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Momentum balance and radial electric fields in axisymmetric and nonaxisymmetric toroidal plasmas

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Abstract.

It is investigated how symmetry properties of toroidal magnetic configurations influence mechanisms of determining the radial electric field such as the momentum balance and the ambipolar particle transport. Both neoclassical and anomalous transport of particles, heat, and momentum in axisymmetric and nonaxisymmetric toroidal systems are taken into account. Generally, in nonaxisymmetric systems, the radial electric field is determined by the neoclassical ambipolarity condition. For axisymmetric systems with up-down symmetry and quasisymmetric systems with stellarator symmetry, it is shown by using a novel parity transformation that the particle fluxes are automatically ambipolar up to $\mathcal{O}(\delta^2)$ and the determination of the radial electric field E_s requires solving the $\mathcal{O}(\delta^3)$ momentum balance equations, where δ denotes the ratio of the thermal gyroradius to the characteristic equilibrium scale length. In axisymmetric systems with large $\mathbf{E}\times\mathbf{B}$ flows on the order of the ion thermal velocity v_{Ti}, the radial fluxes of particles, heat, and toroidal momentum are dependent on E_s and its radial derivative while the time evolution of the E_s profile is governed by the $\mathcal{O}(\delta^2)$ toroidal momentum balance equation. In nonaxisymmetric systems, $\mathbf{E}\times\mathbf{B}$ flows of $\mathcal{O}(v_{Ti})$ are not generally allowed even in the presence of quasisymmetry because the nonzero radial current is produced by the large flow term in the equilibrium force balance for which the Boozer and Hamada coordinates cannot be constructed.

Keywords: momentum transport, radial electric field, toroidal geometry

1. Introduction

Roles of radial electric fields in improvement of plasma confinement have been intensively investigated in theoretical and experimental researches of tokamaks and helical systems such as stellarators and heliotrons [1, 2]. For example, reduction of turbulent transport due to the large sheared radial electric field is observed in edge regions of H mode tokamak plasmas [3]. In nonaxisymmetric systems, neoclassical ripple transport is significantly reduced in the presence of the radial electric field [4]. It is also well-known that the neoclassical particle fluxes are automatically ambipolar for any radial electric field in axisymmetric systems while they are not so in nonaxisymmetric systems [5, 6, 7, 8]. The mechanism to determine the radial electric field is closely connected to the momentum balance, and there have been numerous theoretical and experimental studies on the momentum transport in recent years [9, 10, 11, 12, 13, 14, 15, 16]. The momentum transport processes are deeply influenced by symmetry properties of the magnetic configuration. In tokamaks, the large toroidal flow velocity on the order of the ion thermal velocity v_{Ti} can be driven by external torque and, in such a case, the toroidal momentum balance equation governs the time evolution of the profile of the radial electric field [17, 18, 19, 20, 21, 22]. Also, theoretical studies about effects of up-down asymmetry on the momentum transport and the radial electric field in tokamaks are reported by several works [23, 24]. As for the nonaxisymmetric systems, if the magnetic field strength satisfies a certain condition called 'quasisymmetry' [25, 26, 27, 28, 29], the large flow is expected to be produced in the direction associated with the quasisymmetry, which may lead to the production of the large radial electric field.

In this work, we comprehensively investigate how the momentum balance as a mechanism to determine the radial electric field is influenced by symmetry properties of toroidal magnetic configurations. In Sec. 2, both neoclassical and anomalous (or turbulent) transport of particles, heat, and momentum in axisymmetric and nonaxisymmetric systems are treated based on the formulation developed in [19, 21, 30, 31], where the basic kinetic and electromagnetic field equations are separated into ensemble-averaged and fluctuating parts. There, the $\mathbf{E}\times\mathbf{B}$ drift velocity is assumed to be on the order of the thermal velocity multiplied by δ, where $\delta \sim \rho/L$ represents the small gyroradius ordering parameter given as the ratio of the gyroradius ρ to the characteristic equilibrium gradient scale length L. In Sec. 3, momentum balance equations for tokamaks and helical systems are expanded with respect to δ, and it is examined order by order in δ whether the particle fluxes are automatically ambipolar or the radial electric field is determined by the ambipolarity condition or the momentum transport equation. In Sec. 4, we prove by employing a novel parity operator that the neoclassical and anomalous radial momentum transport fluxes vanish automatically up to $\mathcal{O}(\delta^2)$ in tokamaks with up-down symmetry and in helical systems with stellarator symmetry, where we need to solve the momentum balance equation up to $\mathcal{O}(\delta^3)$ to determine the radial electric field. In Sec. 5, we explain how the radial electric field is determined from the toroidal momentum transport in tokamaks with the large toroidal flow velocity of

$\mathcal{O}(v_{Ti})$ and discuss a problem arising from the equilibrium force balance including the large flow velocity in helical systems with quasisymmetry. Finally, conclusions are given in Sec. 6.

2. Basic kinetic equations and radial transport of particles and heat

In this section, it is briefly reviewed how classical, neoclassical, and anomalous (or turbulent) transport fluxes of particles and heat are defined in terms of the corresponding parts of the distribution function. We start from a basic kinetic equation that is written as

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{e_a}{m_a}\left\{\left(\mathbf{E}+\hat{\mathbf{E}}\right) + \frac{1}{c}\mathbf{v}\times\left(\mathbf{B}+\hat{\mathbf{B}}\right)\right\}\cdot\frac{\partial}{\partial \mathbf{v}}\right](f_a + \hat{f}_a)$$
$$= C_a(f_a + \hat{f}_a) \qquad (1)$$

where $C_a \equiv \sum_b C_{ab}$ denotes a collision term and the distribution function for species a (the electromagnetic fields) is divided into the ensemble average part f_a ($\mathbf{E} = -\nabla\Phi - c^{-1}\partial\mathbf{A}/\partial t, \mathbf{B} = \nabla\times\mathbf{A}$) and the fluctuating part $\hat{f}_a$ ($\hat{\mathbf{E}} = -\nabla\hat{\phi} - c^{-1}\partial\hat{\mathbf{A}}/\partial t, \hat{\mathbf{B}} = \nabla\times\hat{\mathbf{A}}$). The ensemble-averaged and fluctuating parts of the electromagnetic fields are governed by the corresponding sets of the Maxwell equations obtained by using f_a and $\hat{f}_a$, respectively, to define charge densities and currents. We consider a toroidal plasma, in which the magnetic field $\mathbf{B}$ is written in terms of the flux coordinates (s,θ,ζ) as

$$\mathbf{B} = \psi'\nabla s\times\nabla\theta + \chi'\nabla\zeta\times\nabla s = B_s\nabla s + B_\theta\nabla\theta + B_\zeta\nabla\zeta, \qquad (2)$$

where s is an arbitrary label of a flux surface, θ and ζ represent the poloidal and toroidal angles, respectively, and $' \equiv \partial/\partial s$ denotes the derivative with respect to s. The toroidal and poloidal fluxes within the volume inside the surface with the label s are given by $2\pi\psi(s)$ and $2\pi\chi(s)$, respectively.

Taking an ensemble average $\langle\cdots\rangle_{\rm ens}$ of (1) gives the kinetic equation for f_a as

$$\frac{d}{dt}f_a \equiv \left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}}\right]f_a = \langle C_a\rangle_{\rm ens} + \mathcal{D}_a, \qquad (3)$$

where the right-hand side consists of the collision term and the fluctuation-particle interaction term $\mathcal{D}_a$ defined by [31, 32]

$$\mathcal{D}_a = -\frac{e_a}{m_a}\left\langle\left(\hat{\mathbf{E}} + \frac{1}{c}\mathbf{v}\times\hat{\mathbf{B}}\right)\cdot\frac{\partial\hat{f}_a}{\partial\mathbf{v}}\right\rangle_{\rm ens}. \qquad (4)$$

The differential operator on the left-hand side of (3) is written as

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}}$$
$$\equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla' + \dot{\varepsilon}\frac{\partial}{\partial\varepsilon} + \dot{\mu}\frac{\partial}{\partial\mu} + \dot{\xi}\frac{\partial}{\partial\xi}. \qquad (5)$$

Here, the phase space variables $(\mathbf{x}',\varepsilon,\mu,\xi)$ are defined in terms of $(\mathbf{x},\mathbf{v})$ as $\mathbf{x}' = \mathbf{x}$, $\varepsilon = \frac{1}{2}mv^2 + e_a\Phi_1(s)$, $\mu = m_a v_\perp^2/2B$, and $\mathbf{v}_\perp = \mathbf{e}_1\cos\xi + \mathbf{e}_2\sin\xi$, where $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{b}\equiv\mathbf{B}/B)$

are unit vectors which forms a right-handed orthogonal system at each point, and $\mathbf{v} = v_\parallel \mathbf{b} + \mathbf{v}_\perp$ with $v_\parallel = \mathbf{v}\cdot\mathbf{b}$.

