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Determination of uncertainties in the machine variables of LHD and CHS for confinement time scaling

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Abstract

Detailed descriptions of the uncertainties in measurements of the effective minor radius, electron density and the diamagnetic energy content together with the uncertainty of the estimated absorbed power are presented for LHD and CHS. Data entries of both devices to the International Stellarator Confinement Database [ISCDB] are examined to study the impact of uncertainties on scaling expressions of the energy confinement time. It is proposed to employ Bayesian inference as a statistical tool for the determination of the scaling exponents.

Keywords: energy confinement time, international stellarator confinement database, Bayesian probability theory, confinemet scaling, uncertainty of data

Introduction

The International Stellarator Confinement Data-Base (ISCDB) [ISCDB] contains global variables of discharges from various stellarators, among these LHD and CHS. In order to classify data values against each other and to compare machines a thorough discussion of the uncertainties of the entries is inevitable. For example neglecting the uncertainties of data in the regression method employed to derive a scaling relation is equivalent to assign same weight to every entry in a data-base. However, the diagnostics which measure these data, i.e. machine variables, may perform differently among the fusion devices, and, moreover, for a machine itself the conditions may change over the years. This should be given credit to in the analysis of the data. A further reason for the consideration of the uncertainties originates from the fact that the uncertainties of some machine variables, e.g. minor radius or absorbed heating power, are of comparable size to the uncertainty of the quantity of interest, e.g. plasma energy content. Ordinary least squares fitting fails in this case, because it focuses only on the deviations between response variable and model value but does not incorporate uncertainties of the input variables. An approach to overcome this problem has been performed for tokamak scaling by an errors in variables technique [Kardaun et al.(1989), Cordey et al.(2004)]. For the stellarators a discussion of the uncertainties of the $\bar{\iota} = 1/3$ data of W7-AS was performed within a probability theoretical approach, i.e. Bayesian inference [Dose et al.(1998), Preuss et al.(1999)]. This paper expands the discussion to the heliotrons LHD and CHS.

Confinement time scaling

The scaling function for the plasma confinement time of a fusion device in the stellarator/heliotron line is

$$\tau = 10^{\alpha_c} a^{\alpha_a} R^{\alpha_R} P^{\alpha_P} n^{\alpha_n} B^{\alpha_B} \bar{\iota}^{\alpha_{\bar{\iota}}} \quad , \tag{1}$$

with the effective minor radius a, the major radius R, absorbed heating power P, electron density n, toroidal magnetic field B, rotational transform $\bar{\iota}$ and linear constant c. In order to linearize this power law ansatz it is common practice to consider the decadic logarithm of Eq. (1):

$$\log \tau_i = \vec{\alpha} \cdot \vec{x}_i \quad . \tag{2}$$

The index i denotes a single data point. For convenience we use vector notation with the regression parameters $\vec{\alpha}^T = (\alpha_a, \alpha_R, \alpha_P, \alpha_n, \alpha_B, \alpha_{\bar{\iota}}, \alpha_c)$ and the logarithm of the machine variables $\vec{x}_i^T = (\log a_i, \log R_i, \log P_i, \log n_i, \log B_i, \log \bar{\iota}_i, 1)$.

Bayesian probability theory [Jaynes(2005)] provides a straightforward and unique recipe to the problem of uncertainties in input ($\vec{x}_i^T$) and response ($\log \tau_i$) variables in the regression for deriving the scaling exponents. In this probabilistic framework, the results for the parameters are given as expectation values over posterior probability distributions of the data, e.g. for scaling exponent α_k one has to evaluate

$$\langle\alpha_k\rangle = \frac{\int \alpha_k p(\vec{x}_i, \log\tau_i|\vec{\alpha}, \vec{\sigma}_i) p(\vec{\alpha}) \mathrm{d}\vec{\alpha}}{\int p(\vec{x}_i, \log\tau_i|\vec{\alpha}, \vec{\sigma}_i) p(\vec{\alpha}) \mathrm{d}\vec{\alpha}} \quad . \tag{3}$$

