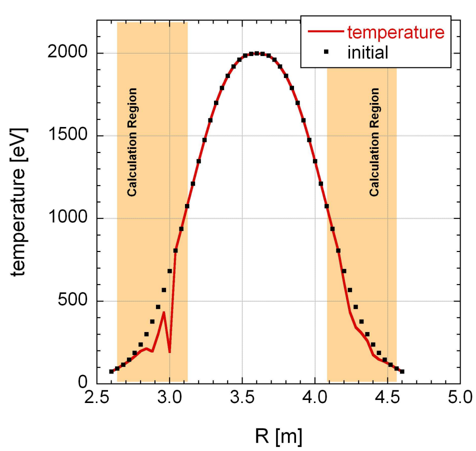

Fig. 3 Temperature profile along the line of $Z = 0$ in Fig.1, where the initial profile is given as solid squares and the profile in the steady-state is given as the solid line. The distribution f is fixed in the outside of the calculation region.

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