In order to expand physical variables perturbatively, we use the small gyroradius ordering parameter $\delta \sim \rho_a/L$, where $\rho_a = v_\perp/\Omega_a$ and L represents the gyroradius and the characteristic equilibrium gradient scale length, respectively. When the time differential operator $\partial/\partial t$ acts on ensemble-averaged quantities, the transport time scale ordering [5, 6] is used to write $\partial/\partial t = \mathcal{O}(\delta^2)$ which represents that the characteristic frequency is on the order of δ^2 multiplied by the transit frequency $\omega_T = v_{Ta}/L$. In this and next sections, we assume that the $\mathbf{E}\times\mathbf{B}$ drift velocity driven by the lowest-order equilibrium electrostatic potential Φ_1 is tangential to the flux surface and is on the order of the diamagnetic drift velocity $\sim \delta v_{Ta}$, where $v_{Ta} \equiv (2T_a/m_a)^{1/2}$ is the thermal velocity. Then, $\Phi_1 = \Phi_1(s)$ is a flux-surface function and $e\Phi_1/T_i = \mathcal{O}(1)$. The fast gyro-frequency $\Omega_a \equiv e_a B/(m_a c) = \mathcal{O}(\delta^{-1})$ is contained in $\dot{\xi}$ as $\dot{\xi} = -\Omega_a + \dot{\xi}_0$ [$\dot{\xi}_0 = \mathcal{O}(\delta^0)$, $\dot{\xi}_0/\Omega_a = \mathcal{O}(\delta)$], and we subtract the fast gyro-frequency from d/dt to define the operator [33], $\mathcal{L} \equiv d/dt + \Omega_a \partial/\partial\xi$. Then, rewriting (1) in the phase space variables $(\mathbf{x}', \varepsilon, \mu, \xi)$ and separating it into the average and oscillating parts with respect to the gyrophase angle ξ, we obtain [31]

$$\overline{\mathcal{L}(\overline{f}_a + \widetilde{f}_a)} = \left\langle \overline{C}_a \right\rangle_{\rm ens} + \overline{\mathcal{D}}_a, \qquad \Omega_a \frac{\partial \widetilde{f}_a}{\partial \xi} = \widetilde{\mathcal{L} f_a} - \left\langle \widetilde{C}_a \right\rangle_{\rm ens} - \widetilde{\mathcal{D}}_a \tag{6}$$

where the average and oscillating parts of an arbitrary function $F(\xi)$ with respect to ξ are denoted by $\overline{F} \equiv (2\pi)^{-1} \oint d\xi\, F$ and $\widetilde{F} \equiv F - \overline{F}$, respectively.

The radial particle flux Γ_a and the radial heat fluxes q_a are defined from $\widetilde{f}_a$ as

$$\begin{aligned} \Gamma_a &\equiv \langle \mathbf{\Gamma}_a \cdot \nabla s\rangle \equiv \left\langle \int d^3v\, \widetilde{f}_a \mathbf{v}\cdot\nabla s \right\rangle, \\ \frac{q_a}{T_a} &\equiv \frac{\langle \mathbf{q}_a \cdot \nabla s\rangle}{T_a} \equiv \left\langle \int d^3v\, \widetilde{f}_a \left(\frac{m_a v^2}{2T_a} - \frac{5}{2}\right) \mathbf{v}\cdot\nabla s \right\rangle, \end{aligned} \tag{7}$$

where $\langle\cdots\rangle$ represents the flux-surface average. The radial momentum transport flux is treated in the next section. The lowest-order distribution function f_{a0} is given by the local Maxwellian, $f_{a0} = n_{a0}(m_a/2\pi T_{a0})^{3/2} \exp(-m_a v^2/2T_{a0})$, where the lowest-order density $n_{a0} = n_{a0}(s)$ and temperature $T_{a0} = T_{a0}(s)$ are flux surface functions. From (6), the gyrophase dependent part of the ensemble-averaged distribution function is given to the lowest order in δ by $\widetilde{f}_{a1} = \Omega_a^{-1} \int^\xi d\xi\, \widetilde{\mathcal{L} f_{a0}} = -\boldsymbol{\rho}_a \cdot \nabla' f_{a0} = -f_{a0}\,(\boldsymbol{\rho}_a \cdot \nabla s)\,[(\partial \ln p_{a0}/\partial s) + (e_a/T_a)(\partial\Phi_1/\partial s) + (\partial \ln T_{a0}/\partial s)(m_a v^2/2T_a - 5/2)]$, where $\boldsymbol{\rho}_a = \mathbf{b}\times\mathbf{v}/\Omega$ is the gyroradius vector, $p_{a0} = n_{a0}T_{a0}$ is the lowest-order pressure and the integration constant related to $\int^\xi d\xi \cdots$ is uniquely determined by the condition $\overline{\int^\xi d\xi \cdots} = 0$. Substituting $\widetilde{f}_{a1}$ into (6) and retaining the $O(\delta)$ terms lead to the linearized drift kinetic equation [5, 6] which governs the $O(\delta)$ part $\overline{f}_{a1}$ of $\overline{f}_a$.

The particle and heat flows calculated from $\widetilde{f}_{a1}$ give $\mathbf{E}\times\mathbf{B}$ and diamagnetic flows associated with the equilibrium pressure and temperature gradients although they have no components in the direction perpendicular to the flux surface. Therefore, Γ_a and q_a

are of $O(\delta^2)$ as is well known. The $O(\delta^2)$ part of $\widetilde{f}_a$ is written as

$$\widetilde{f}_{a2} = \widetilde{f}_a^N + \widetilde{f}_a^H + \widetilde{f}_a^C + \widetilde{f}_a^A \equiv \frac{1}{\Omega_a}\int^{\xi} d\xi \left[\mathcal{L}\widetilde{\overline{f}}_{a1} + \widetilde{\mathcal{L}\widetilde{f}_{a1}} - C_a^L(\widetilde{f}_{a1}) - \widetilde{D}_a\right], \quad (8)$$

and C_a^L denotes the linearized collision operator [6]. Here, $\widetilde{f}_a^H$ is an even function of $\mathbf{v}$ up to $\mathcal{O}(\delta^2)$ and accordingly it makes no contribution to Γ_a and q_a. Using $\widetilde{f}_a^N$, $\widetilde{f}_a^C$, and $\widetilde{f}_a^A$ in (7), we obtain the $\mathcal{O}(\delta^2)$ radial particle and heat fluxes,

$$\Gamma_a = \Gamma_a^{\rm ncl} + \Gamma_a^{\rm cl} + \Gamma_a^{\rm anom}, \qquad q_a = q_a^{\rm ncl} + q_a^{\rm cl} + q_a^{\rm anom}$$

where the superscripts 'ncl', 'cl', and 'anom' represents the neoclassical, classical, and anomalous (or turbulent) parts derived from $\widetilde{f}_a^N$, $\widetilde{f}_a^C$, and $\widetilde{f}_a^A$, respectively. The neoclassical fluxes $\Gamma_a^{\rm ncl}$ and $q_a^{\rm ncl}$ are rewritten in terms of $\overline{f}_{a1}$ given as the solution of the linearized drift kinetic equation. On the other hand, the anomalous fluxes $\Gamma_a^{\rm anom}$ and $q_a^{\rm anom}$ are rewritten in terms of the turbulent distribution function and the turbulent electromagnetic fields which are assumed to take the eikonal form [see (34)]. The detailed expressions for $\Gamma_a^{\rm anom}$ and $q_a^{\rm anom}$ in terms of the fluctuations are found in [31]. For example, the anomalous radial particle flux is given by

$$\Gamma_a^{\rm anom} \equiv \left\langle\left\langle \int d^3v \sum_{\mathbf{k}_\perp} {\rm Im}(\hat{h}^*_{a\mathbf{k}_\perp}\hat{\psi}_{a\mathbf{k}_\perp})(\mathbf{k}_\perp \times \mathbf{b})\cdot\nabla s \right\rangle\right\rangle, \quad (9)$$