The first probability function in both integrals is the so-called likelihood function being a measure for the model description of the data. It reads for data point i with uncertainties in all variables

$$p(\vec{x}_i, \log\tau_i|\vec{\alpha}, \vec{\sigma}_i) = \frac{1}{(2\pi)^{\frac{7}{2}} \sigma_{\log\tau_i} \prod_{k=1}^{6} \sigma_{ki}} \exp\left\{ -\frac{1}{2} \frac{(\log\tau_i - \vec{\alpha}\cdot\vec{x}_i)^2}{\sigma_{\tau_i}^2 + \sum_{k=1}^{6} \alpha_k^2 \sigma_{ki}^2} \right\} , \tag{4}$$

with the uncertainty vector $\vec{\sigma}_i^T = (\sigma_{\log a_i}, \sigma_{\log R_i}, \sigma_{\log P_i}, \sigma_{\log n_i}, \sigma_{\log B_i}, \sigma_{\log \bar{\tau}_i})$ of the logarithm of the machine variables. Note that the denominator in the argument of the exponent makes the difference to ordinary least squares fitting.

The second function showing up in Eq. (3) is the prior probability function of the parameters entering the problem. Since apriori there is nothing known about the values the parameters should adopt, one has to go back to basic requirements like transformation invariances. Following this path one can derive the so-called hyperplane prior given by (see [Dose(2003)])

$$p(\vec{\alpha}|I) \propto \left(\frac{\alpha_c^2}{\alpha_a^2 + \alpha_R^2 + \alpha_P^2 + \alpha_n^2 + \alpha_B^2 + \alpha_{\bar{\tau}}^2 + 1} \right)^{\frac{7}{2}} \quad . \tag{5}$$

The integrals in Eq. (3) are not analytically solvable, so Markov chain Monte Carlo methods are employed for the calculation.

Uncertainties of control parameters in LHD and CHS

In the following we present a discussion of the uncertainties of the machine variables of CHS and LHD (see Tab. 1). The major radius R and the toroidal magnetic field B can be accurately measured. In comparison with the other control parameters which enter the scaling relation their uncertainty is negligible. The rotational transform is known to very high accuracy as well.

Effective minor radius: In CHS the uncertainty is an absolute value of $\sigma_a = 0.2$cm. For LHD we have a relative uncertainty depending on the minor radius a_{eff}: $\sigma_a = 0.024 \cdot a_{eff}$.

Absorbed heating power: To determine the uncertainty of the heating power P we have to take a closer look at the constituents of $P = P_{abs,ECR} + P_{abs,NBI}$. For CHS we got an uncertainty estimation of 20% for ECR and 10% for NBI heating. In LHD the uncertainty of the neutral beam deposition corrected for shine through is 8%. Only NBI heating data sets contribute to the database.

	CHS	LHD
N_{data}	196	162
a_{min}	0.187	0.519
a_{max}	0.2	0.634
σ_a	1%	2.4%
P_{min}	0.061	1.300
P_{max}	0.945	6.516
σ_P	10-20%	8%
n_{min}	0.241	0.89
n_{max}	7.9	5.44
σ_n	0.7%	0.21-0.77%
W_{min}	178.3	49650
W_{max}	3670	691300
σ_W	1.3-19%	2.2-6.8%

Table 1: Data ranges and uncertainties of the minor radius a [m], the absorbed heating power P[MJ], density $n[10^{19}/\mathrm{m}^3]$ and the energy content W[J].

Electron density: The line integral over the electron density, $\int n_e dl = 2an$, is obtained with very high accuracy (within 2 %) for all machines. The uncertainty in the density n depends therefore mainly from error propagation of the uncertainty in a. For CHS the measurement is over a larger distance than the minor radius, reducing the uncertainty to 0.7%. In LHD we face an absolute uncertainty of $6 \cdot 10^{16}\mathrm{m}^{-3}$ due to the mechanical changes in the dimensions of the vessel and a relative uncertainty of 0.1% due to the microwave interferometer diagnostic itself.

