Simulation Study of Interaction between Energetic Ions and Alfvén Eigenmodes in LHD

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Abstract. Interaction between energetic ions and Alfvén eigenmodes (AE modes) in LHD was investigated with numerical approaches. The spatial profile and frequency of the AE modes in an LHD plasma #47645, where the creation of holes and clumps in the energetic ion energy spectrum associated with AE modes was observed with the neutral particle analyzer (NPA) [M. Osakabe et al., Nucl. Fusion **46**, S911 (2006)], were analyzed with the AE3D code. The phase space structures of the energetic ions on the NPA line-of-sight were investigated with the Poincaré plots where an oscillating AE mode for each plot is employed. The radial width of the phase space regions trapped by the AE modes corresponds to the transport distance of energetic ions. As island width depends on AE mode amplitude, it was found that AE mode amplitude of $\delta B_r / B \sim 10^{-3}$ is consistent with the energetic-ion transport over 10% of the minor radius that is suggested by the difference in slowing-down time between the holes and clumps observed with the NPA in the LHD experiment. Furthermore, a numerical code which simulates the time evolution of energetic particles and AE mode amplitude and phase in a self-consistent way has been newly developed for three-dimensional equilibria such as LHD. Alfvén eigenmode bursts in LHD plasma were simulated with neutral-beam injection and collisions taken into account.

1. Introduction

When we try to understand and predict behavior of energetic particles (EP) and Alfvén eigenmodes (AE modes) in magnetically confined plasmas, we need extensive physical knowledge such as AE mode linear properties (spatial profile, frequency, and damping rate), energetic particle orbits with and without AE mode, AE mode nonlinear properties which include generation of higher-order harmonics, and nonlinear evolution of the EP-AE system with source, sink and collisions. Numerical analyses on interaction between energetic particles and AE modes are more challenging in three-dimensional equilibria such as LHD plasmas than in the axisymmetric equilibria. In this paper, we report on the AE3D code [1,2] analysis of Alfvén eigenmodes in LHD plasma #47645 where the creation of hole and clump pairs in energetic ion energy spectra associated with the AE modes was observed with the neutral particle analyzer (NPA) [3]. In the experiment, the frequencies of the AE modes are roughly 55 kHz and 68 kHz, respectively. Both of the AE modes have toroidal mode number n=1. The hole and clump pairs are created around an energy of 150 keV. The difference in the slowing-down times between the holes and clumps suggests that the energetic ions were transported over 10% of the plasma minor radius.

We also investigated the AE mode amplitude that is consistent with the energetic-ion transport, as suggested by the NPA data. The Poincaré plots were made to investigate the phase space structures related to the energetic ions arriving at the NPA line-of sight [4]. The phase space regions trapped by the AE modes appear as islands in the Poincaré plots. The radial width of the islands corresponds to the effective transport distance of the energetic ions. As the island width depends on the AE mode amplitude, we can determine the amplitude that is consistent with the NPA observation.

Furthermore, a numerical code that simulates the time evolution of energetic particles and AE mode amplitude and phase in a self-consistent way, which is similar to the code for tokamaks [5], has been newly developed for three-dimensional equilibria such as LHD. In the simulation code, AE mode spatial profiles are assumed constant. The three-dimensional magnetohydrodynamic (MHD) equilibrium calculated with the HINT code [6] and the AE modes calculated with the AE3D code are used in this simulation. The AE mode instability due to energetic particles in the LHD plasma was simulated with this code. Alfvén eigenmode bursts in LHD plasma were also simulated with neutral-beam injection and collisions taken into account.

2. Analysis of Alfvén eigenmodes

The Alfvén eigenmodes with toroidal mode number n=1 in the LHD shot #47645 were analyzed with the AE3D code [1,2], which is based on a Galerkin approach using a combined Fourier mode (poloidal/toroidal angle) finite element (radial) representation in Boozer coordinates. An MHD equilibrium was constructed in the Boozer coordinates. Two toroidal Alfvén eigenmodes (TAE modes) were found with the eigen-frequencies of 42.7 kHz and 79.1 kHz. The eigen-frequencies are comparable to the AE mode frequencies observed in the experiment. The spatial profiles of electrostatic potential are shown in Fig. 1. The primary poloidal harmonics of both the two TAE modes are m=1 and m=2. The primary poloidal harmonics have the same sign for the TAE mode with frequency 42.7 kHz, while they have the opposite signs for the mode with frequency 79.1 kHz.