where $\langle\langle\cdots\rangle\rangle$ represents a double average over the flux-surface and the ensemble. The gyrophase-averaged potential for the turbulent electromagnetic fields is defined by $\hat{\psi}_{a\mathbf{k}_\perp} \equiv J_0(k_\perp v_\perp/\Omega_a)[\hat{\phi}_{\mathbf{k}_\perp} - (v_\parallel/c)\hat{A}_{\parallel\mathbf{k}_\perp}] + J_1(k_\perp v_\perp/\Omega_a)(v_\perp/c)(\hat{B}_{\parallel\mathbf{k}_\perp}/k_\perp)$ with J_n $(n = 0, 1)$ denoting the nth-order Bessel functions. Here, the nonadiabatic part $\hat{h}_{a\mathbf{k}_\perp}$ of the perturbed distribution function and the turbulent field variables $(\hat{\phi}_{\mathbf{k}_\perp}, \hat{A}_{\parallel\mathbf{k}_\perp}, \hat{B}_{\parallel\mathbf{k}_\perp})$ are given as the solutions of a coupled system of the gyrokinetic Boltzmann and Maxwell equations [31, 34]. The gyrokinetic Boltzmann and Maxwell equations are obtained from the fluctuating parts of the kinetic equation (1) and the Maxwell equations. It is shown in [31] by using the gyrokinetic Poisson equation and Ampère's law that, for both axisymmetric and nonaxisymmetric systems, the lowest-order or $\mathcal{O}(\delta^2)$ turbulent radial particle fluxes $\Gamma_a^{\rm anom}$ satisfy the ambipolarity condition, $\sum_a e_a\Gamma_a^{\rm anom} = 0$, automatically for any radial electric field $\mathbf{E}_s = \mathcal{O}(\delta)$. The classical particle fluxes are automatically ambipolar, $\sum_a e_a\Gamma_a^{\rm cl} = 0$, too, because of the momentum conservation in collisions. However, in nonaxisymmetric systems, the ambipolarity of the neoclassical particle fluxes, $\sum_a e_a\Gamma_a^{\rm cl} = 0$, is not automatically satisfied but it puts a constraint on the radial electric field as explained in Sec. 3.3.

3. Momentum balance and radial electric fields in toroidal plasmas with $\mathbf{E}\times\mathbf{B}$ drift velocities of $\mathcal{O}(\delta v_T)$

In this section, it is examined how the symmetry of the magnetic geometry influences the momentum balance, the ambipolarity condition of radial particle fluxes, and the mechanism to determine the radial electric field E_s in axisymmetric and nonaxisymmetric toroidal systems. Throughout this section, we assume the $\mathbf{E} \times \mathbf{B}$ drift velocity to be on the order of the diamagnetic drift velocity ($\sim \delta_i v_{Ti} \sim \delta_e v_{Te}$). Results obtained in this section are summarized in Table 1.

3.1. Momentum balance equations

Multiplying (3) with $m_a \mathbf{v}$, the ensemble-averaged momentum balance equation is derived as

$$\frac{\partial}{\partial t}(n_a m_a \mathbf{u}_a) = -\nabla \cdot \mathbf{P}_a + n_a e_a \left(\mathbf{E} + \frac{\mathbf{u}_a}{c} \times \mathbf{B} \right) + \mathbf{F}_{a1} + \mathbf{K}_{a1} \tag{10}$$

where $n_a \equiv \int d^3v \ f_a$, $n_a \mathbf{u}_a \equiv \int d^3v \ f_a \mathbf{v}$, $\mathbf{P}_a \equiv \int d^3v \ f_a m_a \mathbf{v}\mathbf{v}$, $\mathbf{F}_{a1} \equiv \int d^3v \ C_a(f_a) m_a \mathbf{v}$, and $\mathbf{K}_{a1} \equiv \int d^3v \ \mathcal{D}_a \mathbf{v}$. Taking the species summation and flux-surface average of the inner product of (10) and $\mathbf{B}$, we obtain

$$\frac{\partial}{\partial t} \sum_a \langle n_a m_a u_{a\|} B \rangle = -\sum_a \langle \mathbf{B} \cdot (\nabla \cdot \mathbf{P}_a) \rangle + \sum_a \left\langle B K_{a1\|} \right\rangle , \tag{11}$$

where $\sum_a n_a e_a = 0$ and $\sum_a \mathbf{F}_{a1} = 0$ are used, and $\partial \mathbf{B}/\partial t = 0$ is assumed. Similarly, taking the inner product of (10) and $c_1 \partial \mathbf{x}/\partial \theta + c_2 \partial \mathbf{x}/\partial \zeta$, where c_1 and c_2 are arbitrary constants, and taking its species summation and flux-surface average, we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \sum_a \langle n_a m_a (c_1 u_{a\theta} + c_2 u_{a\zeta}) \rangle = & -\sum_a \left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot (\nabla \cdot \mathbf{P}_a) \right\rangle \\ & + \sum_a \langle c_1 K_{a1\theta} + c_2 K_{a1\zeta} \rangle \\ & + \frac{1}{c}(-c_1 \psi' + c_2 \chi') \sum_a e_a \langle n_a u_a^s \rangle , \end{aligned} \tag{12}$$

where contravariant and covariant components of vectors and tensors with respect to the flux coordinates (s, θ, ζ) are represented by using superscripts and subscripts, respectively. In terms of the perturbed density $\hat{n}_a \equiv \int d^3v \ \hat{f}_a$ and the perturbed current $\hat{\mathbf{j}}_a \equiv \sum_a e_a n_a \hat{\mathbf{u}}_a \equiv \sum_a e_a \int d^3v \ \hat{f}_a \mathbf{v}$, the forces due to the turbulent electromagnetic fields in (12) are rewritten as

$$\begin{aligned} \sum_a \mathbf{K}_{a1} &= \left\langle \left(\sum_a e_a \hat{n}_a \right) \hat{\mathbf{E}} + \frac{\hat{\mathbf{j}}}{c} \times \hat{\mathbf{B}} \right\rangle_{\text{ens}} = \frac{1}{4\pi} \left\langle \left(\nabla \cdot \hat{\mathbf{E}} \right) \hat{\mathbf{E}} + \left(\nabla \times \hat{\mathbf{B}} - \frac{1}{c} \frac{\partial \hat{\mathbf{E}}}{\partial t} \right) \times \hat{\mathbf{B}} \right\rangle_{\text{ens}} \\ &= \nabla \cdot \left\langle \frac{1}{4\pi} \left(\hat{\mathbf{E}}\hat{\mathbf{E}} + \hat{\mathbf{B}}\hat{\mathbf{B}} \right) - \frac{1}{8\pi} \left(\hat{E}^2 + \hat{B}^2 \right) \mathbf{I} \right\rangle_{\text{ens}} - \frac{1}{4\pi c} \frac{\partial}{\partial t} \left\langle \hat{\mathbf{E}} \times \hat{\mathbf{B}} \right\rangle_{\text{ens}} \\ &= \nabla \cdot \mathbf{T}_{EM} - \frac{\partial}{\partial t} \left(\frac{\mathbf{S}_{EM}}{c^2} \right) , \end{aligned} \tag{13}$$

where $\mathbf{T}_{EM}$ and $\mathbf{S}_{EM}$ represent the Maxwell stress tensor and the Poynting vector due to the electromagnetic fluctuations, respectively. Using the Ampère's law with the displacement current and the formula $\langle (\nabla \times \mathbf{B}) \cdot \nabla s \rangle = \langle \nabla \cdot (\mathbf{B} \times \nabla s) \rangle = 0$, the surface-averaged radial current on the right-hand side of (12) is rewritten as

$$\sum_a e_a \Gamma_a \equiv \sum_a e_a \langle n_a u_a^s \rangle = -\frac{1}{4\pi} \frac{\partial}{\partial t} \langle E^s \rangle . \tag{14}$$

3.2. Axisymmetric systems and up-down symmetry

In this subsection, we consider axisymmetric toroidal systems, in which the magnetic field is written as $\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla\chi$. Here, $I = I(s)$ is a flux-surface function given by $I = RB_T$ with the major radius R and the toroidal field B_T. In axisymmetric systems, we have $\nabla(\partial\mathbf{x}/\partial\zeta) = R^{-1}[(\nabla R)(\partial\mathbf{x}/\partial\zeta) - (\partial\mathbf{x}/\partial\zeta)(\nabla R)]$, $(\nabla\mathbf{P}_a) : [\nabla(\partial\mathbf{x}/\partial\zeta)] = 0$, and $\langle(\partial\mathbf{x}/\partial\zeta)\cdot(\nabla\cdot\mathbf{P}_a)\rangle = \langle\nabla\cdot(\mathbf{P}_a\cdot\partial\mathbf{x}/\partial\zeta)\rangle = (V')^{-1}\partial\left[V'\left\langle(P_a)^s_\zeta\right\rangle\right]/\partial s$, where $V' = dV(s)/ds$ is the radial derivative of the volume $V(s)$ inside the flux surface with the label s. Then, (12) with $(c_1, c_2) = (0, 1)$ reduces to the toroidal momentum balance equation given by

$$\frac{\partial}{\partial t}\left\langle\sum_a n_a m_a u_{a\zeta} + \frac{(S_{EM})_\zeta}{c^2}\right\rangle = -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle\sum_a (P_a)^s_\zeta - (T_{EM})^s_\zeta\right\rangle\right] + \frac{\chi'}{c}\sum_a e_a\langle n_a u^s_a\rangle. \tag{15}$$