Confinement energy: In the process of calculating the confinement time the total plasma energy as determined by diamagnetic measurements (ISCDB column name: WDIA) is used. In CHS its uncertainty is 40 $\times$ BT (with BT as the ISCDB column name of the toroidal magnetic field). For LHD the diamagnetic energy is calculated from the sum of toroidal, helical and paramagnetic/diamagnetic fluxes, termed PHI_TOR, PHI_PARA and PHI_HEL, respectively. The total uncertainty results from error propagation of the single uncertainties of those fluxes.

$$\begin{aligned}\sigma_{WDIA} &= \left\{(0.02 \cdot \mathrm{WDIA})^2 + (8 \cdot 10^7 \cdot \mathrm{BT}/3)^2 \cdot \right. \\ &\left. \left[(0.01 \cdot \mathrm{PHI\_TOR})^2 + (0.01 \cdot \mathrm{PHI\_PARA})^2 + (0.02 \cdot \mathrm{PHI\_HEL}/0.07)^2\right]\right\}^{\frac{1}{2}} \quad (6)\end{aligned}$$

Eventually, we get the uncertainty in the confinement time $\tau = WDIA/PTOT$ from error propagation. Note, that all above discussed uncertainties have to be transformed to decadic

	α_a	α_R	α_P	α_n	α_B	$\alpha_{\bar{\iota}}$	α_c
1	2.28±0.02	0.64±0.02	-0.61±0.01	0.54±0.01	0.84±0.01	0.41±0.01	-0.87±0.02
2	2.06±0.03	1.15±0.03	-0.60±0.02	0.57±0.01	0.99±0.02	-0.03±0.03	-1.49±0.09
3	2.39±0.04	1.22±0.03	-0.77±0.02	0.69±0.02	0.88±0.02	0.04±0.04	-1.32±0.10

Table 2: Case study results: (1) ISS04 restated for comparison; (2) ISCDB subset with W7-AS, W7-A, CHS and LHD not using uncertainties; (3) same as (2), but using uncertainties of the machine variables.

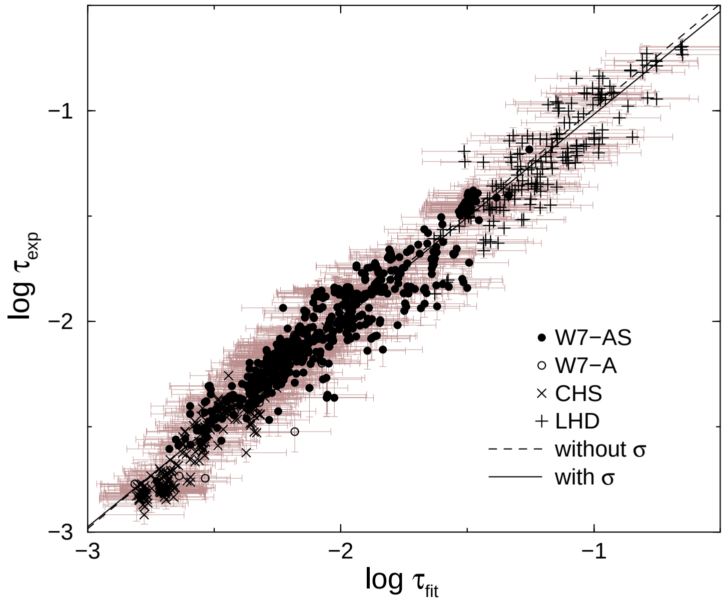

Figure 1: Plot of the confinement time from experiment vs. fit. The dotted line represents the result for the analysis not employing the uncertainties. The error bars are stated for both $\log \tau_{exp}$ and $\log \tau_{fit}$.

logarithmic scale, e.g. in the case of the confinement time one has to calculate:

$$\sigma_{LOGT} = \sqrt{\left[\frac{\sigma_{WDIA}}{10^{LOG\_TAU} \cdot PTOT}\right]^2 + \left[\frac{\sigma_{PTOT}}{PTOT}\right]^2} / log_2(10) \qquad (7)$$

Results and final remarks

Tab. 2 shows the ISS04 result [Yamada et al.(2005)] in the first row. Note that it was obtained with an configuration dependent parameter. The second row is the result for the small subset of W7-AS, W7-A, CHS and LHD neglecting the uncertainties of the data. The discrepancies in α_R and $\alpha_{\bar{\iota}}$in comparison with ISS04 can be explained by the choice of this subset without paying attention to configuration. Eventually, the third row shows the result of the present study. If we compare the results of the subsets in row 2 and 3, the most prominent differences are found in exactly those scaling exponents where the discussion of the uncertainties took place, i.e. α_a, α_P and α_n. This demonstrates the importance of considering the uncertainties of the machine variables. As can be seen in Fig. 1 the result of the fit $\log\tau_{fit}$ and the measured $\log\tau_{exp}$ agree well within the abscissa error bars, which is a direct measure for the reliability of the prediction.

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