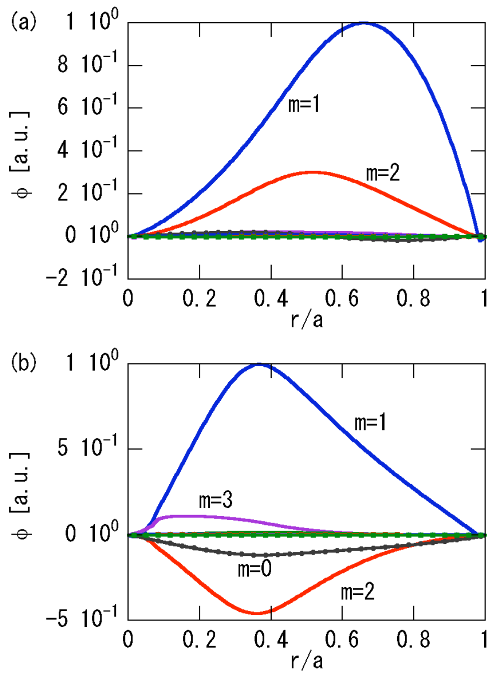

FIG. 1. Spatial profiles of the n=1 toroidal Alfvén eigenmodes analyzed with the AE3D code for the LHD shot #47645. The frequencies are (a) 42.7 kHz and (b) 79.1 kHz, respectively.

3. Analysis of energetic-ion orbit

The energetic-ion orbits were calculated with different starting points on the NPA line-of sight. The orbits were calculated in an MHD equilibrium constructed with the HINT code. The energetic-ion energy is 150 keV at which hole and clump creation in energy spectrum was observed with the NPA. The pitch angle is determined by the direction of the NPA line-of-sight. The drift-kinetic description is employed for the energetic ions. The guiding-center velocity is given by,

$$\mathbf{u} = \mathbf{v}_{//}^{*} + \mathbf{v}_{E} + \mathbf{v}_{B} \ ,$$

$$\mathbf{v}_{//}^{*} = \frac{v_{//}}{B^{*}}\left[\mathbf{B} + \rho_{//} B \nabla \times \mathbf{b}\right],$$

$$\mathbf{v}_{E} = \frac{1}{B^{*}}\left[\mathbf{E} \times \mathbf{b}\right],$$

$$\mathbf{v}_{B} = \frac{1}{q_h B^{*}}\left[-\mu \nabla B \times \mathbf{b}\right],$$

$$\rho_{//} = \frac{m_h v_{//}}{q_h B} \ ,$$

$$\mathbf{b} = \mathbf{B}/B \ ,$$

$$B^{*} = B\left(1 + \rho_{//}\mathbf{b} \cdot \nabla \times \mathbf{b}\right) \ ,$$

$$m_h v_{//} \frac{dv_{//}}{dt} = \mathbf{v}_{//}^{*} \cdot \left[q_h \mathbf{E} - \mu \nabla B\right],$$

where $v_{//}$ is the velocity parallel to the magnetic field, μ is the magnetic moment which is an adiabatic invariant, m_h and q_h are energetic ion mass and electric charge. In the MHD equilibrium used in the orbit calculation reported in this section, the electric field represented by $\mathbf{E}$ is zero.

The poloidal orbit-frequency is defined by $\omega_\theta = 2\pi / T_\theta$ where T_θ is the poloidal circulation time. The toroidal orbit-frequency is defined by $\omega_\varphi = \Delta\varphi / T_\theta$ where $\Delta\varphi$ is the toroidal angle covered by the energetic ion during a time T_θ. In the experiment, the magnetic field was reversed so that the toroidal field is positive $B_\varphi > 0$ and the poloidal field is negative $B_\theta < 0$. The toroidal field is counter-clockwise when viewed from the top side. The energetic ions which arrived on the NPA line-of-sight were moving in the direction of the magnetic field. The orbit frequency of the energetic ions is defined by

$$f_{m/n} = \left(n\omega_\varphi - m\omega_\theta\right) / 2\pi \ ,$$

where n and m are toroidal and poloidal mode numbers. We take n=1 and m=1 which are identical to the primary poloidal harmonic of the TAE modes shown in Fig. 1. In Fig. 2, the orbit frequency is shown versus the major radius of the starting point on the NPA line-of-sight. We see that energetic ions exist with the orbit frequencies 55kHz and 68kHz same as the AE mode frequencies observed in the LHD experiment. These energetic ions resonate with the AE modes. The major radius and pitch angle of the particles at the starting point on the NPA line-of-sight with the orbit frequencies 55kHz and 68kHz are ($R = 4.19$m, $v_{//} / v_{\mathrm{total}} = 0.91$) and ($R = 4.10$m, $v_{//} / v_{\mathrm{total}} = 0.89$), respectively.