Noting the transport time scaling $\partial/\partial t = \mathcal{O}(\delta^2)$, the first and second terms on the left-hand side of (15) are of $\mathcal{O}(\delta^3)$ and $\mathcal{O}(\delta^4)$, respectively. We see from (14) that the radial current is of $\mathcal{O}(\delta^3)$. Then, the $\mathcal{O}(\delta^2)$ ambipolarity condition is derived as

$$\frac{\chi'}{c}\sum_a e_a\langle n_a u^s_a\rangle = \frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle\sum_a (P_a)^s_\zeta - (T_{EM})^s_\zeta\right\rangle\right] = 0, \tag{16}$$

where the $\mathcal{O}(\delta^2)$ radial transport of the toroidal momentum is written as

$$\left\langle\sum_a (P_a)^s_\zeta - (T_{EM})^s_\zeta\right\rangle = \sum_a\left(\Pi^{\rm ncl}_a + \Pi^H_a + \Pi^A_a\right) + \frac{1}{4\pi}\sum_{\mathbf{k}_\perp}\left\langle\left\langle(\mathbf{k}_\perp\mathbf{k}_\perp) : (R^2\nabla\zeta)(\nabla s)\left(|\phi_{\mathbf{k}_\perp}|^2 - |\mathbf{A}_{\mathbf{k}_\perp}|^2\right)\right\rangle\right\rangle. \tag{17}$$

Here, as shown in [20, 21, 35],

$$\Pi^{\rm ncl}_a + \Pi^H_a = -\frac{m_a c}{2e_a}\left\langle\int d^3v\left[m_a\left(\frac{Iv_\parallel}{B}\right)^2 + \mu\frac{|\nabla\chi|^2}{B}\right]C^L_a(\bar{f}_{a1})\right\rangle \tag{18}$$

represents the collisional radial transport flux of the toroidal momentum transport derived from the solution $\overline{f}_{a1}$ of the drift kinetic equation and

$$\Pi^A_a \equiv \mathrm{Re}\left\langle\left\langle\int d^3v\sum_{\mathbf{k}_\perp}\hat{h}^*_{a\mathbf{k}_\perp}\left[m_a c\left(\frac{Iv_\parallel}{B}\right)i\mathbf{k}_\perp\cdot(R^2\nabla\zeta)\,\hat{\psi}_{a\mathbf{k}_\perp} + e_a\frac{1}{k^2_\perp}(\mathbf{k}_\perp\mathbf{k}_\perp) : (R^2\nabla\zeta)(\nabla\chi)\,\hat{\chi}_{a\mathbf{k}_\perp}\right]\right\rangle\right\rangle \tag{19}$$

is the turbulent radial transport flux of the toroidal momentum transport. Here, $\hat{\psi}_{a\mathbf{k}_\perp}$ is defined after (9), and $\hat{\chi}_{a\mathbf{k}_\perp} = -\gamma_a J_1(\gamma_a)[\hat{\phi}_{\mathbf{k}_\perp} - (v_\parallel/c)\hat{A}_{\parallel\mathbf{k}_\perp}] + [\gamma_a J_0(\gamma_a) - J_1(\gamma_a)](v_\perp/c)(\hat{B}_{\parallel\mathbf{k}_\perp}/k_\perp)$ is used, where $\gamma_a \equiv k_\perp v_\perp/\Omega_a$.

Generally, when up-down symmetry is broken [23, 24], the $\mathcal{O}(\delta^2)$ ambipolarity condition shown in (16) imposes a nontrivial constraint to give the radial electric field $E_s = \mathcal{O}(\epsilon_A)$, where ϵ_A denotes a measure of up-down asymmetry which, for example, is given by $\epsilon_A \sim \langle \partial \ln B / \partial \theta \rangle$. Therefore, if up-down asymmetry is strong ($\epsilon_A \gg \delta$), we need treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ as discussed in Sec. 5.

Now, we consider axisymmetric toroids with up-down symmetry. As shown in Sec. 4, in this case, the collisional and turbulent radial fluxes of the toroidal momentum shown in (17)–(19) vanish automatically up to $\mathcal{O}(\delta^2)$. Then, the lowest-order terms in the toroidal momentum balance equation (15) are of $\mathcal{O}(\delta^3)$. Up to $\mathcal{O}(\delta^3)$, (15) is written as

$$\frac{\partial}{\partial t}\left[\frac{\chi'}{4\pi c}\left(\langle|\nabla s|^2\rangle + 4\pi c^2 \sum_a n_a m_a \frac{\langle R^2\rangle}{(\chi')^2}\right) E_s + \sum_a m_a \left(-\frac{c\langle R^2\rangle}{e_a \chi'}\frac{\partial p_a}{\partial s} + n_a I \frac{u_a^\theta}{B^\theta}\right)\right]$$
$$= -\frac{1}{V'}\frac{\partial}{\partial s}\left[V'\left\langle \sum_a (P_a)^s_\zeta - (T_{EM})^s_\zeta \right\rangle^{(3)}\right], \tag{20}$$

which describes the temporal variation of the toroidal momentum or the E_s profile caused by the radial toroidal momentum transport due to collisions and turbulent fluctuations. On the right-hand side of (20), $\langle\cdots\rangle^{(3)}$ represents that these toroidal momentum transport terms are of $\mathcal{O}(\delta^3)$. In deriving (20), we have used

$$\langle n_a m_a u_{a\zeta}\rangle = \frac{n_a m_a c}{\chi'}\langle R^2\rangle\left(-\frac{1}{n_a e_a}\frac{\partial p_a}{\partial s} + E_s\right) + n_a m_a I \frac{u_a^\theta}{B^\theta}, \tag{21}$$

which is derived from the incompressible-flow condition and is valid to $\mathcal{O}(\delta)$. In the axisymmetric system, u_a^θ / B^θ is a flux-surface function which can be written in terms of the thermodynamic forces other than E_s by using the $\mathcal{O}(\delta)$ parallel momentum balance equations based on the neoclassical theory [5, 6]. The difficulty in determining E_s arises from the right-hand-side terms in (20). In order to evaluate the $\mathcal{O}(\delta^3)$ toroidal momentum transport terms $\left\langle \sum_a (P_a)^s_\zeta - (T_{EM})^s_\zeta \right\rangle^{(3)}$, we need the solutions of drift kinetic and gyrokinetic equations with higher-order accuracy than the conventional ones [12]. However, it should be emphasized that, in axisymmetric systems, the lowest-order neoclassical and anomalous fluxes of particle and heat and the parallel current are independent of E_s. Here, the lowest-order anomalous transport fluxes are considered to be uninfluenced by the $\mathcal{O}(\delta v_T)$ $\mathbf{E} \times \mathbf{B}$ velocity profile with the gradient scale length L because the $\mathbf{E} \times \mathbf{B}$ shearing rate of $\mathcal{O}(\delta v_T / L)$ is slower than the typical gyrokinetic turbulence frequency of $\mathcal{O}(v_T / L)$ by a factor of δ. Therefore, we can calculate the time evolution of the density and temperature profiles without knowing E_s when the $\mathbf{E} \times \mathbf{B}$ drift velocity is on the order of δv_T. If the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(v_{Ti})$ exists, the neoclassical and anomalous transport fluxes are explicitly dependent on E_s and the toroidal momentum balance equation governing the time evolution of the E_s profile is given in $\mathcal{O}(\delta^2)$ as explained in Sec. 5.

3.3. Nonaxisymmetric systems, quasisymmetry, and stellarator symmetry

In nonaxisymmetric systems without quasisymmetry, we need to specify the radial electric field E_s to evaluate the neoclassical particle and heat fluxes. In both axisymmetric and nonaxisymmetric systems, the lowest-order nontrivial parallel momentum balance equation given from (11) is of $\mathcal{O}(\delta)$ and written as

$$\sum_a \left\langle \mathbf{B} \cdot \left(\nabla \cdot \mathbf{P}_a^{(1)} \right) \right\rangle = \sum_a \left[B^\theta \left\langle \frac{\partial \mathbf{x}}{\partial \theta} \cdot \left(\nabla \cdot \mathbf{P}_a^{(1)} \right) \right\rangle + B^\zeta \left\langle \frac{\partial \mathbf{x}}{\partial \zeta} \cdot \left(\nabla \cdot \mathbf{P}_a^{(1)} \right) \right\rangle \right] = 0, \quad (22)$$

where the Hamada [36] coordinates (s, θ, ζ) are used to regard B^θ and B^ζ as flux-surface functions. Here, the $\mathcal{O}(\delta)$ pressure tensor $\mathbf{P}_a^{(1)}$ takes the form of $\mathbf{P}_a^{(1)} = \int d^3v \overline{f}_{a1}[v_\parallel^2 \mathbf{bb} + (v_\perp^2/2)(\mathbf{I} - \mathbf{bb})] = P_{\parallel a}^{(1)} \mathbf{bb} + P_{\perp a}^{(1)} (\mathbf{I} - \mathbf{bb})$. The $\mathcal{O}(\delta)$ parallel viscosity $\left\langle \mathbf{B} \cdot \left(\nabla \cdot \mathbf{P}_a^{(1)} \right) \right\rangle$ can be determined from the neoclassical theory [37, 38]. This parallel momentum balance does not give a sufficient condition to determine E_s from other thermodynamic forces such as pressure and temperature gradients. For nonaxisymmetric systems without quasisymmetry, (12) is rewritten to the lowest order in δ as

$$\sum_a \left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \left(\nabla \cdot \mathbf{P}_a^{(1)} \right) \right\rangle = \frac{1}{c} (-c_1 \psi' + c_2 \chi') \sum_a e_a \langle n_a u_a^s \rangle = 0. \quad (23)$$

The poloidal and toroidal viscosity terms of $\mathcal{O}(\delta)$ appearing on the left-hand side of (23) can be evaluated by the neoclassical theory using the solution of the drift kinetic equation [39]. When $c_1/c_2 \neq B^\theta / B^\zeta$, (23) gives another constraint independent of (22) and represents the $\mathcal{O}(\delta)$ ambipolarity condition that determines the radial electric field $E_s = -\partial \Phi / \partial s$ from the radial gradients of equilibrium pressures and temperatures and the parallel electric field $\langle B E_\parallel \rangle$ [4, 39, 40, 41].

We now consider nonaxisymmetric systems with quasisymmetry, which satisfy

$$c_1 \frac{\partial B}{\partial \theta} + c_2 \frac{\partial B}{\partial \zeta} = 0. \quad (24)$$

The quasi-axisymmetry and quasi-poloidal-symmetry correspond to $(c_1, c_2) = (0, 1)$ and $(1, 0)$, respectively. It is shown in [39] that the quasisymmetry condition (24) written in the Hamada coordinates is equivalent to the one written in the Boozer coordinates [42]. Using the quasisymmetry condition in (24), we obtain

$$\begin{aligned} & \left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \left[\nabla \cdot \left\{ P_{\parallel a} \mathbf{bb} + P_{\perp a} (\mathbf{I} - \mathbf{bb}) \right\} \right] \right\rangle \\ &= - \left\langle \left(P_{\parallel a} - P_{\perp a} \right) \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \nabla \ln B \right\rangle = 0. \end{aligned} \quad (25)$$

Therefore, we find that the $\mathcal{O}(\delta)$ ambipolarity condition shown in (23) holds automatically for an arbitrary value of E_s in the same way as in the axisymmetric case. Then, in (12), the lowest-order terms $\langle (c_1 \partial \mathbf{x}/\partial \theta + c_2 \partial \mathbf{x}/\partial \zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle$ and $\langle c_1 K_{a1\theta} + c_2 K_{a1\zeta} \rangle$ are of $\mathcal{O}(\delta^2)$. Recalling (14) which shows that the radial current is of $\mathcal{O}(\delta^3)$, the $\mathcal{O}(\delta^2)$ ambipolarity condition is obtained from (12) and (13) as

$$\begin{aligned} & \frac{1}{c} (-c_1 \psi' + c_2 \chi') \sum_a e_a \langle n_a u_a^s \rangle \\ &= \sum_a \left\langle \left(c_1 \frac{\partial \mathbf{x}}{\partial \theta} + c_2 \frac{\partial \mathbf{x}}{\partial \zeta} \right) \cdot \left[\nabla \cdot \left(\sum_a \mathbf{P}_a - \mathbf{T}_{EM} \right)^{(2)} \right] \right\rangle = 0, \end{aligned} \quad (26)$$

where $(\cdots)^{(2)}$ represents that the pressure tensor and turbulent Maxwell stress terms are of $\mathcal{O}(\delta^2)$. Similarly to the axisymmetric case with up-down asymmetry, if a quasisymmetric system does not satisfy stellarator symmetry, the $\mathcal{O}(\delta^2)$ ambipolarity condition shown in (26) imposes a nontrivial constraint to give $E_s = \mathcal{O}(\epsilon_A)$, where ϵ_A denotes a measure of stellarator-symmetry breaking which, for example, is given by $\epsilon_A \sim \langle (c_2 \partial \ln B/\partial\theta - c_1 \partial \ln B/\partial\zeta)\rangle$. Therefore, if stellarator symmetry is strongly breaking ($\epsilon_A \gg \delta$), treatment of the large $\mathbf{E}\times\mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ seems to be required. However, in Sec. 5, we discuss the difficulty arising from the large $\mathbf{E}\times\mathbf{B}$ drift velocity in quasisymmetric systems.

We now consider quasisymmetric systems with stellarator symmetry. As shown in Sec. 4, in the presence of stellarator symmetry, all $\mathcal{O}(\delta^2)$ terms in (12) vanish automatically. Then, keeping terms of $\mathcal{O}(\delta^3)$, (12) reduces to

$$\begin{aligned}
&\frac{\partial}{\partial t}\left[\frac{(c_2\chi' - c_1\psi')}{4\pi c}\left\{\langle|\nabla s|^2\rangle + \frac{4\pi c^2 \sum_a n_a m_a}{(c_2\chi' - c_1\psi')^2}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle\right\} E_s\right.\\
&\left. + \sum_a \frac{m_a}{(c_2\chi' - c_1\psi')}\left\{-\frac{c}{e_a}\frac{\partial p_a}{\partial s}\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle + \frac{n_a V'}{4\pi^2}\langle c_1 B_\theta + c_2 B_\zeta\rangle \left\langle c_2 u_a^\theta - c_1 u_a^\zeta\right\rangle\right\}\right]\\
&= \left\langle\left(c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right)\cdot\left[\nabla\cdot\left(-\sum_a \mathbf{P}_a + \mathbf{T}_{EM}\right)^{(3)}\right]\right\rangle, \qquad (27)
\end{aligned}$$

which describes the temporal variation of E_s profile caused by the radial toroidal momentum transport due to collisions and turbulent fluctuations. On the right-hand side of (27), $(\cdots)^{(3)}$ represents that the pressure tensor and turbulent Maxwell stress terms are of $\mathcal{O}(\delta^3)$.

In deriving (27), we have used (14) and the $\mathcal{O}(\delta)$ formula for the flow in the quasisymmetry direction,

$$\begin{aligned}
\langle n_a m_a (c_1 u_{a\theta} + c_2 u_{a\zeta})\rangle = \frac{n_a m_a}{(c_2\chi' - c_1\psi')}&\left[c\left(-\frac{1}{n_a e_a}\frac{\partial p_a}{\partial s} + E_s\right)\left\langle\left|c_1\frac{\partial \mathbf{x}}{\partial\theta} + c_2\frac{\partial \mathbf{x}}{\partial\zeta}\right|^2\right\rangle\right.\\
&\left. + \frac{V'}{4\pi^2}\langle c_1 B_\theta + c_2 B_\zeta\rangle\left\langle c_2 u_a^\theta - c_1 u_a^\zeta\right\rangle\right]. \qquad (28)
\end{aligned}$$

Equations (27) and (28) are valid whichever of Boozer or Hamada coordinates are used. In the quasisymmetric system, $\left\langle c_2 u_a^\theta - c_1 u_a^\zeta\right\rangle$ can be expressed in terms of the thermodynamic forces other than E_s by using the $\mathcal{O}(\delta)$ parallel momentum balance equations based on the neoclassical theory [39]. Similarly to the axisymmetric case with up-down symmetry, it is difficult to evaluate the $\mathcal{O}(\delta^3)$ terms on the right-hand side of (27) which is necessary for determination of E_s. However, it is emphasized again that the lowest-order transport fluxes necessary for determining the density and temperature profiles can be calculated without knowing E_s for quasisymmetric systems with the $\mathbf{E}\times\mathbf{B}$ drift velocity of $\mathcal{O}(\delta v_T)$.

4. Axisymmetric systems with up-down symmetry and helical systems with stellarator symmetry

Here, we consider axisymmetric systems with up-down symmetry and helical systems with stellarator symmetry. The purpose of this section is to show that, in these systems, the ambipolarity condition $\sum_a e_a \Gamma_a = 0$ is automatically satisfied up to $\mathcal{O}(\delta^2)$ by the classical, neoclassical, and anomalous particle fluxes for any value of the radial electric field E_s of $\mathcal{O}(\delta v_T)$. In the following, mathematical analyses for stellarator symmetry are described although those of up-down symmetry of axisymmetric systems are immediately obtained by just dropping the ζ-dependence ($\partial/\partial\zeta \to 0$) from the case of stellarator symmetry.