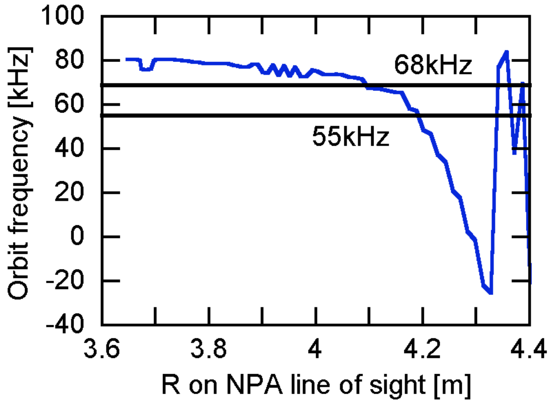

FIG. 2. Blue curve represents energetic-ion orbit frequency with m/n=1/1 versus major radius of the starting point on the NPA line-of-sight. Black lines denote Alfvén eigenmode frequencies observed in the LHD experiment.

4. Phase space structure of energetic ions

We now consider how far the energetic ions are transported by the finite amplitude eigenmodes. To study this, we investigate Poincaré plots where only one eigenmode is taken into account and the amplitude of the eigenmode is at a constant value. We need a conserved variable to make Poincaré plots interpretable. In axisymmetric equilibria, we can find a conserved variable $E' \equiv E - \omega P_\varphi / n$ in the interaction of a constant amplitude wave with frequency ω and toroidal mode number n. Here, E is particle energy and P_φ is canonical toroidal momentum. We choose particles that have a constant value of E'. Then with a single mode we have a conserved variable and we can make Poincaré plots that are easily interpretable. The phase space regions trapped by the eigenmodes appear as islands in the Poincaré plots. As island width depends on Alfvén eigenmode amplitude, we can determine the amplitude that is consistent with the energetic ion transport. On the other hand, in nonaxisymmetric equilibria, we cannot find such a conserved variable. Then, we investigate an axisymmetric equilibrium comparable to the LHD equilibrium. Nevertheless, we would like to emphasize that this is the first attempt to understand the phase space structures in the LHD plasmas with finite amplitude waves.

We investigate an axisymmetric equilibrium comparable to the LHD plasma #47645. The parameters of the axisymmetric equilibrium are major radius $R_0 = 3.67$m, minor radius $a = 0.54$m, toroidal magnetic field 0.5T, and safety factor profile $q(r) = 2.11 - 1.23(r/a)^2$. This safety factor profile is a good approximation for that of the LHD plasma. The value of E' is defined on the outer edge mid-plane at $(R - R_0)/a = 0.7$ with energy 150keV and pitch angle $v_{//}/v_{\rm total} = 1$. As the harmonics of the two TAE modes other than n=1 are negligibly small, the major four poloidal harmonics with n=1 were mapped to the axisymmetric equilibrium. It is illustrated in Ref. [5] how to calculate the electromagnetic field of the Alfvén eigenmodes from their potential harmonics. The frequencies are renormalized to the experimental values 55kHz for the lower frequency TAE shown in Fig. 1(a) and 68kHz for the higher frequency TAE shown in Fig. 1(b). The energetic-particle orbits were calculated in the electromagnetic field that is a sum of the equilibrium magnetic field and the electromagnetic field of the Alfvén eigenmodes. In the Poincaré plot we print the major radius $(R-R_0)/a$ and phase, $n\varphi-\omega t$, of a particle each time the poloidal angle of the particle reaches $\theta=0°$.

The Poincaré plots for the lower frequency TAE are shown in Fig. 3. The phase space regions trapped by the Alfvén eigenmodes appear as the islands in the Poincaré plots. The radial width of the islands corresponds to the transport distance of the energetic ions. The island width depends on the Alfvén eigenmode amplitude. For the lower frequency TAE, the amplitude $\delta B_r/B=2\times10^{-3}$ transports energetic ions over 10% of the minor radius. The phase space structures for the higher frequency TAE are analyzed in Fig. 4. Comparing Fig. 4(a) and (b), we see that the amplitude of $\delta B_r/B=10^{-3}$ is consistent with the transport of energetic ions over 10% of the minor radius.