First, we describe the conditions satisfied in the toroidal systems with stellarator symmetry. In this stage, we do not assume quasisymmetric toroidal systems necessarily. Figure 1 shows an example of nonaxisymmetric toroidal systems with stellarator symmetry. We see from Fig. 1 that there exists an axis lying on the equatorial plane such that the system looks unchanged by a rotation about it by 180 degrees. When defining the origin $(\theta, \zeta) = (0, 0)$ at the position where this symmetry axis intersects the flux surface, the magnetic field strength $B(s, \theta, \zeta)$ has the same value at the points (s, θ, ζ) and $(s, -\theta, -\zeta)$. Thus, we have

$$B(s, -\theta, -\zeta) = B(s, \theta, \zeta). \tag{29}$$

We also find that the contravariant and covariant components of the magnetic field in the flux coordinates (s, θ, ζ) have the following parities with respect to the change in the signs of θ and ζ,

$$\begin{aligned}
&B^\theta(s, -\theta, -\zeta) = B^\theta(s, \theta, \zeta), \qquad B^\zeta(s, -\theta, -\zeta) = B^\zeta(s, \theta, \zeta), \\
&B_\theta(s, -\theta, -\zeta) = B_\theta(s, \theta, \zeta), \qquad B_\zeta(s, -\theta, -\zeta) = B_\zeta(s, \theta, \zeta), \\
&B_s(s, -\theta, -\zeta) = -B_s(s, \theta, \zeta),
\end{aligned} \tag{30}$$

where we should recall that $B^s = 0$. Furthermore, the covariant components of the metric tensor show even or odd parities as follows,

$$\begin{aligned}
&g_{ss}(s, -\theta, -\zeta) = g_{ss}(s, \theta, \zeta), \qquad g_{\theta\theta}(s, -\theta, -\zeta) = g_{\theta\theta}(s, \theta, \zeta), \\
&g_{\theta\zeta}(s, -\theta, -\zeta) = g_{\theta\zeta}(s, \theta, \zeta), \qquad g_{\zeta\zeta}(s, -\theta, -\zeta) = g_{\zeta\zeta}(s, \theta, \zeta), \\
&g_{s\theta}(s, -\theta, -\zeta) = -g_{s\theta}(s, \theta, \zeta), \qquad g_{s\zeta}(s, -\theta, -\zeta) = -g_{s\zeta}(s, \theta, \zeta), \\
&g(s, -\theta, -\zeta) = g(s, \theta, \zeta),
\end{aligned} \tag{31}$$

where $g = \det(g_{\alpha\beta})$. The contravariant components $g^{\alpha\beta}$ have the same parities as the corresponding covariant components $g_{\alpha\beta}$ shown in (31).

We now introduce a formal expansion parameter η which represents the same order of magnitude as the small gyroradius expansion parameter δ. Therefore, $\mathcal{O}(\eta) = \mathcal{O}(\delta)$. This new notation η is used in order to clarify the role played by its sign that is changed by a certain parity operator associated with stellarator symmetry as explained later. It is useful to represent the small-gyroradius or large-gyrofrequency ordering by writing the electric charge as $\eta^{-1}e_a$ [43]. The above-mentioned rule for using η is adopted in the basic kinetic equation (1) as well as in the Maxwell equations. Then, the distribution functions and the electromagnetic fields which are given as the solutions of these equations are considered to include η as a parameter. For example, the ensemble-averaged distribution function f_a is regarded as a function of $(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta)$ and is expanded with respect to the parameter η as

$$\begin{aligned} f_a(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta) &= f_{aM}(s, v, \eta^2 t) + \eta f_{a1}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t) \\ &\quad + \eta^2 f_{a2}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t) + \cdots, \end{aligned} \tag{32}$$

where the lowest-order of f_a is given by the local Maxwellian function, $f_{aM}(s, v, \eta^2 t) = n_{a0}(m_a/2\pi T_{a0})^{3/2}\exp(-m_a v^2/2T_{a0})$ with the density $n_{a0} = n_{a0}(s, \eta^2 t)$ and the temperature $T_{a0} = T_{a0}(s, \eta^2 t)$. Here, t enters the functions through the form $\eta^2 t$ which assures that the temporal variation is consistent with the transport time scale ordering, $\partial/\partial t = \mathcal{O}(\eta^2)$. The electrostatic potential Φ associated with the ensemble-averaged electric field $\mathbf{E}$ of $\mathcal{O}(\delta v_T)$ is written as

$$\Phi(s, \theta, \zeta, t, \eta) = \eta\Phi_1(s, \eta^2 t) + \eta^2\Phi_2(s, \theta, \zeta, \eta^2 t) + \cdots. \tag{33}$$

We assume turbulent fluctuations to be expressed by using the eikonal representation [44]. For example, the perturbed distribution function $\hat{f}_a$ is written as

$$\hat{f}_a = \sum_{\mathbf{k}_\perp} \hat{f}_{a\mathbf{k}_\perp} \exp\left(i\eta^{-1}S_{\mathbf{k}_\perp}\right), \tag{34}$$

where $S_{\mathbf{k}_\perp} = S_{\mathbf{k}_\perp}(s, \theta, \zeta)$ is the eikonal, and the amplitude part $\hat{f}_{\mathbf{k}_\perp}$ is written in terms of the parameter η as

$$\hat{f}_{a\mathbf{k}_\perp} = \eta\hat{f}^{(1)}_{a\mathbf{k}_\perp}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t) + \eta^2\hat{f}^{(2)}_{a\mathbf{k}_\perp}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t) + \cdots. \tag{35}$$

The gradient of the eikonal gives the perpendicular wave number vector as

$$\mathbf{k}_\perp = \nabla S_{\mathbf{k}_\perp} = k_s\nabla s + k_\alpha\nabla[\zeta - q(s)\theta]. \tag{36}$$

We also note that $\partial/\partial t = \mathcal{O}(\eta^0)$ for turbulent fluctuations. The lowest-order perturbed distribution function $\hat{f}^{(1)}_{\mathbf{k}_\perp}$ in (35) and the lowest-order turbulent electromagnetic fields are given as the solutions of a closed system of the gyrokinetic Boltzmann and Maxwell equations [31].

We now define the parity operator $\mathcal{P}$ acting on an arbitrary function $Q(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta)$ by

$$(\mathcal{P}Q)(s, \theta, \zeta, v^s, v^\theta, v^\zeta, t, \eta) \equiv Q(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, t, -\eta). \tag{37}$$

Then, it is found for the equilibrium magnetic field geometry with stellarator symmetry satisfying (29)–(31) that, if $f_a + \hat{f}_a$, $E_s + \hat{E}_s$, $E_\theta + \hat{E}_\theta$, $E_\zeta + \hat{E}_\zeta$, $B_s + \hat{B}_s$, $B_\theta + \hat{B}_\theta$, and $B_\zeta + \hat{B}_\zeta$ are solutions of the basic kinetic and Maxwell equations, $\mathcal{P}(f_a + \hat{f}_a)$, $-\mathcal{P}(E_s + \hat{E}_s)$, $\mathcal{P}(E_\theta + \hat{E}_\theta)$, $\mathcal{P}(E_\zeta + \hat{E}_\zeta)$, $-\mathcal{P}(B_s + \hat{B}_s)$, $\mathcal{P}(B_\theta + \hat{B}_\theta)$, and $\mathcal{P}(B_\zeta + \hat{B}_\zeta)$ are also solutions. Note that the contravariant components of the electromagnetic fields have the same symmetry as the covariant components because of (31). This symmetry with respect to the parity transformation $\mathcal{P}$ is also shown to be valid for the ensemble-averaged and perturbed parts, separately. Here, the distribution function f_a and the electrostatic potential Φ given by (32) and (33) are considered as solutions, the lowest-order parts of which are given as the local Maxwellian $f_{aM}(s, v, \eta^2 t)$ and $\eta\Phi_1(s, \eta^2 t)$, respectively. Since $\mathcal{P} f_{aM} = f_{aM}$ and $-\mathcal{P}(\eta\Phi_1) = \eta\Phi_1$, we see that $\mathcal{P} f_a$ and $-\mathcal{P}(\eta\Phi_1)$ are also the solution which has the same lowest-order parts as f_a and Φ. Then,

$$\mathcal{P} f_a = f_a, \qquad -\mathcal{P}\Phi = \Phi \tag{38}$$

are derived from uniqueness of those solutions which have the same lowest-order parts. Since the perturbed parts of the Boltzmann and Maxwell equations have the same symmetry with respect to $\mathcal{P}$, the probability distribution of the ensemble of the perturbed solutions is considered to be invariant under the parity transformation,

$$(\hat{f}_a, \hat{E}_s, \hat{E}_\theta, \hat{E}_\zeta, \hat{B}_s, \hat{B}_\theta, \hat{B}_\zeta) \longrightarrow (\mathcal{P}\hat{f}_a, -\mathcal{P}\hat{E}_s, \mathcal{P}\hat{E}_\theta, \mathcal{P}\hat{E}_\zeta, -\mathcal{P}\hat{B}_s, \mathcal{P}\hat{B}_\theta, \mathcal{P}\hat{B}_\zeta). \tag{39}$$