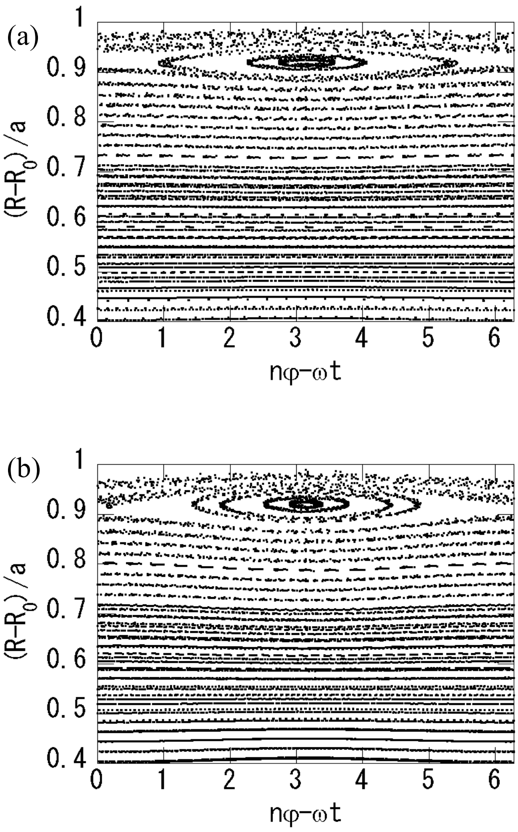

FIG. 3. Poincaré plots for the lower frequency TAE mode with amplitude (a) $\delta B_r/B=10^{-3}$ *and (b)* $\delta B_r/B=2\times10^{-3}$.

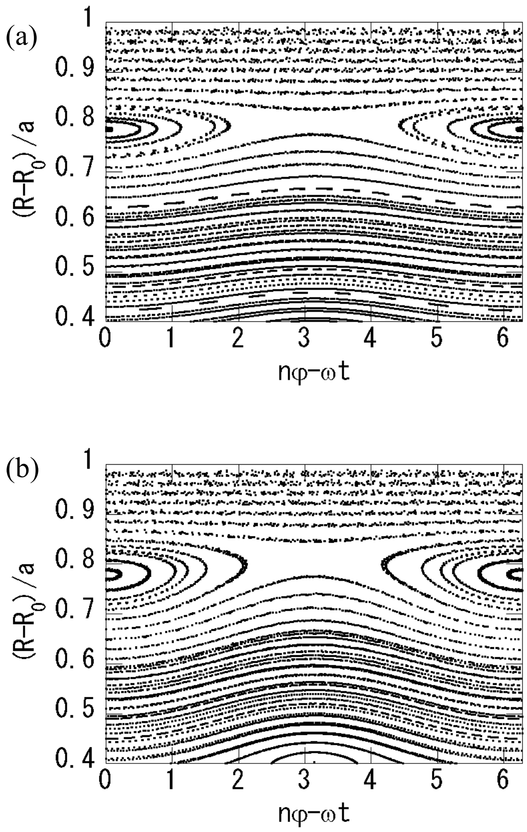

FIG. 4. Poincaré plots for the higher frequency TAE mode with amplitude (a) $\delta B_r/B=10^{-3}$ *and (b)* $\delta B_r/B=2\times10^{-3}$.

5. Reduced simulation of Alfvén eigenmode bursts

Computer simulation is a powerful tool to study self-consistent evolution of the system of energetic particles and AE modes. Simulation methods can be divided into two types. The first type is a hybrid simulation where full or reduced MHD equations are coupled with energetic-particle pressure or current density while energetic-particle orbits are followed in the MHD electromagnetic field. The major advantage of the EP-MHD hybrid simulation is that MHD nonlinearity is naturally taken account. Hybrid simulation codes for nonaxisymmetric equilibria have been developed [7, 8]. In the second type of simulation, the time evolution of AE mode amplitude and phase is followed in a way consistent with the energetic-particle evolution while the AE mode spatial profile is assumed constant. This reduced simulation method has an advantage that the time step is not restricted by the Courant condition of the MHD equations and it suits simulation for long time-scale comparable to the EP slowing-down time.