Therefore, we obtain $\mathcal{P}\mathcal{D}_a = \mathcal{D}_a$ in the ensemble-averaged kinetic equation (3), which shows the symmetry with respect to $\mathcal{P}$ in consistence with (38). The gyrokinetic equation for the lowest-order perturbed distribution function $\hat{f}_{a1}$ written in (34) also retains the same symmetry. Defining $\overline{\mathbf{k}}_\perp$ from $\mathbf{k}_\perp$ in (36) by

$$\overline{\mathbf{k}}_\perp \equiv \nabla S_{\overline{\mathbf{k}}_\perp} \equiv -\nabla(\mathcal{P} S_{\mathbf{k}_\perp}) = -k_s \nabla s + k_\alpha \nabla[\zeta - q(s)\theta], \tag{40}$$

$\hat{f}_{a1}$ shown in (34) is transformed by $\mathcal{P}$ as

$$\mathcal{P}\hat{f}_{a1} = \sum_{\mathbf{k}_\perp} \left(\mathcal{P}\hat{f}_{a\mathbf{k}_\perp}\right) \exp\left(i\eta^{-1} S_{\overline{\mathbf{k}}_\perp}\right) = \sum_{\mathbf{k}_\perp} \left(\mathcal{P}\hat{f}_{a\overline{\mathbf{k}}_\perp}\right) \exp\left(i\eta^{-1} S_{\mathbf{k}_\perp}\right). \tag{41}$$

From the invariance with respect to the parity transformation in (39), we can derive

$$\begin{aligned} &\langle K^{(j)}_{a1\theta}\rangle = \langle K^{(j)}_{a1\zeta}\rangle = \langle (T^{(j)}_{EM})^s_\theta\rangle = \langle (T^{(j)}_{EM})^s_\zeta\rangle \\ &= \langle (S^{(j)}_{EM})_\theta\rangle = \langle (S^{(j)}_{EM})_\zeta\rangle = 0 \qquad \text{(for even } j), \end{aligned} \tag{42}$$

where the subscript (j) is used to represent the $\mathcal{O}(\delta^j)$ part of the quantities.

Using (32), (33), and (38), we obtain

$$f_{aj}(s, -\theta, -\zeta, v^s, -v^\theta, -v^\zeta, \eta^2 t) = (-1)^j f_{aj}(s, \theta, \zeta, v^s, v^\theta, v^\zeta, \eta^2 t), \tag{43}$$

and

$$\Phi_j(s, -\theta, -\zeta, \eta^2 t) = (-1)^{j-1} \Phi_j(s, \theta, \zeta, \eta^2 t), \tag{44}$$

where $j = 1, 2, \cdots$. We find from the (43) that, when j is an even number, f_{aj} is an even function with respect to the parity transformation $\mathcal{P}$. Therefore, we finally obtain

$$\left\langle \left(P^{(j)}_a\right)^s_\theta \right\rangle = \left\langle \left(P^{(j)}_a\right)^s_\zeta \right\rangle = 0 \qquad \text{(for even } j), \tag{45}$$

and

$$\left\langle \frac{\partial \mathbf{x}}{\partial \theta} \cdot \left(\nabla \cdot \mathbf{P}^{(j)}_a\right) \right\rangle = \left\langle \frac{\partial \mathbf{x}}{\partial \zeta} \cdot \left(\nabla \cdot \mathbf{P}^{(j)}_a\right) \right\rangle = 0 \qquad \text{(for even } j), \tag{46}$$

where $\mathbf{P}^{(j)}_a \equiv \int d^3v\; f_{aj} m_a \mathbf{v}\mathbf{v}$. As seen in (25), $\left\langle (c_1 \partial\mathbf{x}/\partial\theta + c_2 \partial\mathbf{x}/\partial\zeta) \cdot \left(\nabla \cdot \mathbf{P}^{(1)}_a\right)\right\rangle = 0$ is automatically satisfied in systems with quasisymmetry $c_1 \partial B/\partial\theta + c_2 \partial B/\partial\zeta = 0$. Then, we find from (26), (42), and (46) that the ambipolarity condition $\sum_a e_a \Gamma_a = 0$

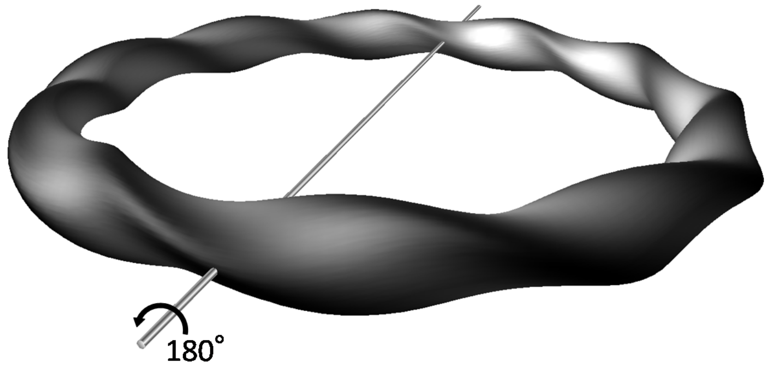

Figure 1. A nonaxisymmetric toroidal system with stellarator symmetry. The system looks unchanged by a rotation about the shown axis by 180 degrees.

and $\langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a)\rangle = 0$ are automatically satisfied up to $\mathcal{O}(\delta^2)$ in quasisymmetric systems with stellarator symmetry.

In the same way as described above, it is shown from (16), (42), and (45) that the ambipolarity condition $\sum_a e_a \Gamma_a = 0$ and $\langle (P_a)^s_\zeta \rangle = \langle (T_{EM})^s_\zeta \rangle = 0$ are automatically satisfied up to $\mathcal{O}(\delta^2)$ in axisymmetric systems with up-down symmetry.

5. Toroidal plasmas with $\mathbf{E} \times \mathbf{B}$ drift velocities of $\mathcal{O}(v_{Ti})$

In the previous sections, we have treated the toroidal systems with $\mathbf{E} \times \mathbf{B}$ drift velocities of $\mathcal{O}(\delta v_T)$. In such cases, if axisymmetry or quasisymmetry exists, E_s does not influence the lowest-order transport of particles Γ_a, heat q_a, and parallel current $\langle J_\parallel B \rangle$. In this section, we consider toroidal plasmas with $\mathbf{E} \times \mathbf{B}$ drift velocities of $\mathcal{O}(v_{Ti})$. Actually, it is noted in Sec. 3.2 and Table 1 that, if up-down symmetry is strongly broken in axisymmetric systems ($\epsilon_A \gg \delta$), the large radial electric field $E_s = \mathcal{O}(\epsilon_A)$ driven by the momentum balance requires treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity.

The equilibrium flow in the axisymmetric system with the $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(v_{Ti})$ is in the toroidal direction and it is written as[17, 18, 19, 20, 21, 22]

$$\mathbf{V} = V^\zeta \frac{\partial \mathbf{x}}{\partial \zeta}, \qquad V^\zeta = -c \frac{\Phi_0'(s)}{\chi'(s)} = \mathcal{O}(v_{Ti}). \tag{47}$$

Here, $\Phi_0(s)$ is the lowest-order electrostatic potential satisfying $e_a \Phi_0 / T_a = \mathcal{O}(\delta^{-1})$, which gives the toroidal flow on the order of the ion thermal velocity, $V = RV^\zeta = \mathcal{O}(v_{Ti})$. The inertia term resulting from the toroidal flow $\mathbf{V}$ now enters the equilibrium force balance as

$$\left(\sum_a n_a m_a \right) \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla P. \tag{48}$$

Several past works[17, 18, 19, 20, 21, 22] showed that, for this high-speed rotating plasma, the radial derivative of the toroidal flow [or equivalently the shear of the radial electric field due to (47)] becomes one of thermodynamic forces to drive the radial neoclassical and turbulent transport fluxes of particles Γ_a, heat q_a, and toroidal

Table 1. Remarks on the transport fluxes Γ_a, q_a, $\langle J_\parallel B \rangle$, $\langle (P_a)^s_\zeta \rangle$, $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle$, the viscosity terms $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle$, $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle$, the ambipolarity condition $\sum_a e_a \Gamma_a = 0$, and the radial electric field E_s in toroidal systems with the $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\delta v_T)$. Here, ϵ_A is used to represent a measure of up-down asymmetry or stellarator-symmetry breaking.