A reduced simulation code has been newly developed to study the time evolution of energetic particles and AE modes in three-dimensional equilibria such as LHD. Three-dimensional magnetohydrodynamic (MHD) equilibrium data calculated with the HINT code are used in the reduced simulation. Spatial profile and frequency of Alfvén eigenmodes are needed to analyze in advance of the simulation using linear analysis codes such as AE3D and CAS3D [9] for three-dimensional equilibria. The AE3D code was employed in this work. The numerical algorithm how to calculate the time evolution of EP-AE system is the same as a reduced code for tokamaks [5]. An Alfvén eigenmode instability due to energetic particles was simulated using the new code. The equilibrium employed is the LHD plasma #47645 which was investigated in the previous sections. The higher-frequency TAE shown in Fig. 1(b) was used for the simulation. The major four poloidal harmonics with toroidal mode number n=1 were mapped to the simulation region using the mapping of the Boozer coordinates to the cylindrical coordinates. The simulation result of the time evolution of TAE amplitude is shown in Fig. 5. The linear growth and saturation of the instability was observed. The amplitude oscillation took place after the saturation, suggesting the particle trapping by the TAE.

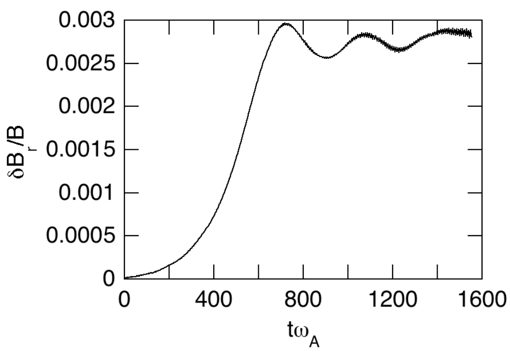

FIG. 5. Amplitude evolution of the TAE destabilized by the energetic particles.

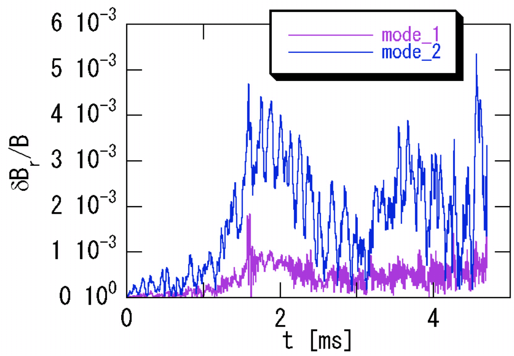

FIG. 6. Time evolution of TAE amplitude in the simulation with neutral beam injection and collsions. The modes 1 and 2 denote the lower and higher frequency TAE modes, respectively.

Alfvén eigenmode bursts in LHD plasma #47645 were simulated with neutral-beam injection (NBI) and collisions taken into account. The two TAE modes shown in Fig. 1 were used. For simplicity, the NBI deposition profile in the simulation is assumed to be a Gaussian profile in radial direction and uniform in poloidal and toroidal directions. The radial scale length of the deposition is assumed to be $0.4a$. The beam energy and injection power in the simulation are 180 keV and 10 MW, respectively. The slowing-down time is assumed 20 ms and the damping rate of the TAE modes is taken to be 2% of the angular frequency of the lower-frequency mode (mode 1). The amplitude evolution in the simulation result is shown in Fig. 6. It is seen in the figure that the AE mode bursts take place.

6. Summary

The spatial profile and frequency of the Alfvén eigenmodes have been examined using the AE3D code for the LHD shot #47645, where the creation of hole/clump pairs in the energetic ion energy spectrum associated with the Alfvén eigenmodes was observed. The difference in slowing-down time between the holes and clumps observed in the experiment suggests that the energetic ions experienced rapid transport over 10% of the plasma minor radius. The phase space structures of the energetic ions on the NPA line-of-sight were investigated with the Poincaré plots where an oscillating Alfvén eigenmode for each plot is employed. The phase space regions trapped by the Alfvén eigenmodes appear as the islands in the Poincaré plots. The radial width of the islands corresponds to the transport distance of the energetic ions. As island width depends on the Alfvén eigenmode amplitude, it was found that the Alfvén eigenmodes with amplitude $\delta B_r / B \sim 10^{-3}$ resulted in island widths at about 10% of the minor radius, in consistency with the experimental observations.

A numerical code that simulates the time evolution of energetic particles and AE mode amplitude and phase in a self-consistent way has been newly developed for three-dimensional equilibria such as LHD. In the simulation code, AE mode spatial profiles are assumed constant. The major harmonics of AE modes are mapped to the simulation region using the mapping of the Boozer coordinates to the cylindrical coordinates. The AE mode instability due to energetic particles in the LHD plasma was simulated with this code. The linear growth and saturation of the instability was observed. The amplitude oscillation took place after the saturation, suggesting the particle trapping by the TAE. Simulation with neutral-beam injection and collisions taken into account was carried out in the LHD equilibrium using the two TAE modes. It was demonstrated that Alfvén eigenmode bursts can be simulated with the new code.

Acknowledgments
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