Axisymmetric system with up-down symmetry
$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = 0$ and $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $\sum_a e_a \Gamma_a = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $E_s = \mathcal{O}(\delta)$ is determined from the $\mathcal{O}(\delta^3)$ toroidal momentum balance equation (20).
Axisymmetric system without up-down symmetry
$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = \mathcal{O}(\epsilon_A \delta^2)$ and $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\epsilon_A \delta^2)$ $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta^2)$ drives $E_s = \mathcal{O}(\epsilon_A)$. $\Longrightarrow$ requires treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ when $\epsilon_A \gg \delta$ (see Secs. 3.2 and 5).
Nonaxisymmetric system without quasisymmetry
$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are dependent on $E_s = \mathcal{O}(\delta)$. $\langle (P_a)^s_\zeta \rangle = \mathcal{O}(\delta)$ and $\sum_a \langle \partial \mathbf{x}/\partial\zeta \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\delta)$ $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta)$ determines E_s.
Quasisymmetric system with stellarator symmetry
$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle = 0$ and $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. $\sum_a e_a \Gamma_a = 0$ holds up to $\mathcal{O}(\delta^2)$ for any $E_s = \mathcal{O}(\delta)$. E_s is determined from the $\mathcal{O}(\delta^3)$ momentum balance equation, (27).
Quasisymmetric system without stellarator symmetry
$\Gamma_a = \mathcal{O}(\delta^2)$, $q_a = \mathcal{O}(\delta^2)$, and $\langle J_\parallel B \rangle = \mathcal{O}(\delta^0)$ are independent of $E_s = \mathcal{O}(\delta)$. $\langle c_1 (P_a)^s_\theta + c_2 (P_a)^s_\zeta \rangle = \mathcal{O}(\epsilon_A \delta^2)$ and $\sum_a \langle (c_1 \partial \mathbf{x}/\partial\theta + c_2 \partial \mathbf{x}/\partial\zeta) \cdot (\nabla \cdot \mathbf{P}_a) \rangle = \mathcal{O}(\epsilon_A \delta^2)$ $\sum_a e_a \Gamma_a = 0$ up to $\mathcal{O}(\delta^2)$ drives $E_s = \mathcal{O}(\epsilon_A)$. $\Longrightarrow$ requires treatment of the large $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(\epsilon_A v_{Ti})$ when $\epsilon_A \gg \delta$ (see Secs. 3.3 and 5).

momentum Π_a, and the parallel current $\langle BJ_\parallel \rangle$ while the time evolution of the radial electric field is governed by the toroidal momentum balance equation which looks similar to (20) but is $\mathcal{O}(\delta^2)$ instead of $\mathcal{O}(\delta^3)$.

Now, let us consider whether the $\mathbf{E} \times \mathbf{B}$ drift velocity of $\mathcal{O}(v_{Ti})$ can be treated in quasisymmetric systems in the same way as in the axisymmetric case shown above. Here, for simplicity, we restrict our consideration to the quasiaxisymmetric case, in which $\partial B/\partial \zeta = 0$. In this case, we see that the $\mathcal{O}(v_{Ti})$ equilibrium flow should be in the direction of $\partial \mathbf{x}/\partial \zeta$ as given in (47) and that the equilibrium force balance takes the same form as (48). Then, taking the inner product between (48) and $\partial \mathbf{x}/\partial \zeta$, we obtain

$$\frac{1}{2}\left(\sum_a n_a m_a\right)(V^\zeta)^2 \frac{\partial g_{\zeta\zeta}}{\partial \zeta} = \frac{\chi'}{c} J^s = \frac{B^\theta}{c}\left(\frac{\partial B_\zeta}{\partial \theta} - \frac{\partial B_\theta}{\partial \zeta}\right), \tag{49}$$

where $g_{\zeta\zeta} \equiv |\partial \mathbf{x}/\partial \zeta|^2$. In the rigorous axisymmetric case, $g_{\zeta\zeta} = R^2$ leads to $\partial g_{\zeta\zeta}/\partial \zeta = 0$. However, in the quasiaxisymmetric case, $\partial g_{\zeta\zeta}/\partial \zeta \neq 0$ are generally obtained and therefore (49) results in nonzero radial current $J^s \neq 0$. This gives rise to a serious problem because the quasisymmetric system is considered usually by using the Boozer coordinates while neither Boozer nor Hamada coordinates cannot be constructed for the case of $J^s \neq 0$. Thus, the $\mathcal{O}(v_{Ti})$ flow in the $\partial \mathbf{x}/\partial \zeta$-direction does not seem to be allowed by the quasiaxisymmetry condition $\partial B/\partial \zeta = 0$ alone, and this statement can be extended to more general cases with quasisymmetry.

Quasisymmetric systems without stellarator symmetry have not been seen in the literature. However, if stellarator symmetry is strongly broken in quasisymmetric system ($\epsilon_A \gg \delta$), the large radial electric field $E_s = \mathcal{O}(\epsilon_A)$ driven by the momentum balance (see Table 1) may also break the quasisymmetry itself due to the nonzero radial current $J^s \neq 0$ as explained above.

6. Conclusions

In this work, it is shown how symmetry and asymmetry of the magnetic field geometry of toroidal plasmas influence the radial electric field dependence of radial transport of particles, heat, and momentum. Results are summarized in Table 1.

In nonaxisymmetric systems, the radial electric field E_s is generally determined from the ambipolarity condition of only the neoclassical particle fluxes while the turbulent particle fluxes is automatically ambipolar for any E_s. In fact, it is found in the Large Helical Device (LHD) experiments that observed radial electric fields are in reasonable agreement with those predicted from the neoclassical ambipolarity condition even though observed particle fluxes contain significant anomalous parts [45].

When the $\mathbf{E} \times \mathbf{B}$ drift velocity is on the order of the diamagnetic drift velocity, there exist similarities between axisymmetric systems with up-down symmetry and quasisymmetric systems with stellarator symmetry, for which the basic kinetic and Maxwell equations are shown to be invariant under a certain parity transformation. In both systems, the particle fluxes are automatically ambipolar up to $\mathcal{O}(\delta^2)$ and the $\mathcal{O}(\delta^3)$ momentum balance equations determine the time evolution of the E_s profiles while

E_s does not influence the lowest-order particle and heat transport fluxes. Therefore, the determination of E_s requires solutions of the drift kinetic and gyrokinetic equations of higher-order accuracy than the conventional ones, although the time evolution of the equilibrium density and temperature profiles can be determined without E_s. Here, the $\mathcal{O}(\delta v_T)$ $\mathbf{E}\times\mathbf{B}$ velocity profile with the gradient scale length L does not affect the lowest-order anomalous transport because the $\mathbf{E}\times\mathbf{B}$ shearing rate ($\sim \delta v_T/L$) is much slower than the typical gyrokinetic turbulence frequency ($\sim v_T/L$).

In axisymmetric systems with the large $\mathbf{E}\times\mathbf{B}$ flows of $\mathcal{O}(v_{Ti})$, the radial fluxes of particles, heat, and toroidal momentum are dependent on E_s and its radial derivative while the time evolution of the E_s profile is governed by the $\mathcal{O}(\delta^2)$ toroidal momentum balance equation. In quasisymmetric systems, $\mathbf{E}\times\mathbf{B}$ flows of $\mathcal{O}(v_{Ti})$ are not allowed generally because it yields the nonzero radial current from the equilibrium force balance for which the Boozer and Hamada coordinates cannot be constructed.

It is emphasized from the results shown above for both cases of the small and large $\mathbf{E}\times\mathbf{B}$ drift velocity that the easiness of determining E_s and the necessity to calculate E_s for solving the transport equations for the equilibrium density and temperature profiles are closely related and happen simultaneously. When E_s is influential and necessary in evaluating the lowest-order particle and heat transport fluxes, E_s is determined by the low-order ambipolarity condition or the low-order momentum balance equation. If determination of E_s requires solution of the higher-order complicated momentum balance, the lowest-order particle and heat fluxes are independent of E_s. The latter case corresponds to axisymmetric or quasisymmetric systems with low $\mathbf{E}\times\mathbf{B}$ flows while the former corresponds to helical systems without quasisymmetry or toroidally rotating tokamaks with high $\mathbf{E}\times\mathbf{B}$ flows.

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